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ANOTHER EXTENDED POLYNOMIAL LIENARD SYSTEMS: SIMULATIONS AND APPLICATIONS. III

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Abstract: There are a lot of results about the maximum number of limit cycles of Lienard system: $x' = y$; $y' = g(x) + \epsilon f(x)y$. In this article we offer a natural summary of this dynamic model with a polynomial type correction factors $f(x) = f_n(x) = \sum_{i=1}^{\left[\frac{n}{2}\right]}(-1)^{i+1}x^{n-2i} - \frac{x^n}{n}$ for $n = 3, 7, 11, 15, 19, \ldots$, (see [8]) and $\epsilon = \sum_{i=1}^{n} \epsilon_i t^i$. Some modifications based on the models by Cai, Wei and Zhu [4] and Xu [7] are studied.

We give a brief review of simulation techniques as an important tool for solving complex nonlinear problems.

The considered methodological aspects can be successfully applied to study the dynamics of some nonlinear models.

A typical application of the new models (the y component of the planar system) for analysis and simulation of specific emission diagrams and filter characteristics (in appropriate intervals) is also given.

AMS Subject Classification: 65L07, 34A34
Key Words: polynomial Lienard system, Melnikov's approach, extended generalized oscillator model with polynomial type correction factors

1. Introduction

In [4] Cai, Wei and Zhu investigated a Lienard system of type:

$$\begin{cases} \dfrac{dx}{dt} &= y \\ \dfrac{dy}{dt} &= -\frac{1}{7}x(7x+3)(x-1)(x+1)^2 + \epsilon f(x)y \end{cases} \tag{1}$$

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with special elliptic Hamiltonian function

$$H(x,y) = \frac{1}{2}y^2 + \frac{1}{6}x^6 + \frac{2}{7}x^5 - \frac{1}{7}x^4 - \frac{10}{21}x^3 - \frac{3}{14}x^2 \qquad (2)$$

where $0 \leq \epsilon \leq 1$ and

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \qquad (3)$$

$a_i (i = 0, 1, 2, 3)$ are real parameters.

The system (1) leads to

$$x'' - \epsilon f(x) x' + \frac{1}{7} x(7x + 3)(x - 1)(x + 1)^2 = 0. \qquad (4)$$

The system (1) has the degree $n^* = \max[5, n + 1]$.

Let $H^*(5, n)$ denote the maximum number of limit cycles of (1) of degree n^*.

For the limit cycles the following is valid

Theorem [4]. There exist some a_0, a_1, a_2 and a_3 such that system (1) has 9 limit cycles for the type $(5, 4)$ system (1).

There are a lot of results about the maximum number of limit cycles of Lienard system $x' = y;\ y' = g(x) + \epsilon f(x) y$.

For $n = 5$, Hu and Li [5] proved $H(2, 5) \geq 3,\ H(4, 5) \geq 5,\ H(8, 5) \geq 10$.

For $n = 6$, Asheghi and Bakhshalizadeh [6] proved $H(5, 6) \geq 9,\ H(6, 6) \geq 10,\ H(7, 6) \geq 11$.

In [8] we consider a new "extended generalized relaxation oscillator" model of the form:

$$\begin{cases} \dfrac{dx}{dt} & = & c(f_n(x) - y) \\ \dfrac{dy}{dt} & = & \frac{1}{c} x \end{cases} \qquad (5)$$

where $c > 0$ and

$$f_n(x) = \sum_{i=1}^{\left[\frac{n}{2}\right]} (-1)^{i+1} x^{n-2i} - \frac{x^n}{n}$$

for $n = 3, 7, 11, 15, 19, \ldots,$.

In the case $n = 3$ the model coincides with the "classical" Van der Pol's model [1]–[3]. The proof of existence of limit cycle is based on the verification of the conditions in Lienard's theorem. The number of hyperbolic limit cycles with radii r_i in the light of Melnikov's approach is also studied. The hypothetical Van der Pol generalized oscillator in the autonomous regime is also given. The much more generalized nonlinear model based on the ideas given in [4], [7]–[9] and discussed in this article is also of legitimate interest.

A modification by Xu [7] is also studied.

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In this article, we propose an algorithm for generating dynamic models using a technique for embedding polynomial-type correction factors. This makes it attractive for conducting computer simulations. The considered methodological aspects can be successfully applied to study the dynamics of some nonlinear models. We offer a software tool for simulating the dynamics of the new families.

2. Simulations

We consider the following modification of (4):

$$x'' - \epsilon(t) f_n(x) x' + \frac{1}{7} x(7x+3)(x-1)(x+1)^2 = 0 \tag{6}$$

where

$$\epsilon(t) = \sum_{i=0}^{r} \epsilon_i t^i, \tag{7}$$

$$f_n(x) = \sum_{i=1}^{\left[\frac{n}{2}\right]} (-1)^{i+1} x^{n-2i} - \frac{x^n}{n} \tag{8}$$

for $n = 3, 7, 11, 15, 19, \ldots,$.

Example 1. The solution $x(t)$ of (6) for

$$\epsilon(t) = 0.1 + 0.0029t^2 - 0.00015t^3$$

and

a) $n = 3$, i.e. $f_3(x) = x - \frac{x^3}{3}$;

b) $n = 7$, i.e. $f_7(x) = x - x^3 + x^5 - \frac{x^7}{7}$

is visualized on Fig. 1–2.

In [7] Xu consider a Lienard system of type:

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -x(x^2-1)(x^2-\frac{1}{4})(x^2-2) + \epsilon f(x) y \end{cases} \tag{9}$$

where $0 \le \epsilon \le 1$ and

$$f(x) = a_0 + a_1x^2 + a_2x^4 + a_3x^6 + a_4x^8 \tag{10}$$

$a_i (i = 0, 1, 2, 3, 4)$ are real parameters.

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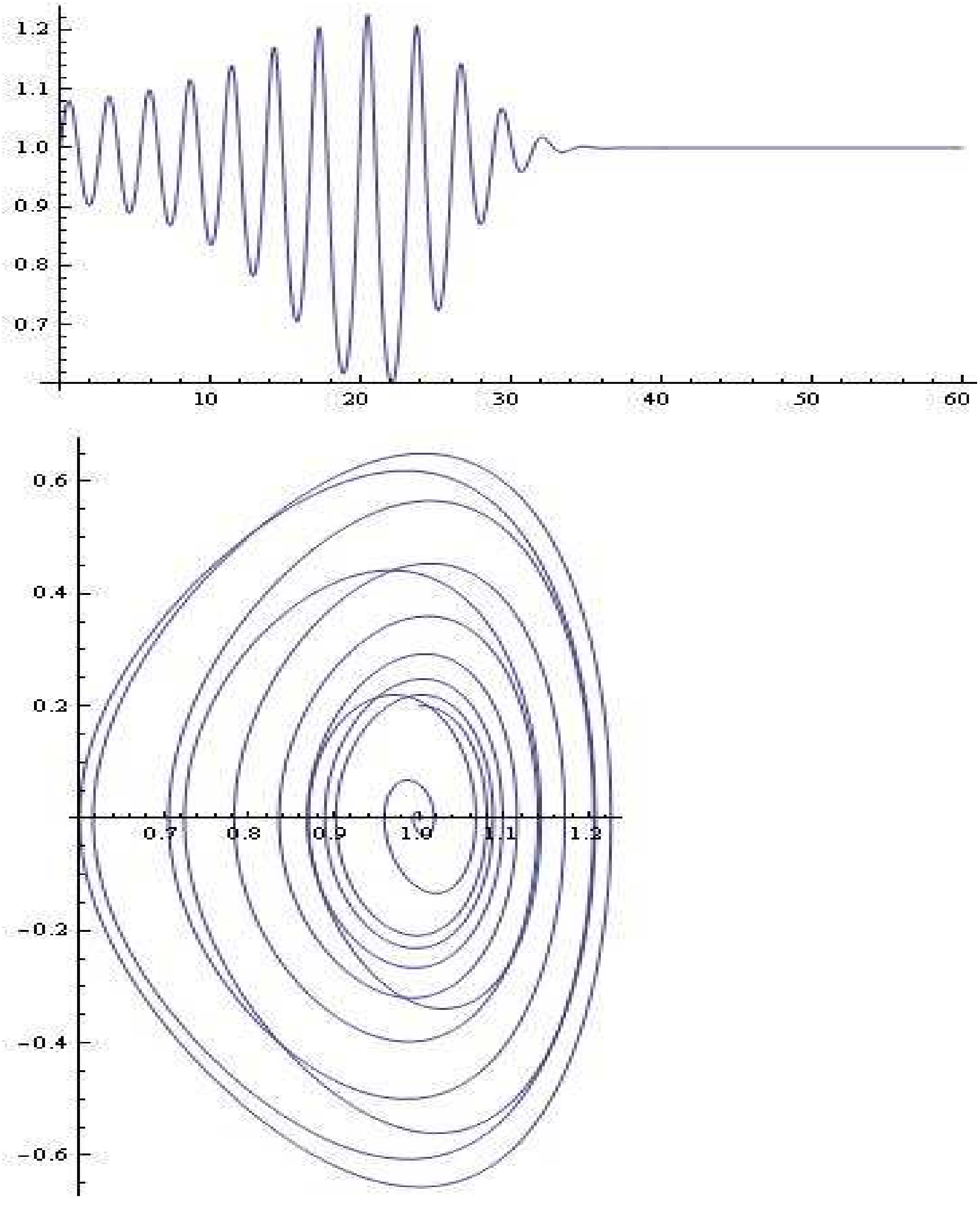
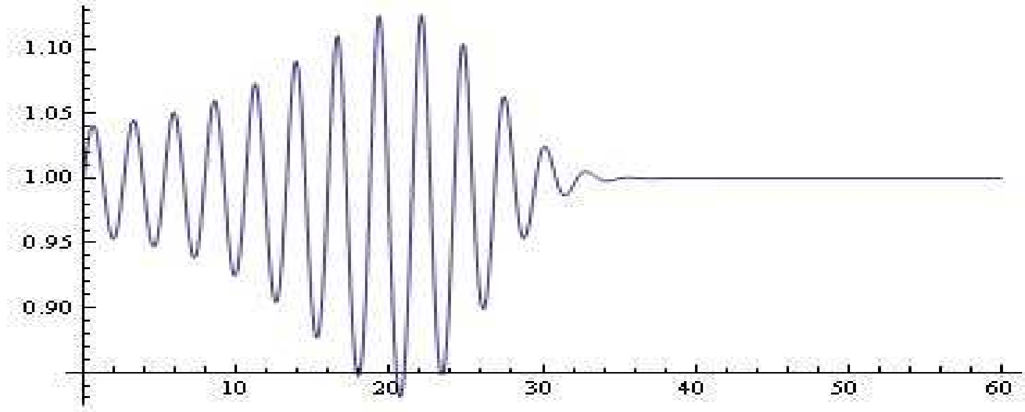


Figure 1: a) The solution $x(t)$ of the equation (6); b) The portrait of the planar system corresponding to (6); (the case $n = 7$).



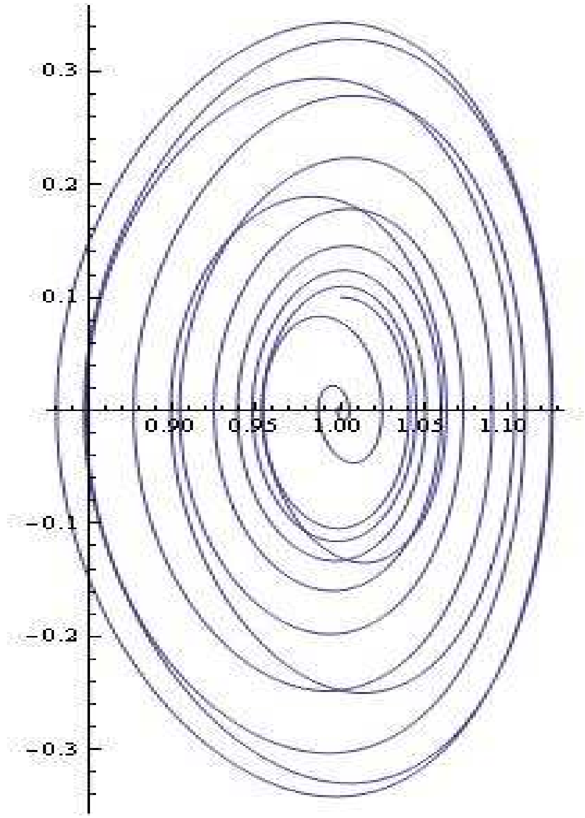


Figure 2: The solution $x(t)$ of the equation (6); b) The portrait of the planar system corresponding to (6); (the case $n = 3$).

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Using the Melnikov approach and bifurcation theory, Xu [7] proved that $H(8,7) \geq 11$.

We consider the following modification

$$\begin{cases} \dfrac{dx}{dt} &= y \\ \dfrac{dy}{dt} &= -x(x^2-1)(x^2-\frac{1}{4})(x^2-2)+\epsilon(t)f_n(x)y \end{cases} \tag{11}$$

where

$$\epsilon(t)=\sum_{i=0}^{r}\epsilon_i t^i,$$

$$f_n(x)=\sum_{i=1}^{[\frac{n}{2}]}(-1)^{i+1}x^{n-2i}-\frac{x^n}{n}$$

for $n=3,7,11,15,19,\ldots,$.

Example 2. The solutions of the Lienard system (11) for

$$\epsilon(t)=0.1+0.0029t^2-0.00015t^3$$

and $n=7$, i.e. $f_7(x)=x-x^3+x^5-\frac{x^7}{7}$ are visualized on Fig. 3.

Example 3. The solutions of the Lienard system (11) for

$$\epsilon(t)=0.01+0.001t-0.0002t^2+0.00001t^3-0.0000001t^4$$

and $n=7$ are visualized on Fig. 4.

3. Applications

We will explicitly note that the models discussed in this article, as well as some of the other modified models described in [8]–[9] may find application in the field of antenna–feeder technique. We will consider an example.

Example 4. A typical application of the new model (11) (the y component of the differential system) for analysis and simulation of specific emission diagrams and filter characteristics (in appropriate intervals) [10] is illustrated in Fig. 5–Fig.6 for $n=7$ and

a) $\epsilon(t)=0.005+0.001t-0.0002t^2+0.00001t^3-0.0000001t^4$

b) $\epsilon(t)=0.0042+0.0012t+-0.0001t^2-0.0000045t^3$.

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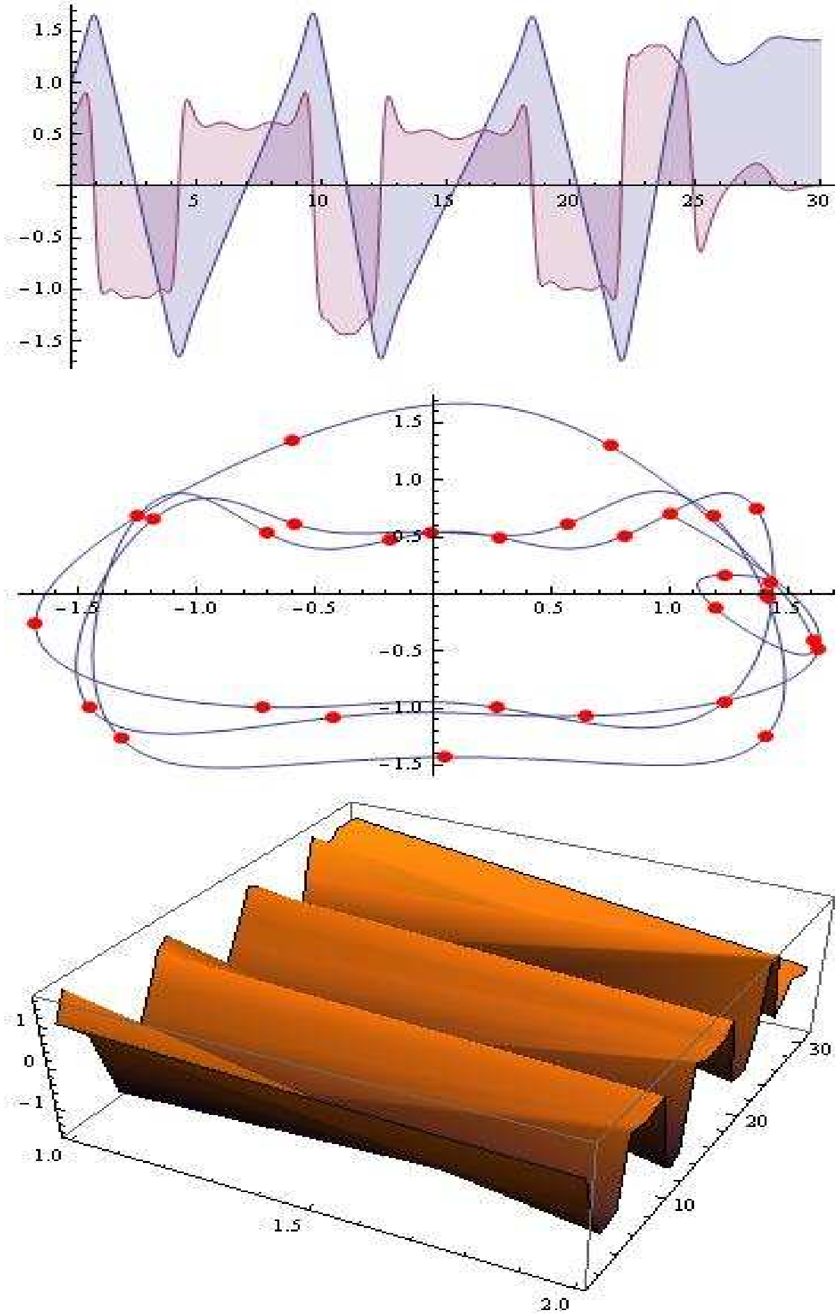

Figure 3: The solutions of the Lienard system (11); b) The portrait of the planar system; (the case $n = 7$).

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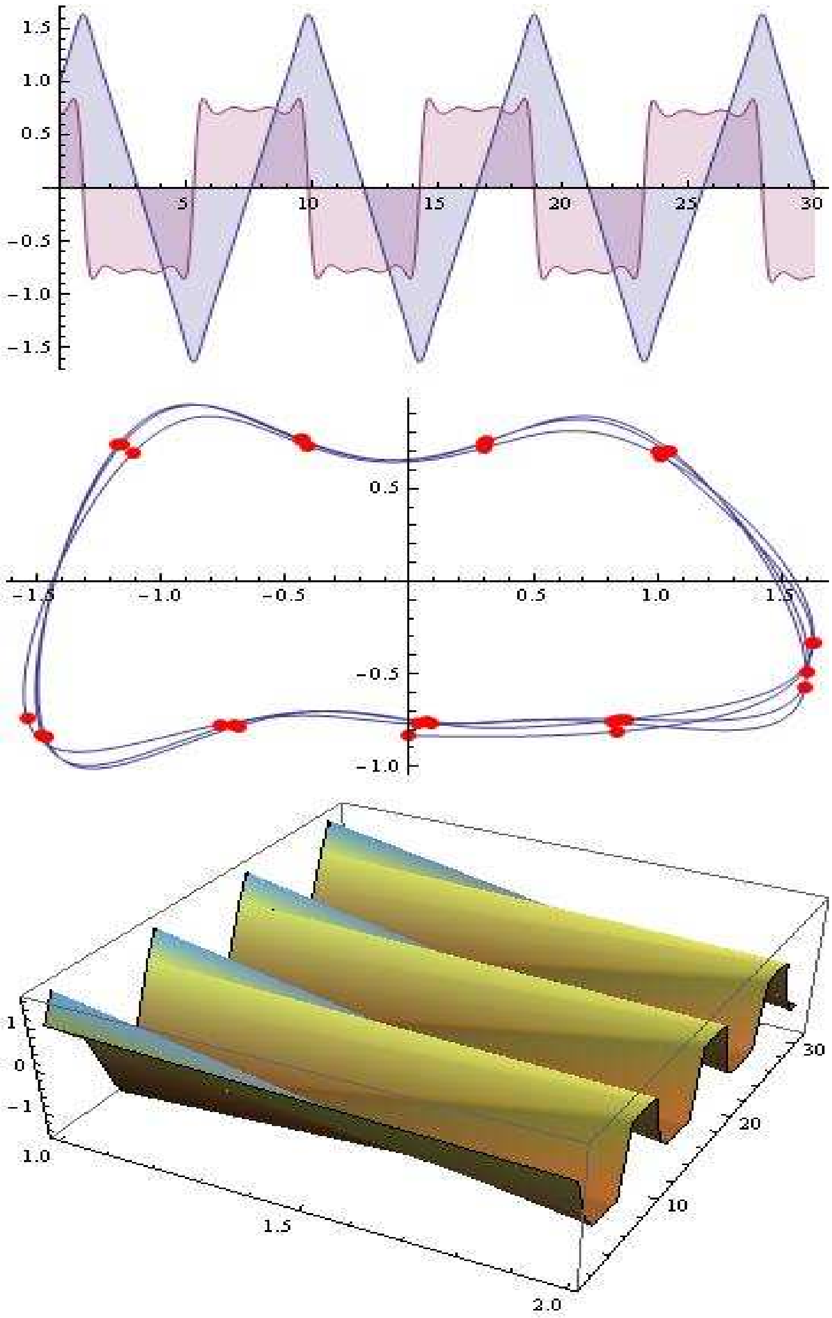

Figure 4: The solutions of the Lienard system (11); b) The portrait of the planar system; (Example 3).

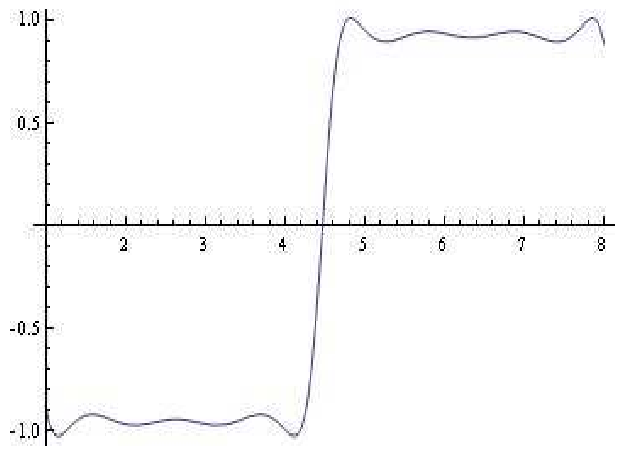

Figure 5: A typical application of the model (the y component of the planar system) for analysis and simulation of specific emission diagrams and filter characteristics (Example 4a).

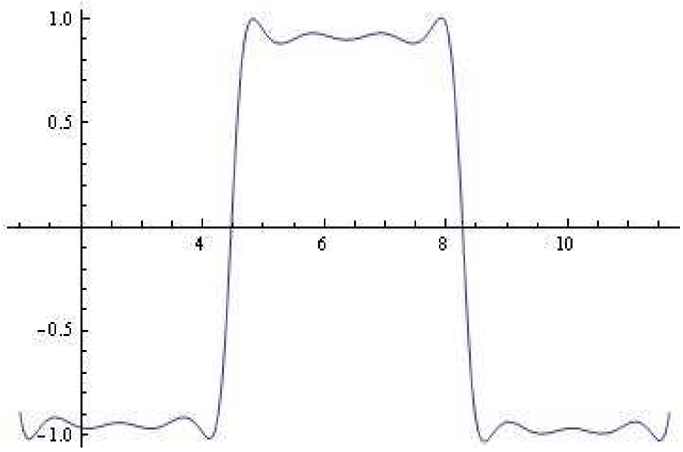

Figure 6: A typical application of the model (the y component of the planar system) for analysis and simulation of specific emission diagrams and filter characteristics (Example 4b).

3.1. Concluding Remarks.

In this article, we propose an algorithm for generating dynamic models using a technique for embedding polynomial-type correction factors.

The considered methodological aspects can be successfully applied to study the dynamics of some nonlinear models.

For other results see [11]–[18].

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