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A NEW COS–G FAMILY WITH BASELINE CUMULATIVE FUNCTION OF VOLMER–TYPE. APPLICATIONS

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Abstract: In this article we study a new class of COS–G family with baseline cumulative function of Volmer–type. We consider also modified transmuted family of "adaptive functions". A justification of the idea for generating the powerful classes of trigonometric–G families is given. Numerical examples, illustrating our results using *CAS MATHEMATICA* are given.

AMS Subject Classification: 41A46
Key Words: generalized Cos–G family with correction of Volmer–type, modified transmuted family, radiation diagrams and filter characteristics

1. Introduction

In the last few years, there have been serious studies in the literature related to the proposed general classes of trigonometric distributions [1]–[13]. In particular, $F_1(t) = \sin\left(\frac{\pi}{2}G(t)\right)$, $F_2(t) = 1 - \cos\left(\frac{\pi}{2}G(t)\right)$, $F_3(t) = \tan\left(\frac{\pi}{4}G(t)\right)$, where $G(t)$ denotes a baseline cdf of a continuous distribution.

Questions related to the synthesis and analysis of transfer functions, radiation diagrams and filters characteristics are elaborated in detail in [14]–[15].

In [13] we study the Hausdorff approximation [16] of the Heaviside function $h_{t_0}(t)$ by special trigonometric –G families with baseline inverted exponential cumulative distribution function (cdf).

Particular attention is paid to the possibility of generating generalized adaptive functions with the so-called "polynomial transfer" and the possibilities that are found for simulating radiation diagrams and filter characteristics.

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These results will be of interest for specialists in this modern scientific branch.

We will note that the "sine and cosine potential corrections" can be used to construct other families of adaptive functions for modeling processes in the field of Growth Theory, Debugging and Test Theory and Computer Viruses Propagation.

In this article we study a new class of COS–G family with baseline cumulative function of Volmer–type:

$$M(t) = \cos\left(\pi G(t)\right); \quad 0 \le t \le \frac{1}{b}; \;\; b > 0. \tag{1}$$

where

$$G(t) = \frac{1}{4}(2+bt)(1-bt)^2 = \frac{1}{2} - \frac{3}{4}bt + \frac{1}{4}(bt)^3.$$

2. Main Results

2.1. A look at the new COS–G family with baseline cdf of Volmer–type

Remark. We note that for $b = 1$ and $t = \cos\theta$ we have the *Volmer's activation function*

$$V(\theta) = G(\theta) = \frac{1}{4}(2+\cos\theta)(1-\cos\theta)^2.$$

The Volmer theory is correct in predicting a dependence of critical supersaturation on contact angle in heterogeneous nucleation.

For some modelling and approximation problems, see [17]–[22].

Evidently

$$G(0) = \frac{1}{2}; \;\; M(0) = 0;$$

$$G\left(\frac{1}{b}\right) = 0; \;\; M\left(\frac{1}{b}\right) = 1.$$

The new family $M(t)$ for a) $b = 0.5$; *b)* $b = 0.6$; *c)* $b = 0.75$; *d)* $b = 1$ is plotted on Figure 1.

2.1.1. Justification of the idea for generating the powerful classes of trigonometric–G families

An antenna array is a cluster of antennas arranged in a specific physical configuration (line, grid, etc.). Each individual antenna is called an element of the array. We initially assume that all array elements (individual antennas) are identical. However, the excitation (both amplitude and phase) applied to each individual element may differ. The far field radiation from the array in a linear medium can be computed

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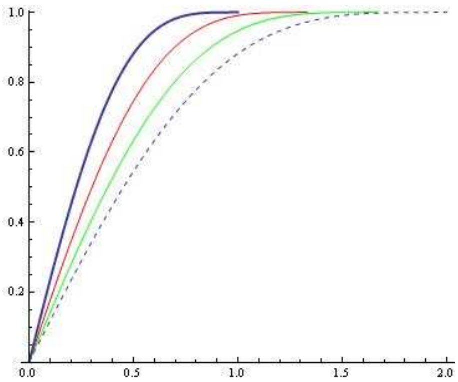

Figure 1: The family $M(t)$ for $b = 0.5$; $b = 0.6$; $b = 0.75$; $b = 1$.

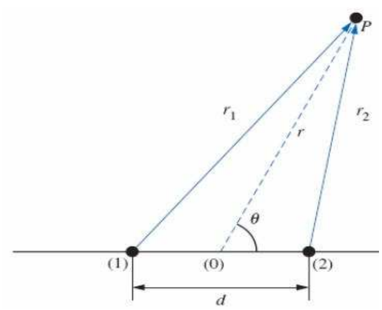

Figure 2: Linear antenna array.

by the superposition of the fields generated by the array elements. We start our discussion from considering a linear array (elements are located in a straight line) consisting of two elements excited by the signals with the same amplitude but with phases shifted by β. Here we will expose from [23]. On arbitrary point P with radius r in the far zone electric field, taking into account the phase difference due to physical separation and difference in excitation, we have

$$E(r) = E_1(r)e^{j\frac{\varphi}{2}} + E_2(r)e^{-j\frac{\varphi}{2}},$$

where E_1 and E_2 are the fields due to antenna 1 and antenna 2 respectively and $\varphi = kd\cos\theta + \beta$, see Figure 2. The phase center is assumed at the array center and the elements are identical

$$E(r) = 2E_1(r)\frac{e^{j\frac{\varphi}{2}} + e^{-j\frac{\varphi}{2}}}{2} = 2E_1(r)\cos\frac{\varphi}{2}.$$

Relocating the phase center point only changes the phase of the result but not

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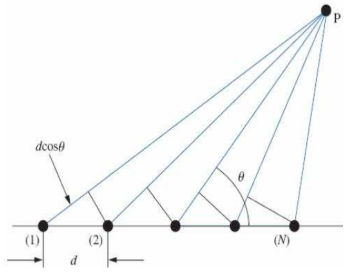

Figure 3: Linear antenna array (N elements)

its amplitude. The radiating pattern can be written as

$$F(\theta) = F_1(\theta)F_a(\theta),$$

where the array factor is

$$F_a(\theta) = \cos\left(\frac{kd\cos\theta + \beta}{2}\right)$$

and $\varphi = kd\cos\theta + \beta$; $k = 2\pi/\lambda$, λ is the wave length; β - the phase difference, θ - azimuthal angle, d - the distance between the emitters. Evidently, for $\beta = 0$ and $d = \frac{\lambda}{2}$ we have

$$F_a(\theta) = \cos\left(\frac{\pi}{2}\cos\theta\right).$$

We consider next an antenna array with more identical elements (see, Figure 3).

There is a linearly progressive phase shift in the excitation signal that feeds N elements. The total field is

$$E(r) = E_0(r)\left(1 + e^{j\varphi} + \cdots + e^{j(N-1)\varphi}\right) = E_0(r)\sum_{n=1}^{N} e^{j(n-1)\varphi}.$$

Using the formula for geometric progression sum

$$\sum_{n=0}^{N-1} q^n = \frac{1-q^N}{1-q}$$

we get for the total radiating electric field

$$E(r) = E_0(r)\frac{1-e^{jN\varphi}}{1-e^{j\varphi}}.$$

Considering the magnitude of the electric field only and using the expression

$$|1-e^{j\varphi}| = |2je^{\frac{j\varphi}{2}}\sin\frac{\varphi}{2}| = 2\sin\frac{\varphi}{2},$$

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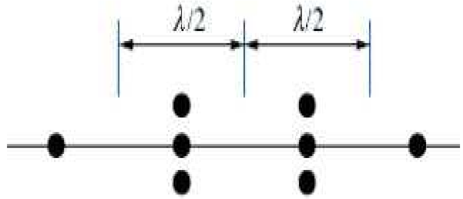

Figure 4: 3-element binomial array)

it follows

$$E(\theta) = E_0(\theta)\frac{\sin\left(\frac{N\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)}.$$

When $\varphi = 0$

$$E(\theta) = E_{\max} = NE_0.$$

Normalized antenna factor F_a for N elements is defined by

$$F_a(\theta) = \frac{\sin\left(\frac{N\varphi}{2}\right)}{N\sin\left(\frac{\varphi}{2}\right)}.$$

Let us consider the initial N-elements binomial array (for $N = 3$, see Figure 4)

The resulting radiating pattern of the binomial array of N elements separated by a half wavelength is

$$F(\theta) = \cos^{N-1}\left(\frac{\pi}{2}\cos\theta\right).$$

At the end of this section let us mention that another interesting class of antenna diagrams. This class is widely used in antenna theory and can be described as:

$$\begin{aligned} F(\theta) &= F_1(\theta)F_a(\theta) = \sin\theta . \frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\left(\frac{kL}{2}\right)}{\sin^2\theta} \\ &= \frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\left(\frac{kL}{2}\right)}{\sin\theta}. \end{aligned}$$

The $F(\theta)$ is the radiating pattern for finite electric dipole antenna. From the above, it is clear how important the task is to generate powerful classes of generalized trigonometric-G families. Let us notice that the array factor depends on the array geometry, amplitudes and phase of the excitation of individual antennas. Another method of modifying the radiating pattern of the array is to change electronically the phase parameter β of the excitation. In this situation, it is possible to change direction of the main lobe in a wide range: the antenna is scanning through certain region of space. Such structure is called a phased-array antenna.

For more details, see [24].

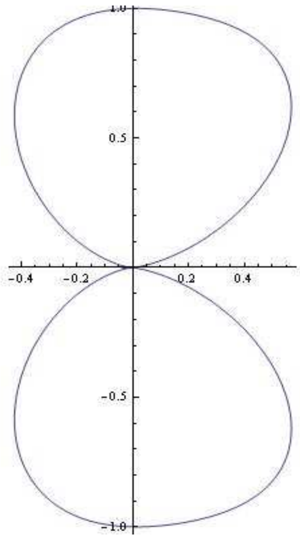

Figure 5: The family $M(\theta)$ for $b = 0.5$; $r = c = 1.74$.

2.1.2. Some Applications

Let $t = r\cos\theta + c$. Consider the new class of COS–G family - adaptive function $M(\theta)$ for $b = 0.5$; $r = c = 1.74$ (see Figure 5).

Thus, we have the typical radiating pattern for "finite electric dipole antenna".
The adaptive function $M(\theta)$ for $b = 0.75$; $r = 0.84$; $c = 0.82$ is plotted on Figure 6.

2.1.3. Parametric adaptive G–family

Following the transformation [25] we define the following new parametric adaptive family:

$$M_1(t) = (1+\lambda)\cos\left(\pi(\frac{1}{2} - \frac{3}{4}bt + \frac{1}{4}(bt)^3)\right) - \lambda\cos^2\left(\pi(\frac{1}{2} - \frac{3}{4}bt + \frac{1}{4}(bt)^3)\right) \quad (2)$$

where $|\lambda| < 1$.
Example 1. The new parametric function $M_1(t)$ for
a) $\lambda = 0.9$; $b = 0.95$;
b) $\lambda = 0.8$; $b = 0.95$;
c) $\lambda = 0.7$; $b = 0.95$
is plotted on Figure 7.

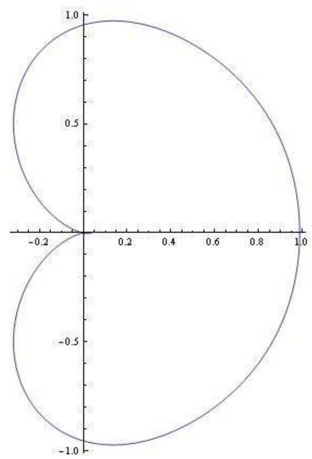

Figure 6: The family $M(\theta)$ for $b = 0.75$; $r = 0.84$; $c = 0.82$.

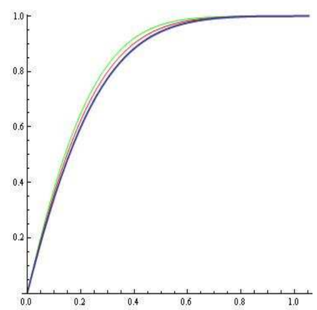

Figure 7: The family $M_1(t)$ (Example 1).

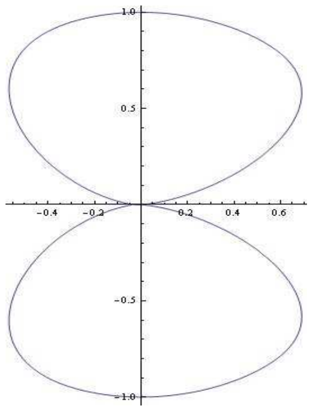

Figure 8: The family $M_1(\theta)$ for $\lambda = 0.95$, $b = 0.089$; $r = 9.7$; $c = 9.8$.

The reader may formulate corresponding approximation problems using $M_1(\theta)$ with applications to the Antenna–feeder Analysis.

For instance, the new adaptive function $M_1(\theta)$ for $\lambda = 0.95$, $b = 0.089$; $r = 9.7$; $c = 9.8$ is plotted on Figure 8.

For some modelling and approximation problems, see [26]–[34].

Acknowledgments

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