International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 16, No. 1 (2022)

International Electronic Journal of Pure and Applied Mathematics
Volume 16 No. 1 2022, 27-37
ISSN: 1314-0744
url: https://www.e.ijpam.eu
doi: 10.12732/iejpam.v16i1.3

A NOTE ON A HYPOTHETICAL PIECEWISE SMOOTH MODIFIED OW–TL–G LOGARITHMIC CDF

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Abstract: Following the ideas given in [8]–[10] in this article, we analyze a understudied model, such as the Oluyede, Chipepa and Wanduku [6] model. It is shown that the model can be modified in view of their possible application for approximation of data from a real test of software modules.

Some numerical examples, using *CAS MATHEMATICA* are also given.

AMS Subject Classification: 41A46

Key Words: modified OW–TL–G Logarithmic cumulative distribution function based on the model by Oluyede, Chipepa and Wanduku, hypothetical piecewise smooth growth function

1. Introduction

Chipepa, Oluyede and Chamunorwa [1] developed a new family of distributions called the odd Weibull–Topp–Leone–G (OW–TLG) family of distributions using the generalized Weibull family by [2] and the Topp–Leone–G distribution. For some recent generalizations see [3]–[5] and references therein. In 2021 Oluyede, Chipepa and Wanduku [6] developed a new generalization of the Weibull–Topp–Leone–G family of distributions called the odd Weibull–Topp–Leone–G–power series (OW–TL–GPS) family. The main motivation of developing this new generalization is to the desirable properties exhibited by the power series generalizations in terms of data fitting.The paper [7] deals with asymptotic behavior of some adaptive functions

Received: January 21, 2022
Revised: March 15, 2022
Published: March 18, 2022

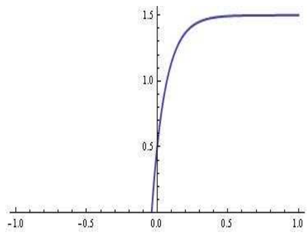

Figure 1: The function $f_1(t)$ for fixed $k = 10$, $a = 0.05$, $A = 1.5$, ($f_1(0) = \frac{1}{2}$).

of the Hausdorff distance between Heaviside function and some novel distribution functions. This study can be very useful for specialists that are working in several scientific fields like insurance, financial mathematics, analysis and approximation of data sets in a various modeling problems and others.

We consider a new "modified OW–TL–G Logarithmic distribution" (M–OW–TL–G).

2. A look at new "modified OW–TL–G Logarithmic distribution" (M–OW–TL–G).

Consider the following new "modified OW–TL–G Logarithmic distribution" (M–OW–TL–G):

$$f_1(t) = A - \frac{\ln(1 - ae^{-kt})}{\ln(1-a)} \tag{1}$$

for $k, a, A > 0$.

Evidently,

$$f_1(0) = A - 1; \quad \lim_{t \to +\infty} f_1(t) = A.$$

For a visualization of the model (1) at fixed values of the parameters k, a, A, see Figure 1.

Suppose that in his experimental work, the researcher observes a series of experimental data that are located at a slightly lower (respectively higher) level than the asymptotic level A guaranteed by the fixed base model (1).

In the field of Growth Theory the problem often arises how to construct a modified model at already fixed values of parameters k, a, A and a change in the dynamics of the growth process for $t > t_0$, in which saturation to the horizontal asymptote at

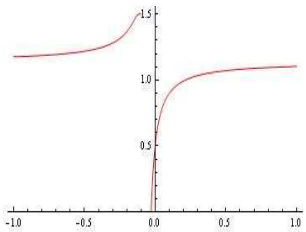

Figure 2: The function $f_2(t)$ for fixed $k = 10$, $a = 0.05$, $A = 1.5$, ($f_2(0) = \frac{1}{2}$).

level B is reached. This can be achieved, for example, with the function $f_2(t)$ (for $t > 0$, see Figure 2)

$$f_2(t) = A - \frac{\ln(1 - ae^{-\frac{kt}{1+kt}})}{\ln(1-a)} \tag{2}$$

for $k, a, A > 0$ and

$$f_2(0) = A - 1; \quad \lim_{t \to +\infty} f_2(t) = A - \frac{\ln(1 - ae^{-1})}{\ln(1-a)} := B.$$

We will note that with the proposed methodology (see, [8]–[10]), the researcher can modify other known (more advanced) models for the needs of his specific research.

This leads us to think of the following hypothetical piecewise smooth growth function

$$F(t) := \begin{cases} A - \dfrac{\ln(1 - ae^{-kt})}{\ln(1-a)} := f_1(t), \ \ t < 0 \\ 0.5, \ \ t = 0 \\ A - \dfrac{\ln(1 - ae^{-\frac{kt}{1+kt}})}{\ln(1-a)} := f_2(t), \ \ t > 0. \end{cases} \tag{3}$$

Evidently, from (3) we have $f_1'(0) = f_2'(0)$.

The model (3) is depicted on Figure 3 for $k = 20$, $a = 0.05$, $A = 1.5$.

Define the following hypothetical piecewise smooth shifted growth function

$$F^*(t) := \begin{cases} A - \dfrac{\ln(1 - ae^{-k(t-r)})}{\ln(1-a)} := f_1^*(t), \ \ t < r \\ 0.5, \ \ t = r \\ A - \dfrac{\ln(1 - ae^{-\frac{k(t-r)}{1+k(t-r)}})}{\ln(1-a)} := f_2^*(t), \ \ t > r. \end{cases} \tag{4}$$

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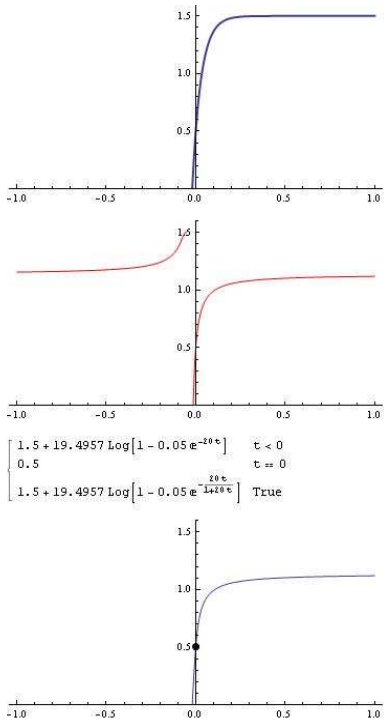

Figure 3: The function $F(t)$ for fixed $k = 20$, $a = 0.05$, $A = 1.5$ (Asymptote at level B is 1.13806).

Evidently, from (4) we have $f_1^{*\prime}(r) = f_2^{*\prime}(r)$.

$F^*(t)$ is depicted on Figure 4 for $k = 10$, $a = 0.05$, $A = 1.5$, $r = 0.06$.

3. Remarks.

1. It can be proved that the function $f_2(t)$ is generated by a real reaction kinetics scheme.

We will not focus on this issue here. Much more general questions in this direction are discussed in [8].

We will only note that the new growth function is a solution of the following differential equation

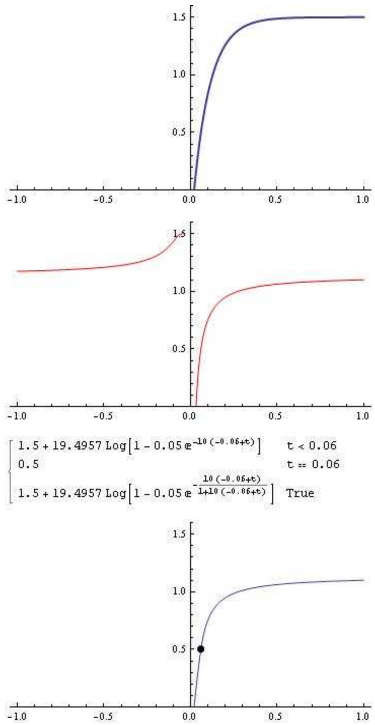

Figure 4: The function $F^*(t)$ for fixed $k = 10$, $a = 0.05$, $A = 1.5$, $r = 0.06$.

$$x'(t) = -\frac{ak}{\ln(1-a)(1+kt)^2}\frac{e^{\frac{-kt}{1+kt}}}{1-ae^{\frac{-kt}{1+kt}}}; \quad x(0) = \frac{1}{2}. \tag{5}$$

Obviously, the function $x(t)$ coincides with the second component $f_2(t)$ of the defined and studied in this article hypothetical piecewise smooth growth function $F(f_1(t), f_2(t))$ (see Figure 5).

2. Model $F^*(f_1^*(t), f_2^*(t))$ can be used successfully in approximating grouped data from the specified scientific field.

Motivated research in this direction is discussed in [].

Of course, $f_2^*(t)$ can be used alone.

For example, we will use the following (abbreviated) test data (t_i, y_i) from the field of debugging theory:

$$DataFailure1 := \{\{0.005, 0.4\}, \{0.05, 0.77\}, \{0.1, 0.89\}, \{0.8, 0.99\},$$

$$\{0.9, 0.995\}, \{1, 1.023\}\}$$

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```
k = Input["k"];(* 20 *)
Print["k = ", k];
A = Input["A"];(* 1.5 *)
Print["A = ", A];
a = Input["a"];(* 0.05 *)
Print["k = ", k];
x0 = Input["Input initial condition - x[0]"]; (* 0.5 *)
Print["Initial condition x0 = ", x0];
t0 = Input["Input t0"];
Print["t0 = ", t0];
t1 = Input["Input t1"];
Print["t1 = ", t1];
Print["The solution of the differential equation"];
NDSolve[{x'[t] == -a*k/(Log[1 - a]*(1 + k*t)^2)*Exp[-k*t/(1 + k*t)]/
      (1 - a*Exp[-k*t/(1 + k*t)]), x[0] == x0}, {x}, {t, t0, t1}];
Plot[Evaluate[{x[t]} /. First[%]], {t, t0, t1}, AxesOrigin → {0, 0}]
```

```
k = 20
A = 1.5
k = 20
Initial condition x0 = 0.5
t0 = 0
t1 = 1
The solution of the differential equation
```

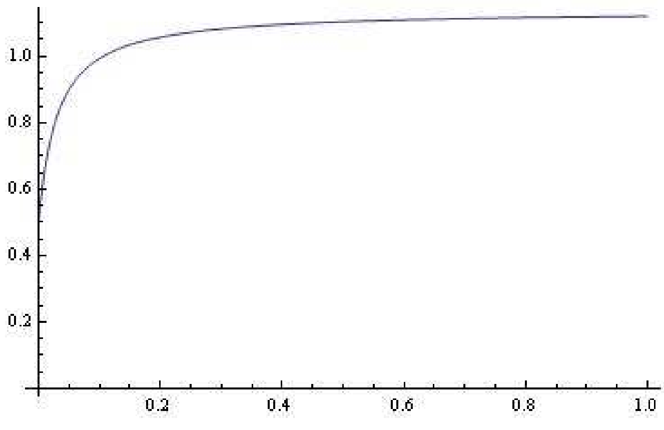

Figure 5: Module in the software environment *CAS Mathematica* for solving and visualizing the solution of the differential equation (5).

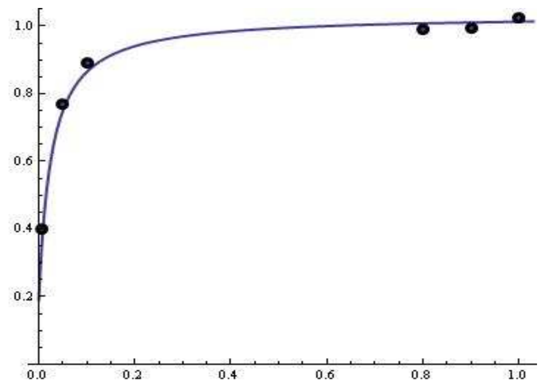

Figure 6: The fitted model $f_2^*(t)$ for for $A = 1.305$; $a = 0.05$, $r = 0.0085$, $k = 17.9957$ (for the "dataFailure1").

From the dataset (t_i, y_i) it can be seen that $\max y_i = 1.023$.

It is appropriate for the researcher to choose in model $f_2^*(t)$ - "saturation level" $B \approx 1.03$.

Let us also fix the parameter $a = 0.05$.

An approximation of the data is visualized in Figure 6.

Figure 6 shows that the result is satisfactory and the error in the root mean square sense is minimal.

3. For the saturation d to the horizontal asymptote at level B in Hausdorff sense [11] we have

$$P(d) := f_2(d) - B + d = 0. \tag{6}$$

For example, for fixed $A = 1.5\,(B = 1.13805)$, $a = 0.05$, $k = 10$ we find from(6) that $d = 0.166818$.

For $A = 1.5\,(B = 1.15752)$, $a = 0.2$, $k = 40$ we have $d = 0.0885262$ (see Figure 7).

Evidently, the function

$$Q(d) := A - B - 1 + \left(1 - \frac{ak}{(1-a)\ln(1-a)}\right) d \tag{7}$$

approximates $P(d)$ with $d \to 0$ as $\mathcal{O}(d^2)$ (see, for example Figure 8).

The reader can get accurate estimates of the value of d.

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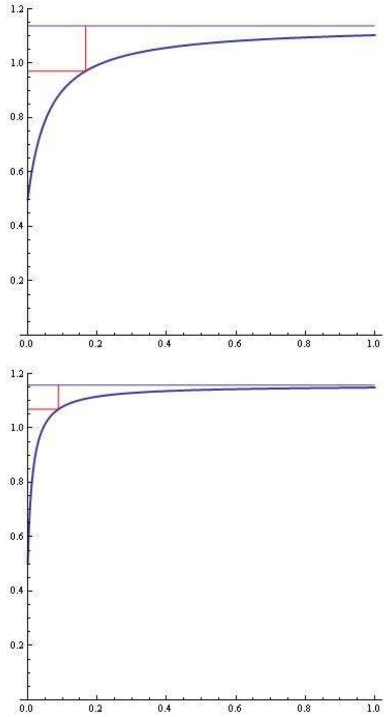

Figure 7: a) $A = 1.5\,(B = 1.13805)$, $a = 0.05$, $k = 10$; H-distance $d = 0.166818$; a) $A = 1.5\,(B = 1.15752)$, $a = 0.2$, $k = 40$; H-distance $d = 0.0885262$.

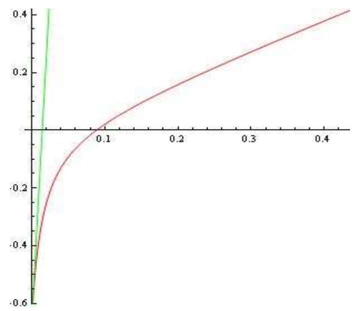

Figure 8: The functions $P(d)$ (red) and $Q(d)$ (green) for fixed $A = 1.5\,(B = 1.15752)$, $a = 0.2$, $k = 40$; H-distance $d = 0.0885262$.

We will not dwell on this question here.

Much more general questions in this direction are discussed in [12].

Concluding Remark.

In conclusion, we will note that with the proposed methodology (see, [8]–[10]), the researcher can modify other known (more advanced) models for the needs of his specific research.

In [7] we develop some dynamic programming modules implemented within the programming environment CAS Wolfram Mathematica and Wolfram Cloud Open Access.

It is planned to upgrade the *Distributed Platform for e-Learning - DisPeL* in view of the possibility of covering emerging theoretical research in this interesting scientific field.

Acknowledgment

This work has been accomplished with the financial support by the Grant No BG05M2OP001-1.001-0003, financed by the Science and Education for Smart Growth Operational Program (2014-2020) and co-financed by the European Union through the European structural and Investment funds.

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