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SOME MORE RESULTS ON ONE MODULO THREE ROOT SQUARE MEAN GRAPHS

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Abstract: A graph G is said to be one modulo three root square mean graph if there is an injective function ϕ from the vertex set of G to the set $\{0, 1, 3, \ldots, 3q-2, 3q\}$ where q is the number of edges of G and ϕ induces a bijection ϕ^* from the edge set of G to $\{1, 4, \ldots, 3q-2\}$ given by $\phi^*(uv) = \left\lceil \sqrt{\frac{[\phi(u)]^2+[\phi(v)]^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{[\phi(u)]^2+[\phi(v)]^2}{2}} \right\rfloor$ and the function ϕ is called one modulo three root square mean labeling of G. The concept of one modulo three root square mean labeling was introduced by C. Jayasekaran and M. Jaslin Melbha and they investigated some path related graphs are one modulo three root square mean graphs.

AMS Subject Classification: 05C78
Key Words: one modulo three root square mean labeling, one modulo three root square mean graphs

1. Introduction

We begin with simple, finite, connected and undirected graph. For standard terminology and notations we follow Harary [3]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a vertex labeling (an edge labeling). Several types of graph labeling and a detailed survey is available in [2].

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C. Jayasekaran and C. David Raj was introduced the concept of one modulo three harmonic mean labeling of graphs in [1]. P. Jeyanthi and A. Maheswari was introduced the concept of one modulo three mean labeling of graphs in [5]. Root square mean labeling was introduced by S.S. Sandhya, S. Somasundaram and S. Anusa in [6]. C. Jayasekaran and M. Jaslin Melbha introduce the concept one modulo three root square mean labeling of graphs in [4]. They investigated the one modulo three root square mean labeling of path related graphs. Not every graph is one modulo three root square mean. For example, star graph $K_{1,n}$, where $n \geq 4$ is not a one modulo three root square mean graph. We are interested to study different classes of graphs, which are one modulo three root square mean graphs.

We will provide a brief summary of definitions and other informations which are necessary for our present investigation.

Definition 1.1. A graph G is said to be one modulo three root square mean graph if there is an injective function ϕ from the vertex set of G to the set $\{0, 1, 3, \ldots, 3q-2, 3q\}$ where q is the number of edges of G and ϕ induces a bijection ϕ^* from the edge set of G to $\{1, 4, \ldots, 3q-2\}$ given by $\phi^*(uv) = \left\lceil \sqrt{\frac{[\phi(u)]^2+[\phi(v)]^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{[\phi(u)]^2+[\phi(v)]^2}{2}} \right\rfloor$ and the function ϕ is called one modulo three root square mean labeling of G.

Definition 1.2. A graph G is said to be complete, if every pair of its distinct vertices are adjacent. A complete graph on p vertices is denoted by K_p.

Definition 1.3. The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where i^{th} vertex of G_1 is adjacent to every vertices in the i^{th} copy of G_2.

Definition 1.4. The graph $P_n \odot K_1$ is called a comb.

Definition 1.5. The product $P_2 \times P_n$ is called a ladder, and it is denoted by L_n. The ladder graph L_n is a planar undirected graph with $2n$ vertices and $3n-2$ edges.

Definition 1.6. The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $E = E_1 \cup E_2$.

Definition 1.7. The graph $W_n = C_{n-1} + K_1$ is called a Wheel with n spokes. A wheel graph W_n is obtained from a cycle C_n by adding a new vertex and joining it to all the vertices of the cycle by an edge, the new edges are called the spokes of the wheel.

Definition 1.8. A triangular snake T_n is obtained from a path $v_1 v_2 \ldots v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $1 \leq i \leq n-1$. That is, every edge of the path is replaced by a triangle C_3.

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Definition 1.9. A double triangular snake consists of two triangular snakes that have a common path. That is, a double triangular snake DT_n is a graph obtained from a path $u_1u_2 \dots u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i for $1 \leq i \leq n-1$.

Definition 1.10. A triangular ladder $TL_n, n \geq 2$ is a graph obtained from L_n by adding the edges $u_iv_{i+1}, 1 \leq i \leq n-1$ where u_i and v_i are the vertices of L_n such that $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ are two paths of length n in the graph L_n.

Definition 1.11. A friendship graph F_n is a one point union of n copies of cycle C_3.

Theorem 1.12. *Any path P_n is a one modulo three root square mean graph [4].*

2. Main Result

Theorem 2.1. *The complete graph K_n is a one modulo three root square mean graph if and only if $n \leq 2$.*

Proof. Suppose K_n is a one modulo three root square mean graph for $n \geq 4$. To get the edge label $3q-2$, we must have $3q-2$ and $3q$ as the vertex labels. Let u and v be the vertices whose labels are $3q-2$ and $3q$ respectively. Again, to get the edge label 1, we must have 0 and 1 as the vertex labels. Let w and z be the vertices whose labels are 0 and 1 respectively then the edges uw, vz receive the same induced label which should not happen. Hence K_n is not a one modulo three root square mean graph for $n \geq 4$. If $n = 2$, the complete graph K_2 is a path P_2, which is a one modulo three root square mean graph. If $n = 3$, the complete graph K_3 is a triangle. By theorem 2.17, triangle is not a one modulo three root square mean graph. Hence The complete graph K_n is a one modulo three root square mean graph if and only if $n \leq 2$. □

Example 2.2. One modulo three root square mean labeling of K_2 is given in figure 1.

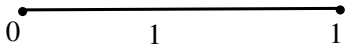

Figure 1: K_2

Theorem 2.3. *$C_4 = L_2$ is not a one modulo three root square mean graph.*

Proof. Let $V(L_2) = \{u, v, x, y\}$ and $E(L_2) = \{uv, vx, xy, yu\}$. Then L_2 has 4 vertices and 4 edges. Define a function $\phi : V(L_2) \to \{0, 1, 3, 4, 6, 7, 9, 10, 12\}$ by $\phi(u) = 0, \phi(v) = 1, \phi(x) = 6, \phi(y) = 7$. Then $\phi^* : E(L_2) \to \{1, 4, 7, 10\}$, where

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$\phi^*(uv) = 1, \phi^*(vx) = 4, \phi^*(xy) = 7, \phi^*(yu) = 4$. Thus two edges get same labeling. Hence L_2 is not a one modulo three root square mean graph. □

Theorem 2.4. *The ladder $L_n = P_n \times K_2$ is a one modulo three root square mean graph for $n \neq 2$.*

Proof. Let $V(L_n) = \{u_i, v_i / 1 \leq i \leq n\}$ and $E(L_n) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_i, u_n v_n / 1 \leq i \leq n-1\}$. Then L_n has 2_n vertices and $3n-2$ edges. Define a function $\phi : V(L_n) \to \{0, 1, 3, \ldots, 3q-2, 3q\}$ by $\phi(u_1) = 0, \phi(u_2) = 1, \phi(u_3) = 6, \phi(u_i) = 9i - 6, 4 \leq i \leq n; \phi(v_i) = 6i + 3, 1 \leq i \leq 3$; $\phi(v_i) = 9i - 9, 4 \leq i \leq n$. Then ϕ induces a bijection $\phi* : E(L_n) \to \{1, 4, \ldots, 3q-2\}$, where $\phi^*(u_1u_2) = 1, \phi^*(u_2u_3) = 4, \phi^*(u_3u_4) = 22, \phi^*(u_iu_{i+1}) = 9i - 2, 4 \leq i \leq n-1; \phi^*(v_iv_{i+1}) = 6i + 7, 1 \leq i \leq 3; \phi^*(v_iv_{i+1}) = 9i - 5, 4 \leq i \leq n-1; \phi^*(u_1v_1) = 7, \phi^*(u_2v_2) = 10, \phi^*(u_3v_3) = 16, \phi^*(u_iv_i) = 9i - 8, 4 \leq i \leq n$. For $n = 2$, by Theorem 2.3, L_2 is not a one modulo three root square mean graph. Hence the ladder L_n is a one modulo three root square mean graph for $n \neq 2$. □

Example 2.5. A one modulo three root square mean labeling of L_9 is given in figure 2.

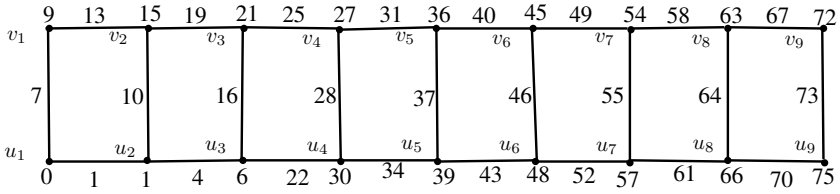

Figure 2: L_9

Theorem 2.6. *$P_m \cup (P_n \odot K_1)$ admits one modulo three root square mean labeling.*

Proof. Let P_n be the path $v_1v_2 \ldots v_n$. Let w_i be the vertex adjacent to $v_i, 1 \leq i \leq n$. The resultant graph is $P_n \odot K_1$. Let $u_1u_2 \ldots u_m$ be the path P_m. Let $G = P_m \cup (P_n \odot K_1)$ with $V(G) = \{u_i, v_j, w_j / 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(G) = \{u_iu_{i+1}, v_jw_j, v_jv_{j+1}, v_nw_n / 1 \leq i \leq m-1, 1 \leq j \leq n-1\}$. Then G has $m + 2n$ vertices and $m + 2n - 3$ edges. Define a function $\phi : V(G) \to \{0, 1, 3, \ldots, 3q-2, 3q\}$ by $\phi(u_1) = 0, \phi(u_2) = 1, \phi(u_i) = 3i - 3, 3 \leq i \leq m-1, \phi(u_m) = 3m - 5; \phi(v_i) = 3m + 6i - 6$ for odd i and $\phi(v_i) = 3m + 6i - 9$ for even $i, i \leq n; \phi(w_i) = 3m + 6i - 9$ for odd i and $\phi(w_i) = 3m + 6i - 6$ for even $i, i \leq n$. Then ϕ induces a bijection $\phi^* : E(L_n) \to \{1, 4, \ldots, 3q-2\}$, where $\phi^*(u_iu_{i+1}) = 3i-2, 1 \leq i \leq n-1; \phi^*(v_iv_{i+1}) = 3m + 6i - 5, 1 \leq i \leq n-1; \phi^*(v_iw_i) = 3m + 6i - 8, 1 \leq i \leq n$. Therefore, ϕ is a one modulo three root square mean labeling. Hence $P_m \cup (P_n \odot K_1)$ admits one modulo three root square mean labeling. □

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Example 2.7. A one modulo three root square mean labeling of $P_7 \cup (P_6 \odot K_1)$ is given in figure 3.

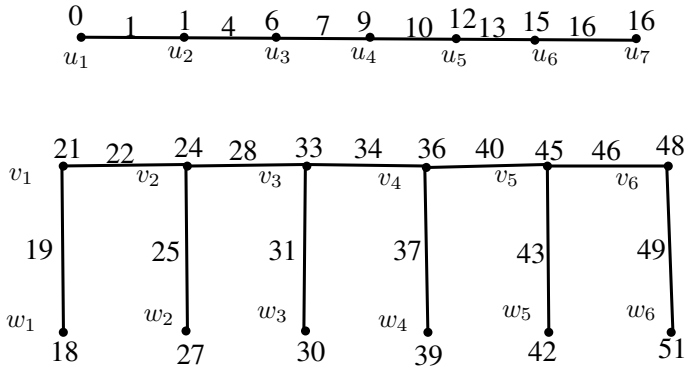

Figure 3: $P_7 \cup (P_6 \odot K_1)$

Theorem 2.8. *$(P_m \odot \bar{K_2}) \cup P_n$ admits one modulo three root square mean labeling.*

Proof. Let P_m be the path $u_1u_2 \dots u_m$. Let v_i, w_i be the vertices of i^{th} copy of $\bar{K_2}$ Join v_i and w_i with the vertex $u_i, 1 \leq i \leq m$. The resultant graph is $P_m \odot \bar{K_2}$. Let $x_1x_2 \dots x_n$ be the path P_n. Let $G = (P_m \odot \bar{K_2}) \cup P_n$ with $V(G) = \{u_i, v_i, w_i, x_j / 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(G) = \{u_iu_{i+1}, u_iw_i, u_iv_i, x_jx_{j+1}, u_mw_m, u_mv_m / 1 \leq i \leq m-1, 1 \leq j \leq n-1\}$. Define a function $\phi : V(G) \to \{0, 1, 3, \dots, 3q-2, 3q\}$ by $\phi(u_1) = 1, \phi(u\phi_2) = 9, \phi(u_i) = 9i - 6, 3 \leq i \leq m; \phi(v_1) = 0; \phi(v_2) = 10; \phi(v_i) = 9i - 11, 3 \leq i \leq n; \phi(w_1) = 6; \phi(w_2) = 15; \phi(w_i) = 9i - 5, 3 \leq i \leq n; \phi(x_i) = 9m + 3i - 6, 1 \leq i \leq n$. Then ϕ induces a bijection $\phi^* : E(G) \to \{1, 4, \dots, 3q-2\}$, where $\phi^*(u_iu_{i+1}) = 9i - 2, 1 \leq i \leq m-1; \phi^*(u_iv_i) = 9i - 8, 1 \leq i \leq m; \phi^*(u_iw_i) = 9i - 5, 1 \leq i \leq m; \phi^*(x_ix_{i+1}) = 9m + 3i - 5, 1 \leq i \leq n-1$. Therefore, ϕ is a one modulo three root square mean labeling. Hence $(P_m \odot \bar{K_2}) \cup P_n$ admits one modulo three root square mean labeling. □

Example 2.9. A one modulo three root square mean labeling of $(P_5 \odot \bar{K_2}) \cup P_7$ is given in figure 4.

Theorem 2.10. *Let G be a graph obtained by joining a pendent vertex with a vertex of degree two of a comb $P_n \odot K_1$. Then G is one modulo three root square mean graph.*

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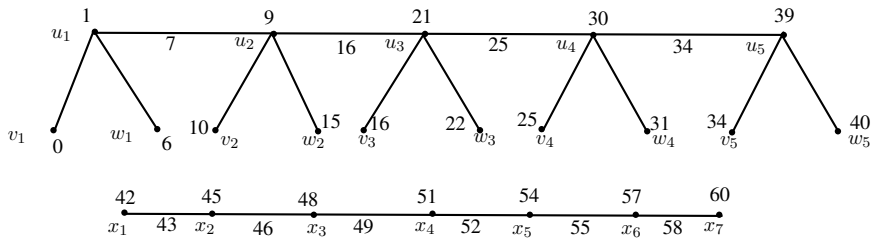

Figure 4: $(P_5 \odot \bar{K_2}) \cup P_7$

Proof. Let P_n be the path $u_1u_2 \dots u_n$. Let v_i be the vertex adjacent to $u_i, 1 \leq i \leq n$. The resultant graph is $P_n \odot K_1$. Let w be the vertex joined with u_n. The resultant graph is G. Here $V(G) = \{u_i, v_i, w/1 \leq i \leq n\}$ and $E(G) = \{u_iv_i, u_iu_{i+1}, u_nv_n, u_nw/1 \leq i \leq n-1\}$. Then G has $2n+1$ vertices and $2n$ edges.
Case 1. n is odd.
Define a function $\phi : V(G) \to \{0, 1, 3, \dots, 3q-2, 3q\}$ by $\phi(u_1) = 1, \phi(u_i) = 6i-6, 2 \leq i \leq n; \phi(w) = 6n, \phi(v_1) = 0; \phi(v_i) = 6i-5, 2 \leq i \leq n$. Then ϕ induces a bijection $\phi^* : E(G) \to \{1, 4, \dots, 3q-2\}$, where $\phi^*(u_iu_{i+1}) = 6i-2, 1 \leq i \leq n-1; \phi^*(u_iv_i) = 6i-5, 1 \leq i \leq n$. Thus the edges get distinct labels $1, 4, \dots, 3q-2$. In this case ϕ is a one modulo three root square mean labeling for G.
Case 2. n is even.
Define a function $\phi : V(G) \to \{0, 1, 3, \dots, 3q-2, 3q\}$ by $\phi(u_1) = 0; \phi(u_i) = 6i-5, 2 \leq i \leq n; \phi(w) = 6n, \phi(v_1) = 1, \phi(v_i) = 6i-6, 2 \leq i \leq n$. Then ϕ induces a bijection $\phi^* : E(G) \to \{1, 4, \dots, 3q-2\}$, where $\phi^*(u_iu_{i+1}) = 6i-2, 1 \leq i \leq n-1; \phi^*(u_iv_i) = 6i-5, 1 \leq i \leq n$. Thus the edges get distinct labels $1, 4, \dots, 3q-2$. In this case ϕ is a one modulo three root square mean labeling for G. Therefore, ϕ is a one modulo three root square mean labeling for G. From case 1 and case 2, we conclude that G is a one modulo three root square mean graph. □

Example 2.11. One modulo three root square mean labeling of G when $n = 6$ and $n = 7$ is given in figure 5 and figure 6 respectively.

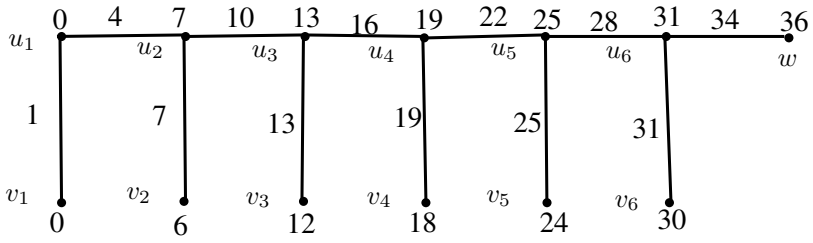

Figure 5

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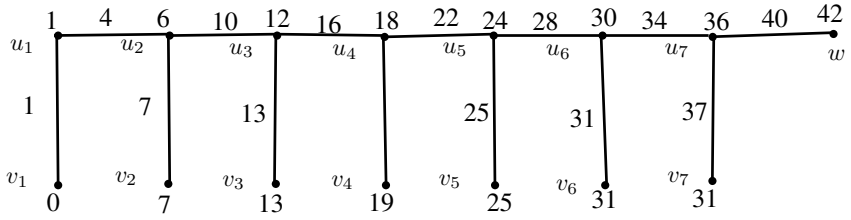

Figure 6

Theorem 2.12. *$L_n \odot K_1$ is one modulo three root square mean graph.*

Proof. Let $u_1u_2 \dots u_n$ and $v_1v_2 \dots v_n$ be two paths of length n. Join u_i and $v_i, 1 \leq i \leq n$. The resultant graph is L_n. For $1 \leq i \leq n$, let x_i be the pendant vertex adjacent to u_i and y_i be the pendant vertex adjacent to v_i. We get the required graph $L_n \odot K_1$ with $V(L_n \odot K_1) = \{u_i, v_i, x_i, y_i / 1 \leq i \leq n\}$ and $E(L_n \odot K_1) = \{u_iv_i, u_ix_i, v_iy_i, v_jv_{j+1}, u_ju_{j+1} / 1 \leq i \leq n, 1 \leq j \leq n-1\}$. Then G has $4n$ vertices and $5n-2$ edges. Define a function $\phi : V(L_n \odot K_1) \to \{0, 1, 3, \dots, 3q-2, 3q\}$ by $\phi(u_1) = 1, \phi(u_2) = 15, \phi(u_3) = 33, \phi(u_4) = 46, \phi(u_i) = 15i - 12, 5 \leq i \leq n; \phi(x_1) = 0, \phi(x_2) = 22, \phi(x_3) = 28, \phi(x_4) = 45, \phi(x_i) = 15i - 17, 5 \leq i \leq n; \phi(v_1) = 6, \phi(v_2) = 18, \phi(v_3) = 34, \phi(v_i) = 15i - 9, 4 \leq i \leq n; \phi(y_1) = 9, \phi(y_2) = 24, \phi(y_i) = 15i - 6, 3 \leq i \leq n$. Then ϕ induces a bijection $\phi^* : E(L_n \odot K_1) \to \{1, 4, \dots, 3q-2\}$, where $\phi^*(u_iu_{i+1}) = 15i - 5, 1 \leq i \leq n-1; \phi^*(u_1x_1) = 1, \phi^*(u_2x_2) = 19, \phi^*(u_ix_i) = 15i - 14, 3 \leq i \leq n; \phi^*(u_1v_1) = 4, \phi^*(u_2v_2) = 16, \phi^*(u_iv_i) = 15i - 11, 3 \leq i \leq n; \phi^*(v_1y_1) = 7, \phi^*(v_2y_2) = 22, \phi^*(v_iy_i) = 15i - 8, 3 \leq i \leq n; \phi^*(v_iv_{i+1}) = 15i - 2, 1 \leq i \leq n-1$. Hence $L_n \odot K_1$ is one modulo three root square mean graph. □

Example 2.13. A one modulo three root square mean labeling of $L_8 \odot K_1$ is given in figure 7.

Construction: Let $L_n = P_n \times K_2$ be the ladder graph. $V(L_n) = \{u_i, v_i / 1 \leq i \leq n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_i, u_nv_n / 1 \leq i \leq n-1\}$. The graph obtained from L_n by deleting the edge u_1v_1 is denoted by G_d.

Theorem 2.14. *The graph G_d is one modulo three root square mean graph.*

Proof. Let $V(G_d) = \{u_i, v_i / 1 \leq i \leq n\}$. $E(G_d) = \{u_iu_{i+1}, v_iv_{i+1} / 1 \leq i \leq n-1\} \cup \{u_iv_i / 2 \leq i \leq n\}$. That is, $V(G_d) = V(L_n)$ and $E(G_d) = E(L_n) - \{u_1v_1\}$. Then G_d has $2n$ vertices and $3(n-1)$ edges. Define a function $\phi : V(G_d) \to \{1, 3, 4, \dots, 3q-2, 3q\}$ by $\phi(u_1) = 0, \phi(u_2) = 1, \phi(u_3) = 15, \phi(u_4) = 22, \phi(u_i) = 9i - 12, 5 \leq i \leq n; \phi(v_1) = 9, \phi(v_2) = 6, \phi(v_3) = 18, \phi(v_i) = 9i - 9, 4 \leq i \leq n$. Then ϕ induces a bijective function $\phi^* : E(G_d) \to \{1, 4, 7, \dots, 3q-2\}$, where $\phi^*(u_1u_2) = 1, \phi^*(u_iu_{i+1}) = 9i - 8, 2 \leq i \leq n-1; \phi^*(v_1v_2) = 7, \phi^*(v_iv_{i+1}) = 9i - 5, 2 \leq i \leq n-1; \phi^*(u_2v_2) = 4, \phi^*(u_iv_i) = 9i - 11, 3 \leq i \leq n$. Thus ϕ provides

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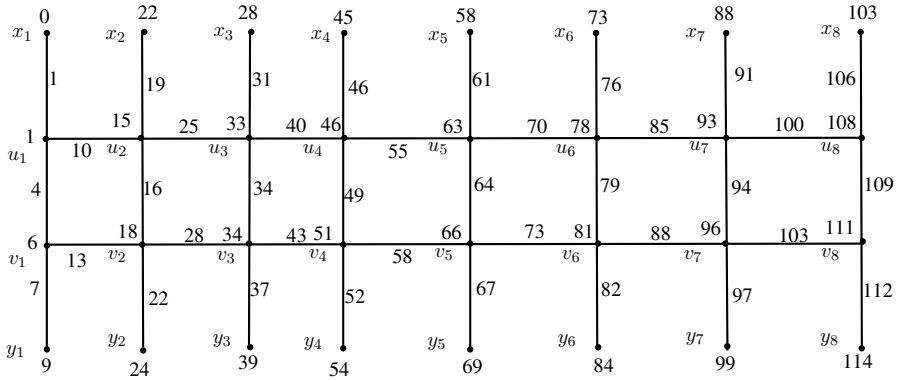

Figure 7: $L_8 \odot K_1$

one modulo three root square mean labeling for G. Hence G_d is an one modulo three root square mean graph. □

Example 2.15. One modulo three root square mean labeling of G_d when $n = 8$ is given in figure 8.

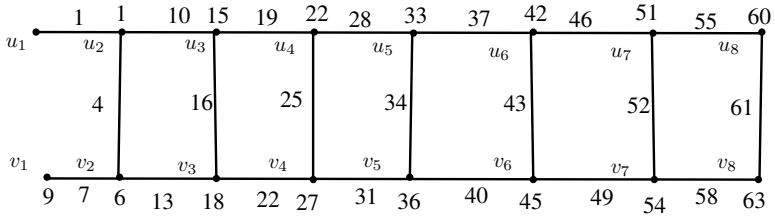

Figure 8

Theorem 2.16. *If G is a graph in which every edge is an edge of a triangle then G is not a one modulo three root square mean graph.*

Proof. Let G be a graph in which every edge is an edge of a triangle. Suppose G is a one modulo three root square mean graph. To get 1 as an edge label, there must be two adjacent vertices x and y such that $\phi(x) = 0$ and $\phi(y) = 1$. Let $xyzx$ be a triangle in which the edge xy lies. To get 4 as an edge label, there must be $\phi(z) = 6$, then xz and yz get the same edge label. This is a contradiction to the fact of one modulo three root square mean labeling. Hence G is not a one modulo three root square mean labeling graph. □

Corollary 2.17. *The wheel graph W_n, flower graph FL_n, triangular snakes, double triangular snakes, triangular ladders, fans $P_n + K_1, n \geq 2$, Double fans $P_n +$*

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$K_2, n \geq 2$, friendship graph C_3^n, windmills $K_m^n, m > 3$, square graph $B_{n,n}^2$, total graph $T(P_n)$ and composition graph $P_n[P_2]$ are not one modulo three root square mean graph.

References

[1] C. David Raj, C. Jayasekaran, Some Results on one modulo three harmonic mean graphs, *International Journal of Mathematical Archieve*, **5**, No. 3 (2014), 203-208.

[2] J.A. Gallian, A dynamic survey of graph labeling, *The Electronics Journal of Combinatorics*, **17**, #DS6 (2014), url: http://www.combinatorics.org/Surveys/ds6.pdf

[3] F. Harary, *Graph Theory*, Narosa Publishing House, New Delhi, 1988.

[4] C. Jayasekaran, M. Jaslin Melbha, One modulo three root square mean labeling of path related graphs, *International Journal of Pure and Applied Mathematics*, **118**, No. 2 (2018), 351-358.

[5] P. Jeyanthi, A. Maheswari, One modulo three mean labeling of graphs, *American Journal of Applied Mathematics and Statistics*, **2**, No. 5 (2014), 302-306.

[6] S.S. Sandhya, S. Somasundaram, S. Anusa, Root square mean labeling of graphs, *International Journal of Contemporary Mathematical Sciences*, **9**, No. 14 (2014), 667-676.