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EXTENDED BASED ON GENERALIZED DAYKIN-HARRIS ALGORITHM USING SGN FUNCTION

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Abstract: In this article we propose extended Daykin-Harris algorithm [7] which calculate coefficients x and y in Bezout's identity [1] i.e. for arbitrary but given integer numbers $a \neq 0$ and $b \neq 0$, so that $x * a + y * b = gcd$, where gcd is greatest common divisor of a and b. This algorithm will be applicable especially for long numbers. From arithmetic operations it implements "addition" and "subtraction".

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Key Words: extended Euclidean algorithm, extended Daykin-Harris algorithm, Bezout's identity, sgn function, long numbers

1. Introduction

Many practically oriented problems are connected to computation of greatest common divisor of two integer numbers $a \neq 0$ and $b \neq 0$, see [4], [16]–[52]. Some new iteration processes directed to this and similar tasks can be found in [16]–[31].

New algorithm presented generalizes iteration processes of Daykin [2], Harris [3] and our recent articles [6], [7].

Many studies are dedicated to similar of Euclidean algorithm iteration processes, see [8]-[15], [32]-[52].

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For testing purposes we will use the following computer: processor – Intel(R) Core(TM) i7-6700HQ CPU 2.60GHz, 2592 Mhz, 4 Core(s), 8 Logical Processor(s), RAM 16 GB, Microsoft Windows 10 Enterprise x64, Microsoft Visual C# 2017 x64.

2. Main Results

We propose the following algorithm, which generalizes extended Daykin-Harris algorithm [7]:

Algorithm 1.

```
x2 = 0; y1 = 0;
if (a > 0) x1 = 1; else if (a < 0) x1 = -1;
if (b > 0) y2 = 1; else if (b < 0) y2 = -1;
a = Math.Abs(a); b = Math.Abs(b);
if (a > b)
{
z = a;
while (a > b) { a -= b; x1 -= y1; x2 -= y2; }
a1 = z - b;
if (a + a1 < z)
do
{
do { a1 += a; y1 -= x1; y2 -= x2; } while (a + a1 < z);
if (a + a1 > z) do { a += a1 - z; x1 -= y1; x2 -= y2; } while (a + a1 > z);
}
while (a + a1 != z);
else if (a + a1 > z)
do
{
do { a += a1 - z; x1 -= y1; x2 -= y2; } while (a + a1 > z);
if (a + a1 < z) do { a1 += a; y1 -= x1; y2 -= x2; } while (a + a1 < z);
}
while (a + a1 != z);
gcd = a; x = x1; y = x2;
}
else
{
z = b;
while (b > a) { b -= a; y1 -= x1; y2 -= x2; }
b1 = z - a;
```

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```
if (b + b1 < z)
do
{
do { b1 += b; x1 -= y1; x2 -= y2; } while (b + b1 < z);
if (b + b1 > z) do { b += b1 - z; y1 -= x1; y2 -= x2; } while (b + b1 > z);
}
while (b + b1 != z);
else if (b + b1 > z)
do
{
do { b += b1 - z; y1 -= x1; y2 -= x2; } while (b + b1 > z);
if (b + b1 < z) do { b1 += b; x1 -= y1; x2 -= y2; } while (b + b1 < z);
} while (b + b1 != z);
gcd = b; x = y1; y = y2;
}
```

and we will see the time of new Algorithm 1. and the time of extended Euclidean using sgn function algorithm [32] (Algorithm 2.):

Algorithm 2.

```
x2 = 0; y1 = 0;
if (a > 0) x1 = 1; else if (a < 0) x1 = -1;
if (b > 0) y2 = 1; else if (b < 0) y2 = -1;
a = Math.Abs(a); b = Math.Abs(b);
while (a != b)
if (a > b) { a -= b; x1 -= y1; x2 -= y2; }
else { b -= a; y1 -= x1; y2 -= x2; }
gcd = a; x = y1; y = y2;
```

3. Numerical Example

For testing purposes of Algorithms 1. and 2. we will use the following main function:

```
long a, b, gcd, d1 = 0, a1, b1, z, x1 = 0, x2, y1, y2 = 0;
for (int i = 1; i < 100000001; i++) { a = i; b = 200000002 - i;
```

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```
//here are placed the source code of Algorithms 1. and 2.
d1 += gcd;
}
Console.WriteLine(d1);
```

CPU time results are:

CPU time of Algorithm 1. is: **98.364 seconds.**

CPU time of Algorithm 2. is: **110.965 seconds.**

Algorithm 1. is much faster than Algorithm 2.

4. Conclusion

We give iteration process which covers the work with numbers which are not only obligatory natural.

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