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ON THE "SATURATION" WITH THE TRANSMUTED FRECHET AND SKEW GENERALIZED INVERSE WEIBULL FAMILIES OF C.D.F.

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Abstract: In [1] the authors considered the following cdf of the transmuted Frechet (GTF) distribution:

$$F(t) = G^a(t)\left((\lambda+1) - \lambda G^b(t)\right),$$

where $-1 \leq \lambda < 1$, $a > 0$ and $b > 0$ and $G(t) = e^{-\beta t^{\alpha}}$ is the cdf of the two–parameter Frechet distribution. In [2] the authors considered the following cdf of the skew generalized inverse Weibull (SGIW) distribution:

$$M(t) = e^{-\gamma\left(\frac{\alpha}{t}\right)^{\beta}(1+\lambda^{-\beta})},$$

where $(\alpha, \beta, \gamma, \lambda) \in R^+$.

Also of interest to the specialists is the task of approximating the Heaviside function

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases}$$

where t_0 is the median with the new cumulative functions in the Hausdorff sense.

Numerical examples, illustrating our results are presented using programming environment *CAS Mathematica*.

AMS Subject Classification: 41A46

Key Words: four–parameter generalized transmuted Frechet c.d.f., four–parameter skew generalized inverse Weibull family of c.d.f, Heaviside step–function $h_{t_0}(t)$, Hausdorff distance

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1. Introduction and Preliminaries

In this note we study the Hausdorff approximation of the Heaviside function $h_{t_0}(t)$ by the cumulative functions $F(t)$ and $M(t)$.

The model $M(t)$ have been tested with real-world data.

The cdf of the two–parameter Frechet distribution is given by:

$$G(t) = e^{-\beta t^{\alpha}},$$

where $\alpha > 0$ is the shape parameter and $\beta > 0$ is scale parameter.

Definition 1. In [1] the authors considered the following cdf of the transmuted Frechet (GTF) distribution:

$$F(t) = G^{a}(t)\left((\lambda+1) - \lambda G^{b}(t)\right), \tag{1}$$

where $-1 \leq \lambda < 1$, $a > 0$ and $b > 0$.

Definition 2. In [2] the authors considered the following cdf of the skew generalized inverse Weibull (SGIW) distribution:

$$M(t) = e^{-\gamma\left(\frac{\alpha}{t}\right)^{\beta}(1+\lambda^{-\beta})}, \tag{2}$$

where $(\alpha, \beta, \gamma, \lambda) \in R^{+}$.

Definition 3. The shifted Heaviside step function is defined by

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{3}$$

Definition 4. [3] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{\sup_{A \in F(f)} \inf_{B \in F(g)} ||A-B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A-B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A-B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

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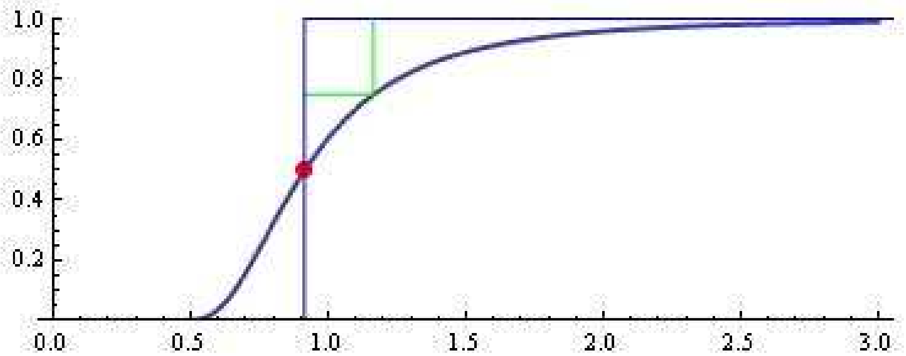

Figure 1: The model (1) for $a = 0.5$, $\lambda = 0.5$, $\alpha = 3.6$, $\beta = 1.1$, $b = 0.1$ and $t_0 = 0.91362$; H–distance $d = 0.250291$.

2. Main Results

2.1. A note on the generalized transmuted Frechet (GTF) cdf (1)

The investigation of the characteristic "supersaturation" of the model (1) to the horizontal asymptote is important.

Sensitive analysis for the "saturation in the Hausdorff sense".

Let t_0 is the value for which $F(t_0) = \frac{1}{2}$.

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the (cdf) $F(t)$ (1) satisfies the relation

$$F(t_0 + d) = 1 - d. \tag{4}$$

For given a, b, α, β, λ and t_0, the nonlinear equation $F(t_0 + d) - 1 + d = 0$ has unique positive root – d.

The model (1) for $a = 0.5$, $\lambda = 0.5$, $\alpha = 3.6$, $\beta = 1.1$, $b = 0.1$ and $t_0 = 0.91362$ is visualized on Fig. 1.

From the nonlinear equation (4) we have: $d = 0.250291$.

The model (1) for $a = 0.099$, $\lambda = -0.99$, $\alpha = 2.75$, $\beta = 0.06$, $b = 0.006$ and $t_0 = 0.180946$ is visualized on Fig. 2.

From the nonlinear equation (4) we have: $d = 0.136902$.

Obviously, the model (1) can also be used for approximating of some "specific data".

2.2. A note on the skew generalized inverse Weibull family of cdf (2)

Sensitive analysis for the "saturation in the Hausdorff sense".

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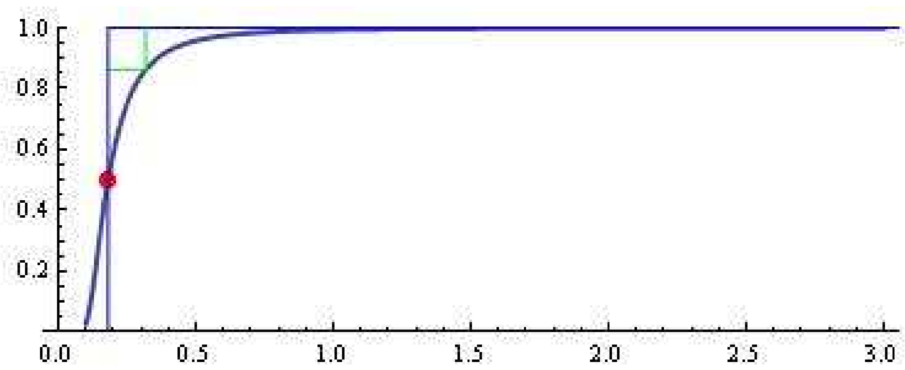

Figure 2: The model (1) for $a = 0.099$, $\lambda = -0.99$, $\alpha = 2.75$, $\beta = 0.06$, $b = 0.006$ and $t_0 = 0.180946$; H–distance $d = 0.136902$.

For the median we have

$$t_0 = \left(\frac{\ln 2}{\gamma\alpha^{\beta}(1+\lambda^{-\beta})}\right)^{-\frac{1}{\beta}}$$

i.e. $M(t_0) = \frac{1}{2}$.

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the (cdf) $M(t)$ (2) satisfies the relation

$$M(t_0 + d) = 1 - d. \tag{5}$$

For given γ, α, β, λ and t_0, the nonlinear equation $M(t_0 + d) - 1 + d = 0$ has unique positive root – d.

The model (2) for $\gamma = 0.15$, $\alpha = 0.03$, $\beta = 4.5$, $\lambda = 0.1$ and $t_0 = 0.213504$ is visualized on Fig. 3.

From the nonlinear equation (5) we have: $d = 0.106345$.

The model (2) for $\gamma = 0.1$, $\alpha = 0.01$, $\beta = 5.5$, $\lambda = 0.08$ and $t_0 = 0.0879089$ is visualized on Fig. 4.

From the nonlinear equation (5) we have: $d = 0.052129$.

The model (2) for $\gamma = 0.09$, $\alpha = 0.005$, $\beta = 6.5$, $\lambda = 0.05$ and $t_0 = 0.073047$ is visualized on Fig. 5.

From the nonlinear equation (5) we have: $d = 0.0399214$.

2.3. Applications

Example. We consider the following "dataModBlaster" worm:

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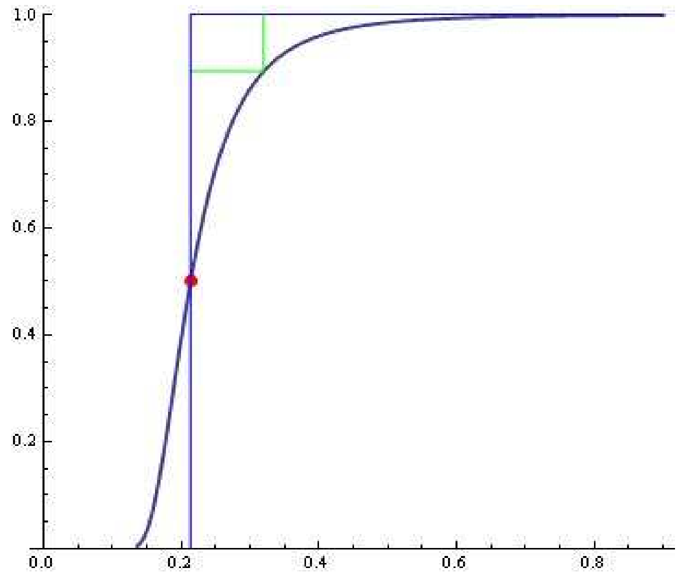

Figure 3: The model (2) for $\gamma = 0.15$, $\alpha = 0.03$, $\beta = 4.5$, $\lambda = 0.1$ and $t_0 = 0.213504$; H–distance $d = 0.106345$.

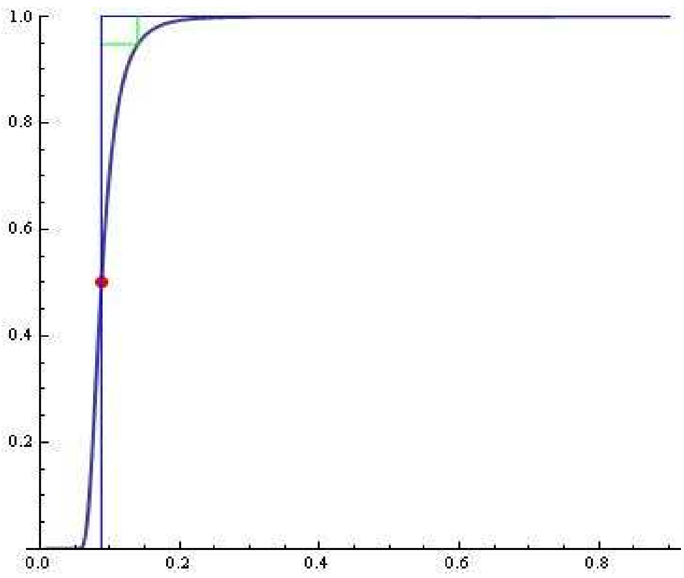

Figure 4: The model (2) for $\gamma = 0.1$, $\alpha = 0.01$, $\beta = 5.5$, $\lambda = 0.08$ and $t_0 = 0.0879089$; H–distance $d = 0.052129$.

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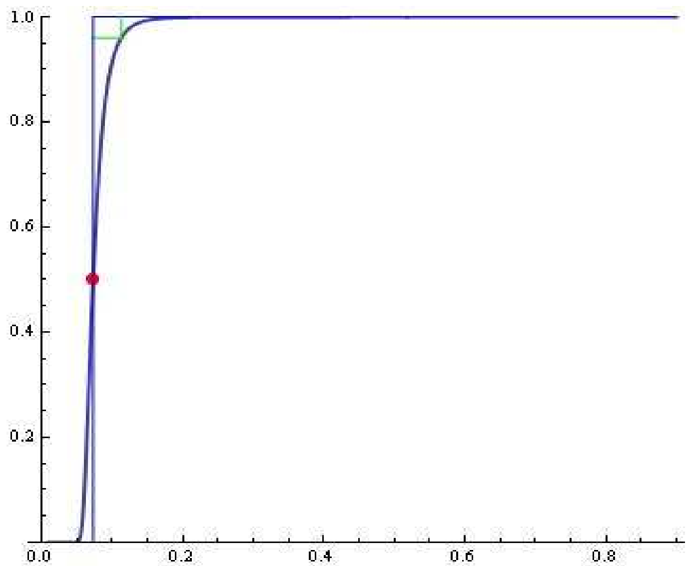

Figure 5: The model (2) for $\gamma = 0.09$, $\alpha = 0.005$, $\beta = 6.5$, $\lambda = 0.05$ and $t_0 = 0.073047$; H–distance $d = 0.0399214$.

$dataModBlaster$

$:= \{\{1, 10\}, \{100, 410\}, \{200, 4103\}, \{300, 5517\}, \{400, 95345\}, \{500, 472414\},$

$\{600, 565517\}, \{700, 581034\}, \{800, 590345\}\};$

The model $M^*(t) = \omega M(t)$ for

$$\alpha = 4.85, \lambda = 0.5, \beta = 8.6, \gamma = 1.33984 \times 10^{14}, \omega = 590345$$

is visualized on Fig. 6.

Obviously, the model (2) can also be used for approximating of some "specific data".

For other approximation and modelling results, see [4]–[26].

We hope that the results will be useful for specialists in this scientific area.

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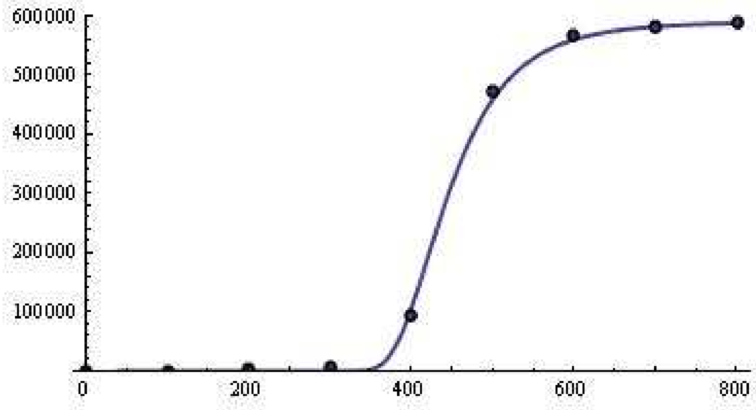

Figure 6: The model M^*.

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