International Electronic Journal of Pure and Applied Mathematics
Volume 17 No. 1 2023, 1-10
ISSN: 1314-0744
url: https://www.e.ijpam.eu
doi: 10.12732/iejpam.v17i1.1

ONE POSSIBLE APPLICATION OF THE q–LOMMEL POLYNOMIALS ASSOCIATED WITH THE JACKSON q-BESSEL FUNCTION

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Abstract: In this article we consider the q–Lommel polynomials associated with the Jackson q-Bessel function generated by three–term recurrence relation $h^{(2)}_{m+1,\nu}(x;q) = 2x\left(1-q^{m+\nu}\right)h^{(2)}_{m,\nu}(x;q) - q^{m+\nu+1}h^{(2)}_{m-1,\nu}(x;q)$; $h^{(2)}_{0,\nu}(x;q)=1$; $h^{(2)}_{1,\nu}(x;q)=2x\left(1-q^{\nu}\right)$ with possible applications in two directions – the study the dynamics of differential systems and the generation of special classes of radiation diagrams. Numerical examples, illustrating our results using *CAS MATHEMATICA* are given.

Key Words: Lienard differential system, q–Lommel polynomials associated with the Jackson q-Bessel function, emitting chart.

1. Associated q–Lommel polynomials

Three q–versions of Lommel polynomials are studied in [1].

The q–Lommel polynomials associated with the Jackson q–Bessel function satisfy the three–term recurrence relation [2]–[3]

$$h^{(2)}_{m+1,\nu}(x;q) = 2x\left(1-q^{m+\nu}\right)h^{(2)}_{m,\nu}(x;q) - q^{m+\nu+1}h^{(2)}_{m-1,\nu}(x;q)$$

with

$$h^{(2)}_{0,\nu}(x;q)=1; \quad h^{(2)}_{1,\nu}(x;q)=2x\left(1-q^{\nu}\right).$$

and $0 < q < 1$.

Received: May 10, 2023
Revised: June 6, 2023
Published: June 8, 2023

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 17, No. 1 (2023)

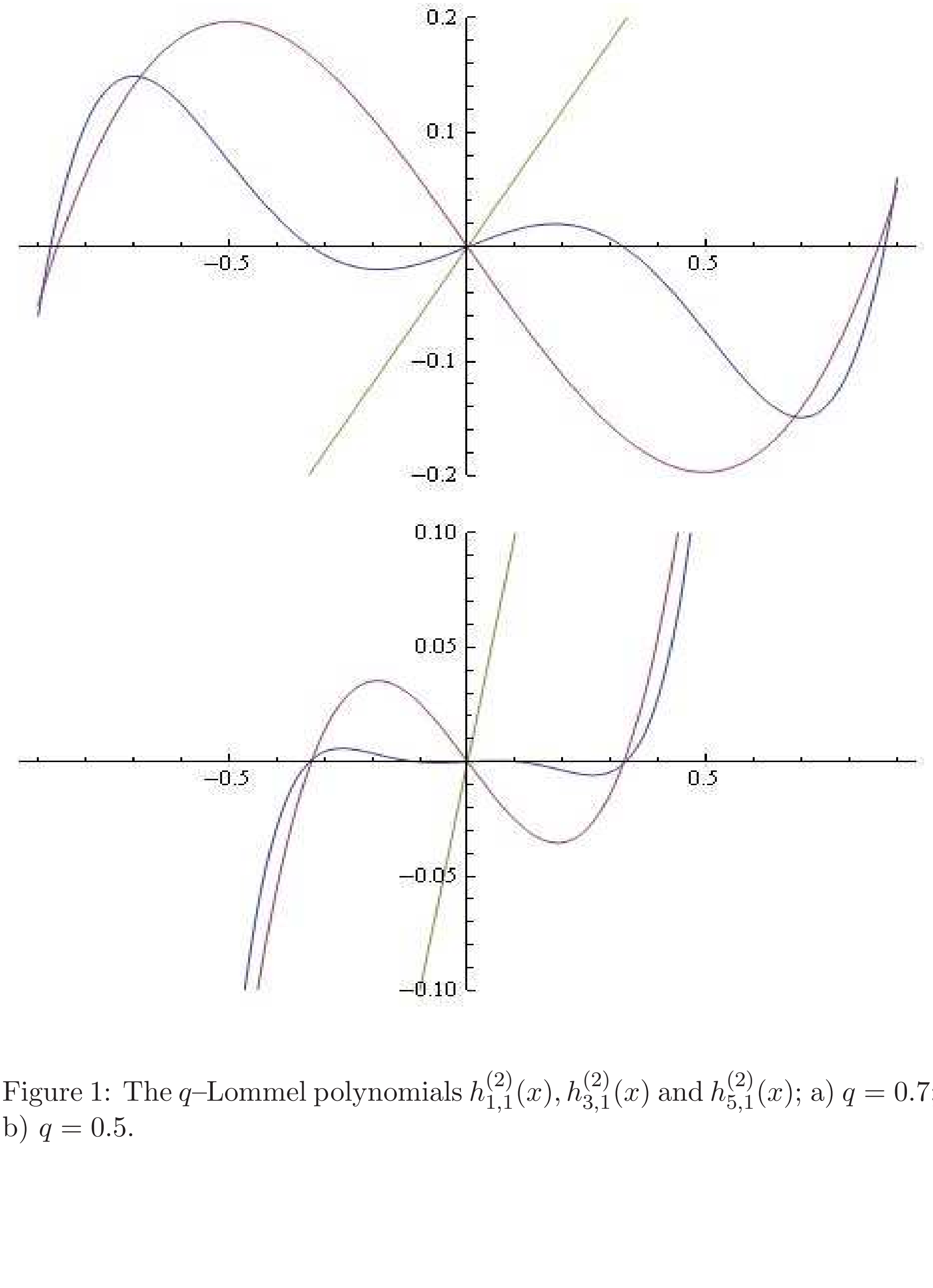

Figure 1: The q–Lommel polynomials $h^{(2)}_{1,1}(x), h^{(2)}_{3,1}(x)$ and $h^{(2)}_{5,1}(x)$; a) $q = 0.7$; b) $q = 0.5$.

We have (see Fig. 1)

$$\begin{cases} h^{(2)}_{1,1}(x;q) &= 2x(1-q) \\ h^{(2)}_{3,1}(x;q) &= 8x^3(1-q)(1-q^2)(1-q^3) - 2xq^3(1+q-q^2-q^3) \\ h^{(2)}_{5,2}(x) &= 232x^5(1-q)(1-q^2)(1-q^3)(1-q^4)(1-q^5)- \\ & -8x^3q^3(1+q-q^3-3q^4-2q^5+2q^7+3q^8+q^9-q^{11}-q^{12})+ \\ & +2xq^8(1+q+q^2-q^3-q^4-q^5) \end{cases}$$

1.1. Some simulations and applications

The simulation on the Lienard system:

$$\begin{cases} \dfrac{dx}{dt} &= y \\ \dfrac{dy}{dt} &= -h^{(2)}_{5,2}(x;q) + \epsilon F(x) y \end{cases} \tag{1}$$

where

$$F(x) = x - \frac{1}{3}x^3$$

and $q = 0.7$ for
1.) $x_0 = 0.6$, $y_0 = 0.5$, $b = 0.8$, $c = 0.9$, $\epsilon = 0.0001$
2.) $x_0 = 0.5$, $y_0 = 0.4$, $b = 1.06$, $c = 0.9$, $\epsilon = 0.0001$ are depicted on Fig. 2 – Fig. 4.

Let $q = 0.9$.

The simulations on the system (1) for 3.) $x_0 = 1.2$, $y_0 = 0.9$, $b = 1.04$, $c = 0.89$, $\epsilon = 0.0001$ are depicted on Fig. 5 – Fig. 6.

Acknowledgment

The work of the first author in this study is financed by the project No FP23-FMI-002 "Intelligent software tools and applications in research in Mathematics, Informatics, and Pedagogy of Education" of the Scientific Fund of the Paisii Hilendarski University of Plovdiv, Bulgaria.

The work of the second author in this study is financed by the Fund "Scientific Research" of Plovdiv University Paisii Hilendarski, within the project: "A study of natural, demographic and cultural resources in the Western Rhodopes: potential for sustainable development of the region" contract No FP 23- FS 003.

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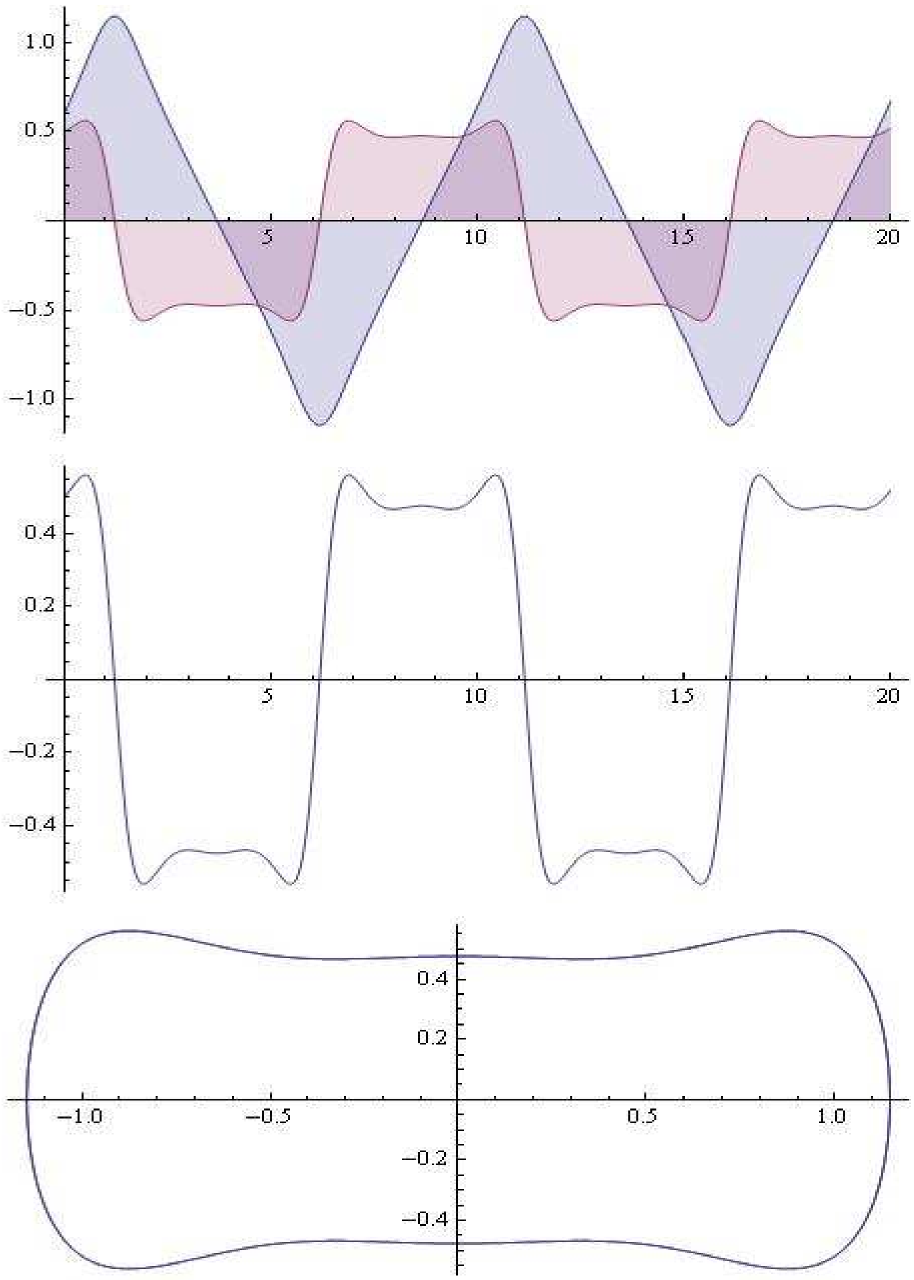

Figure 2: The simulations (the case 1.)): a) solutions of the system; b) y-component of the solution; c) the portrait.

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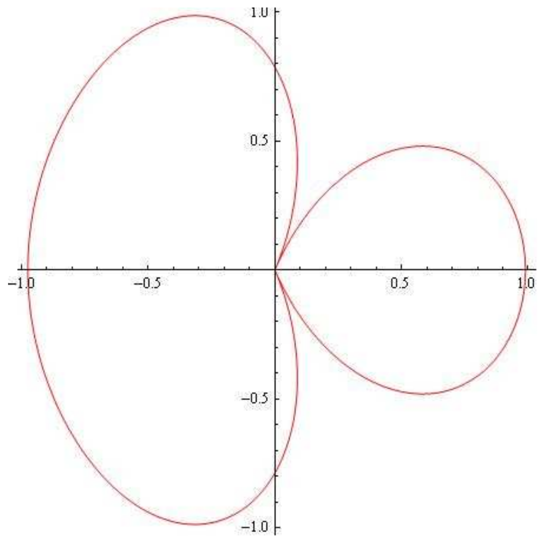

Figure 3: the case 1.): emitting chart (normalized).

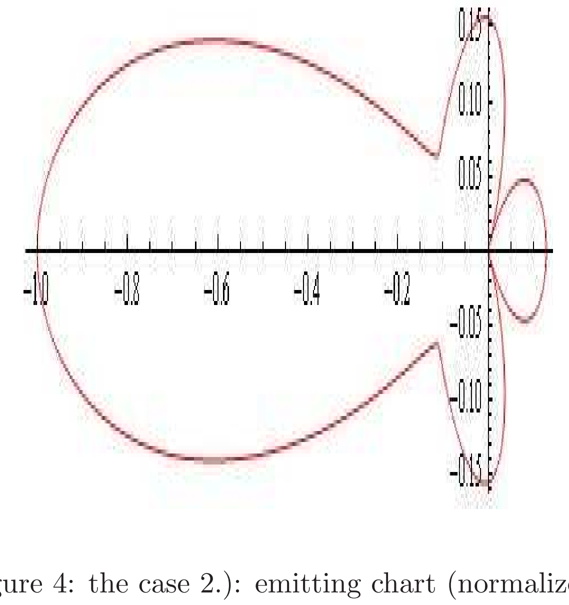

Figure 4: the case 2.): emitting chart (normalized).

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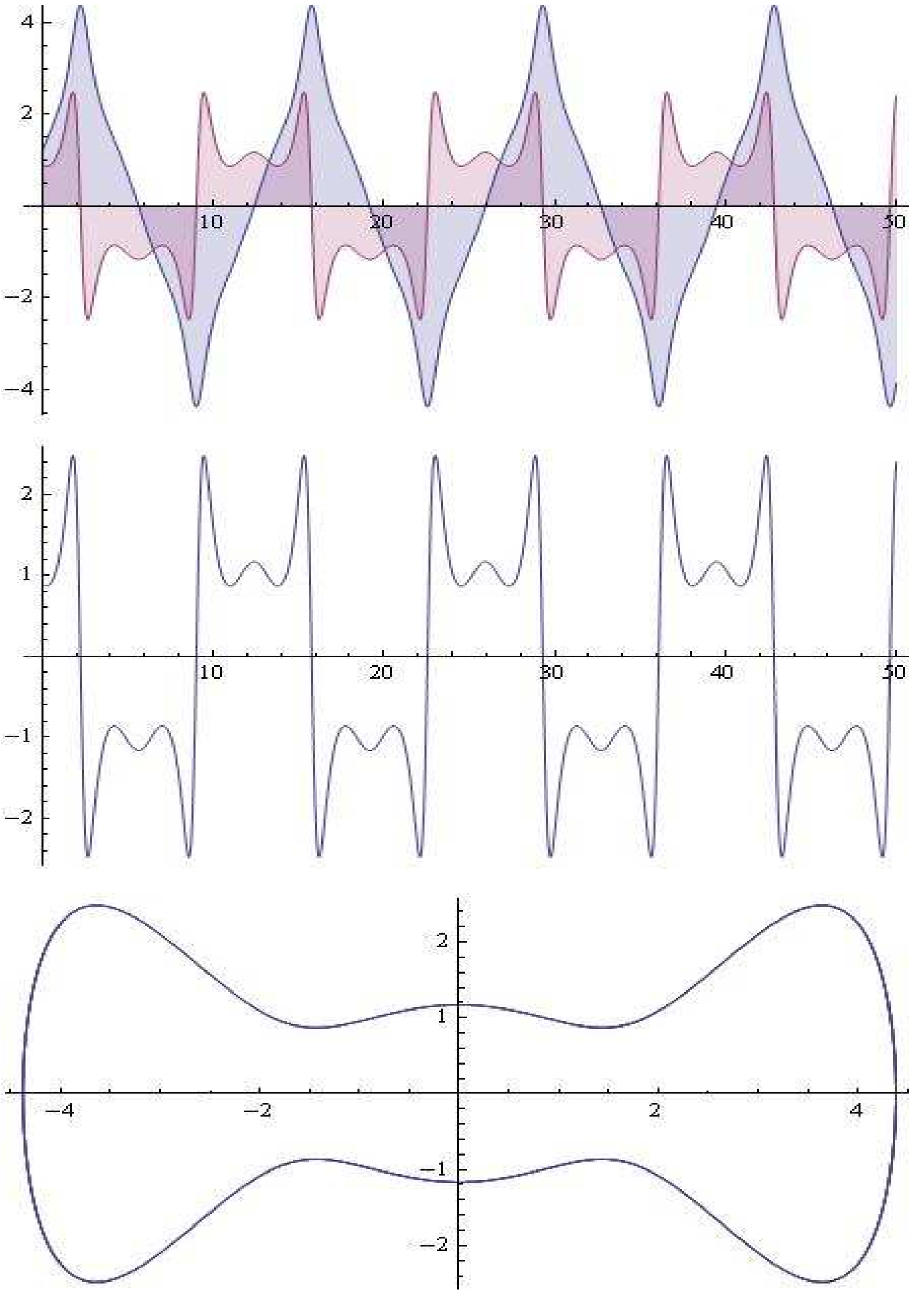

Figure 5: The simulations (the case 3.)): a) solutions of the system; b) y-component of the solution; c) the portrait.

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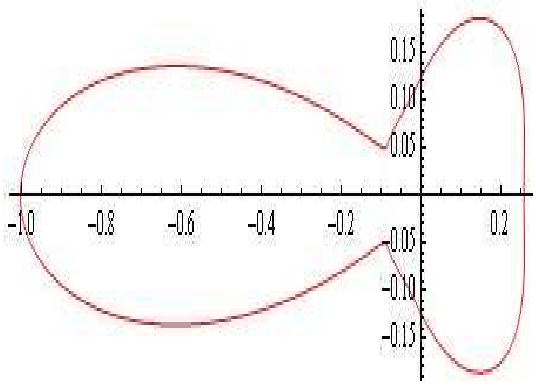

Figure 6: the case 3.): emitting chart (normalized).

References

[1] J. Kim, D. Stanton, Three families of q-Lommel polynomials, arXiv:2105.10096V3 [math.CO] 29 Oct 2021.

[2] M. Ismail, The zeros of basic Bessel functions, The functions $J(x)$, and associated orthogonal polynomials, *J. Math. Anal. Appl.*, 86 (1982), 1–19.

[3] H. T. Koelink, Some basic Lommel polynomials, *J. of Appr. Theory*, 96 (1999), 345–365.

[4] D. Dickinson, On Lommel and Bessel polynomials, *Proc. Amer. Math. Soc.* 5 (1954), 946–956.

[5] H. Koelink, W. Van Assche, Orthogonal polynomials and Laurent polynomials related to the Hahn-Exton q-Bessel function, arXiv:math/9502227V1 [math. CA] 14 Feb. 1995.

[6] N. Kyurkchiev, A. Andreev, *Approximation and Antenna and Filters synthesis. Some Moduli in Programming Environment MATHEMATICA*, LAP LAMBERT Academic Publishing, Saarbrucken, 150 pp., (2014), ISBN: 978-3-659-53322-8.

[7] N. Kyurkchiev, Some Intrinsic Properties of Tadmor-Tanner Functions: Related Problems and Possible Applications, *Mathematics* (2020), **8**, 1963.

[8] V. Kyurkchiev, A.Iliev, A.Rahnev, N. Kyurkchiev, Simulations on the Lienard polynomial system with Dickson-type polynomial corrections. The level curves, *International Journal of Differential Equations and Applications*, **21**, No. 2 (2022), 31–44.

[9] V. Kyurkchiev, A.Iliev, A.Rahnev, N. Kyurkchiev, A note on the extended Lienard system with Dickson polynomials of the third kind as corrections. The level curves, *International Journal of Differential Equations and Applications*, **21**, No. 2 (2022), 73–87.

[10] V. Kyurkchiev, N. Kyurkchiev, On an extended relaxation oscillator model: number of limit cycles, simulations. I, *Communications in Applied Analysis*, **26**, No. 1 (2022).

[11] V. Kyurkchiev, A. Iliev, A. Rahnev, N. Kyurkchiev, A technique for simulating the dynamics of some extended relaxation oscillator models. II, *Communications in Applied Analysis*, **26**, No. 1 (2022).

[12] V. Kyurkchiev, A. Iliev, A. Rahnev, N. Kyurkchiev, Another extended polynomial Lienard systems: simulations and applications. III, *International Electronic Journal of Pure and Applied Mathematics*, **16**, No. 1 (2022), 55–65.

[13] V. Kyurkchiev, A. Iliev, A. Rahnev, N. Kyurkchiev, Investigations on some polynomial Lienard-type systems: number of limit cycles, simulations, *International Journal of Differential Equations and Applications*, **21**, No. 1 (2022), 117–126.

[14] V. Kyurkchiev, N. Kyurkchiev, A. Iliev, A. Rahnev, *On some extended oscillator models: a technique for simulating and studying their dynamics*, Plovdiv, Plovdiv University Press (2022); ISBN 978-619-7663-13-6.

[15] N. Kyurkchiev, A. Iliev, On the hypothetical oscillator model with second kind Chebyshev's polynomial–correction: number and type of limit cycles, simulations and possible applications, *Algorithms* 15, 12, 2022, ISSN:1999-4893

[16] V. Kyurkchiev, A. Iliev, A. Rahnev, N. Kyurkchiev, Lienard System with First Kind Chebyshev's Polynomial–correction in the Light of Melnikov's Approach. Simulations and Possible Applications, International Scientific Conference IMEA'2022, 23-25 November 2022, Pamporovo, Bulgaria, ISBN: 978-619-7663-33-4, Plovdiv University Press, 105–112.

[17] E. Angelova, V. Arnaudova, T. Terzieva, A. Malinova, Investigations on a Differential System with Correction of Zernike-type Radial Polynomials. Simulations, International Scientific Conference IMEA'2022, 23-25 November 2022, Pamporovo, Bulgaria, ISBN: 978-619-7663-33-4, Plovdiv University Press, 87–92.

[18] N. Kyurkchiev, The effects on the dynamics of Lienard equation with Morse–type corrections: level curves, *International Journal of Differential Equations and Applications*, **21**, No. 2 (2022).

[19] A. Malinova, T. Terzieva, O. Rahneva, E. Angelova, Legendre polynomials as "correction factors" in the Lienard differential system. Simulations, *Communications in Applied Analysis*, **26**, No. 1 (2022).

[20] A. Malinova, T. Terzieva, O. Rahneva, E. Angelova, A. Golev, Associated Legendre polynomials as corrections in the classical Lienard system, *Communications in Applied Analysis*, **26**, No. 1 (2022).

[21] A. Golev, V. Arnaudova, Lienard system with "correcting factors" of the type of interpolating polynomials of some basic functions, *International Electronic Journal of Pure and Applied Mathematics*, **16**, No 1 (2022), 67–80.

[22] A. Golev, V. Arnaudova, Higher-Kind Chebyshev Polynomials and Their Applications to Dynamic Model Studies, *International Journal of Differential Equations and Applications*, 21, 2 (2022), 101–108.

[23] V. Kyurkchiev, A.Iliev, A.Rahnev, N. Kyurkchiev, Gegenbauer polynomials as correction in the Lienard planar system: Melnikov's approach, *International Journal of Differential Equations and Applications*, **21**, No. 2 (2022), 45–57.

[24] V. Kyurkchiev, N. Kyurkchiev, A. Iliev, A. Rahnev, *On the Lienard System with Some "Corrections of Polynomial-type": Number of Limit Cycles, Simulations and Possible Applications. Part II.*, Plovdiv, Plovdiv University Press (2022), ISBN: 978-619-7663-41-9.

[25] Kyurkchiev, V., Iliev, A., Rahnev, A., Kyurkchiev, N., Dynamics of the Lienard Polynomial System Using Dickson Polynomials of the (M + 1)-th Kind. The Level Curves, *International Journal of Differential Equations and Applications*, 21, 2, 2022, 109–120.

[26] Kyurkchiev, V., Iliev, A., Rahnev, A., Kyurkchiev, N., On a class of orthogonal polynomials as corrections in Lienard differential system. Applications, *Algorithms*, (2023). (accepted)

[27] Vasileva, M., Kyurkchiev, V., Iliev, A., Rahnev, A., Kyurkchiev, N., Associated Hermite polynomials. Some applications, *International Journal of Differential Equations and Applications*, **22**, No. 1 (2023), 1–17.

[28] A. Golev, V. Arnaudova, Associated Gegenbauer polynomials. Applications, *International Journal of Differential Equations and Applications*, **22**, No. 1 (2023), 19–28.

[29] V. Kyurkchiev, A.Iliev, A.Rahnev, N. Kyurkchiev, Chebyshev polynomials of the fifth kind and their application to study the dynamics of Lienard-type equations, *Communications in Applied Analysis*, **26**, No. 1, (2022), 111–123.

[30] M. Vasileva, A. Iliev, A. Rahnev, N. Kyurkchiev, Applications of Some Associated Orthogonal Polynomials, *Communications in Applied Analysis*, **27**, No. 1 (2023), 27–39.

[31] M. Vasileva, V. Kyurkchiev, A. Iliev, A. Rahnev, N. Kyurkchiev, A Look at the Associated Lommel Polynomials. Applications, *Communications in Applied Analysis*, **27**, No. 1 (2023), 41–52.