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ON $g\alpha r$-CONTINUOUS MAP IN TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce a new class of generalized α regular-continuous map and study some of their properties as well as inter relationship with other continuous maps.

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1. Introduction

Continuous map was studied for different types of closed sets by various researchers for past many years. In 1996, Andrijevic [3, 4] introduced new type called b-open sets. A.A.Omari and M.S.M. Noorani [2] were introduced and studied b-continuous map and b-closed map.

The aim of this paper is to continue the study of generalized α regular-continuous map, generalized α regular-closed map have been introduced and studied their relations with various generalized closed maps. Through out this paper (X, τ) and (Y, σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

Let $A \subseteq X$, the closure of A and interior of A will be denoted by $cl(A)$ and $int(A)$ respectively, union of all b-open sets X contained in A is called b-interior of A and it is denoted by $bint(A)$, the intersection of all b-closed sets of X containing A is called b-closure of A and it is denoted by $bcl(A)$.

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2. Preliminaries

Definition 2.1. Let a subset A of a topological space (X, τ), is called

1) a α-open set [9] if $A \subseteq int(cl(int(A)))$.
2) a generalised-closed set (briefly g-closed) [5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
3) a weakly-closed set(briefly w-closed) [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open.
4) a generalized $*$-closed set (briefly $g*$-closed) [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.
5) a generalized α-closed set (briefly $g\alpha$-closed)[8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.
6) an α generalized-closed set (briefly αg-closed)[7] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
7) a generalized b-closed set (briefly gb-closed) [1] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
8) a semi generalized b-closed set (briefly sgb-closed) [4] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
9) a generalized αb-closed set (briefly $g\alpha b$-closed) [10] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.
10) a regular generalized b-closed set (briefly rgb-closed) [9] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
11) a generalized pre regular-closed set (briefly gpr-closed) [3] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
12) a generalized α regular-closed set (briefly $g\alpha r$-closed) [13] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition 2.2. A function $f : (X, \tau) \to (Y, \sigma)$, is called

1) a generalized-continuous map [17] if $f^{-1}(V)$ is generalized-open in (X, τ) for every open set V of (Y, σ).
2) a generalized $*$-continuous map [14] if $f^{-1}(V)$ is generalized $*$-open in (X, τ) for every open set V of (Y, σ).

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3) an α-continuous map [18] if $f^{-1}(V)$ is α-open in (X, τ) for every open set V of (Y, σ).

4) an α generalized-continuous map [10] if $f^{-1}(V)$ is α generalized-closed in (X, τ) for every open set V of (Y, σ).

5) a generalized α-continuous map [10] if $f^{-1}(V)$ is generalized α-closed in (X, τ) for every open set V of (Y, σ).

6) a generalized b-continuous map [2] if $f^{-1}(V)$ is gb-open in (X, τ) for every open set V of (Y, σ).

7) a generalized αb-continuous map [9] if $f^{-1}(V)$ is $g\alpha b$-open in (X, τ) for every open set V of (Y, σ).

8) a generalized pre regular continuous map [6] if $f^{-1}(V)$ is gpr-open in (X, τ) for every open set V of (Y, σ).

9) a semi generalized b-continuous map [11] if $f^{-1}(V)$ is sgb-open in (X, τ) for every open set V of (Y, σ).

10) a regular generalized b-continuous map [10] if $f^{-1}(V)$ is rgb-open in (X, τ) for every open set V of (Y, σ).

3. On Generalized α Regular-Continuous Map

In this section, we introduce generalized α regular-continuous map ($g\alpha r$-continuous) in topological spaces by using the notions of $g\alpha r$-closed maps and study some of their properties.

Definition 3.1. Let X and Y be two topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is called generalized α regular-continuous map (briefly, $g\alpha r$-continuous map) if the image of every closed set in Y is $g\alpha r$-open in X.

Theorem 3.2. *If a map $f : (X, \tau) \to (Y, \sigma)$ from a topological space X into a topological space Y is continuous, then it is $g\alpha r$-continuous.*

Proof. Let V be an open set in Y. Since f is continuous, then $f^{-1}(V)$ is open in X. As every open set is $g\alpha r$-open, $f^{-1}(V)$ is $g\alpha r$-open in X. Therefore f is $g\alpha r$-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.3. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \}$ and $\sigma = \{Y, \phi, \{a\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b, f(b) = a, f(c) = c$, then f is $g\alpha r$-continuous but not continuously as the inverse image of an open set $\{a, c\}$ in Y is $\{b, c\}$ which is not open set in X.

Theorem 3.4. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha r$-continuous, then it is $g\alpha r$-continuous.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha r$-continuous. Let V be an open set in Y, Since f is $g\alpha r$-continuous then $f^{-1}(V)$ is $g\alpha r$-open. Hence every $g\alpha r$-open is $g\alpha r$-open in X. Therefore f is $g\alpha r$-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.5. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a, f(b) = c, f(c) = b$. the f is $g\alpha r$-continuous but not $g\alpha$-continuous as the inverse image of an open set $\{a, b\}$ in Y is $\{a, c\}$ which is not $g\alpha$-open set in X.

Theorem 3.6. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is αg-continuous then it is $g\alpha r$-continuous.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is αg-continuous. Let V be an open set in Y, Since f is αg-continuous then $f^{-1}(V)$ is αg-open. Hence every αg-open is $g\alpha r$-open in X. Therefore f is $g\alpha r$-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.7. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = b, f(c) = a$, then f is $g\alpha r$-continuous but not αg-continuously as the inverse image of an open set $\{a, c\}$ in Y is $\{a, c\}$ which is not αg-open set in X.

Theorem 3.8. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha r$-continuous, then it is gpr-continuous.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha r$-continuous. Let V be an open set in Y, Since f is $g\alpha r$-continuous then $f^{-1}(V)$ is $g\alpha r$-open. Hence every $g\alpha r$-open is *gpr*-open in X. Therefore f is *gpr*-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.9. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a, b\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$, then f is *gpr*-continuous but not $g\alpha r$-continuously as the inverse image of an open set $\{b, c\}$ in Y is $\{a, c\}$ which is not $g\alpha r$-open set in X.

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Theorem 3.10. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is g-continuous, then it is $g\alpha r$-continuous.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is g-continuous. Let V be an open set in Y, Since f is g-continuous, then $f^{-1}(V)$ is g-open. Hence every g-open is $g\alpha r$-open in X. Therefore f is $g\alpha r$-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.11. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = b, f(c) = a$, then f is $g\alpha r$-continuous but not g-continuous as the inverse image of an open set $\{b, c\}$ in Y is $\{b, c\}$ which is not g-open set in X.

Theorem 3.12. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is w-continuous, then it is $g\alpha r$-continuous.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is w continuous. Let V be an open set in Y, Since f is w continuous then $f^{-1}(V)$ is w-open. Hence every w-open is $g\alpha r$-open in X. Therefore f is $g\alpha r$-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.13. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b, f(b) = a, f(c) = c$, then f is $g\alpha r$-continuous but not w-continuous as the inverse image of an open set $\{a, c\}$ in Y is $\{b, c\}$ which is not w-open set in X.

Theorem 3.14. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is g^* continuous then it is $g\alpha r$-continuous.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is g^* continuous. Let V be an open set in Y, Since f is g^* continuous then $f^{-1}(V)$ is g^* open. Hence every g^* open is $g\alpha r$ open in X. Therefore f is $g\alpha r$ continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.15. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$, then f is g^*-continuous but not $g\alpha r$ continuous as the inverse image of an open set $\{b, c\}$ in Y is $\{a, c\}$ which is not $g\alpha r$ open set in X.

Theorem 3.16. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha^*$-continuous, then it is $g\alpha r$-continuous.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha^*$-continuous. Let V be an open set in Y, since f is $g\alpha^*$-continuous then $f^{-1}(V)$ is $g\alpha^*$-open. Hence every $g\alpha^*$-open is $g\alpha r$-open in X. Therefore f is $g\alpha r$-continuous. □

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The converse of above theorem need not be true as seen from the following example.

Example 3.17. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{b\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$, then f is $g\alpha r$-continuous but not $g\alpha r$-continuous as the inverse image of an open set $\{b\}$ in Y is $\{b\}$ is not $g\alpha r$-open set in X.

Theorem 3.18. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha r$ continuous, then it is rgb-continuous.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha r$ continuous. Let V be an open set in Y, Since f is $g\alpha r$-continuous then $f^{-1}(V)$ is $g\alpha r$-open. Hence every $g\alpha r$-open is rgb-open in X. Therefore f is rgb-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.19. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = b, f(c) = c$, then f is rgb-continuous but not $g\alpha r$-continuous as the inverse image of an open set $\{b, c\}$ in Y is $\{a, b\}$ which is not $g\alpha r$-open set in X.

Remark 3.20. The following examples show that $g\alpha r$-continuous and gb-continuous maps are independent.

Example 3.21. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a\}, \{b\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b, f(b) = a, f(c) = c$, then f is $g\alpha r$-continuous but not gb-continuous as the inverse image of an open set $\{b, c\}$ in Y is $\{a, c\}$ not gb-open set in X.

Example 3.22. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a, f(b) = b, f(c) = c$, then f is gb-continuous but not $g\alpha r$-continuous as the inverse image of $\{c\}$ in Y is $\{c\}$ which is not $g\alpha r$-open set in X.

Remark 3.23. The following examples show that $g\alpha r$-continuous and sgb-continuous maps are independent.

Example 3.24. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a, f(b) = c, f(c) = b$, then f is $g\alpha r$-continuous but not sgb-continuous as the inverse image of $\{b, c\}$ in Y is $\{b, c\}$ not sgb-open set in X.

Example 3.25. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$, then f is sgb-continuous but not $g\alpha r$-continuous as the inverse image of $\{c\}$ in Y is $\{b\}$ which is not $g\alpha r$-open set in X.

4. Applications

Theorem 4.1. *If a map $f : (X, \tau) \to (Y, \sigma)$ then*

(i) the following are equivalent

(a) f is $g\alpha r$-continuous

(b) The inverse image of open set in Y is $g\alpha r$-open in X.

(ii) If $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha r$-continuous, then $f(\alpha^*(A)) \subset cl(f(A))$ for every subset A of X

Proof. (i) Let us assume that $f : X \to Y$ be $g\alpha r$-continuous. Let F be open in Y. Then F^c is closed in Y. Since f is $g\alpha r$-continuous, $f^{-1}(F^c)$ is $g\alpha r$-closed in X. But $f^{-1}(F^c) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is $g\alpha r$-closed in X. So $f^{-1}(F)$ is $g\alpha r$-open in X. Hence (a) $\Rightarrow$ (b)
Conversely, let us assume that the inverse image of each open set in Y is $g\alpha r$ open in X. Let G be closed in Y. Then G^c is open in Y. By assumption $X - f^{-1}(G)$ is open in X. So $f^{-1}(G)$ is $g\alpha r$-closed in X. Therefore f is $g\alpha r$-continuous. Hence (b) $\Rightarrow$ (a). We have (a) and (b) are equivalent.
(ii) Let us assume that f is $g\alpha r$-continuous. Let A be any subset of X. Then $cl(f(A))$ is closed in Y. Since f is $g\alpha r$-continuous, $f^{-1}(cl(f(A)))$ is $g\alpha r$-closed in X and it contains A. But $\alpha^*(A)$ is the intersection of all α^* closed sets containing. Therefore $\alpha^*(A) \subset f^{-1}(cl(f(A)))$. So that $f(\alpha^*(A)) \subset cl(f(A))$. □

Theorem 4.2 (PASTING LEMMA for $g\alpha r$-continuous maps)**.** *Let $X = A \cup B$ be a topological space with topology τ and Y be a topological space with topology σ. Let $f : (A, \tau/A) \to (Y, \sigma)$ and $g : (B, \tau/B) \to (Y, \sigma)$ be $g\alpha r$-continuous map such that $f(x) = g(x)$ for every $x \in A \cup B$. Suppose that A and B are $g\alpha r$-closed sets in X, then $\alpha : (X, \tau) \to (Y, \sigma)$ is $g\alpha r$-continuous.*

Proof. Let F be any closed set in Y. Clearly $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F) = C \cup D$, where $C = f^{-1}(F)$ and $D = g^{-1}(F)$. But C is $g\alpha r$-closed in A and A is $g\alpha r$-closed in X. So C is $g\alpha r$-closed in X. Since we have prove the result, if $B \subseteq A \subseteq X$,, B is $g\alpha r$-closed in A and A is $g\alpha r$-closed in X, then B is $g\alpha r$-closed in X. Also $C \cup D$ is $g\alpha r$-closed in X. Therefore $\alpha^{-1}(F)$ is $g\alpha r$-closed in X. Hence α is $g\alpha r$-continuous. □

References

[1] Ahmad Al-Omari and Mohd. Salmi Md. Noorani, *On Generalized b-closed sets*, Bull. Malays. Math. Sci. Soc(2) 32(1) (2009), 19-30.

[2] M.Caldas and S.Jafari, *On some applications of b-open sets in topological spaces*, Kochi J.Math. 24(4) (1998), 681-688.

[3] Y.Gnanambal, *On generalized pre-regular closed sets in topological spaces*, Indian J.Pure Appl, Math 28 (1997), 351-360.

[4] D.Iyappan & N.Nagaveni, *On semi generalized b-closed set*, Nat. Sem. On Mat & Comp.Sci, Jan (2010), Proc.6

[5] N.Levine, *Generalized closed sets in topology*, Tend Circ., Mat. Palermo (2) 19 (1970), 89-96.

[6] N.Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly 70 (1963)), 36-41.

[7] H.Maki, R.Devi and K.Balachandran, *Associated topologies of generalized α-closed sets and α-generalized closed sets*, Mem. Fac. Sci. Kochi. Univ. Ser. A.Math. 15 (1994), 51-63.

[8] H.Maki, R.J.Umehara and T.Noiri, *Every topological space is pre-T 1/2* , Mem. Fac. Sci. Kochi. Univ. Ser. A. Math. 17(1996), 33-42.

[9] K.Mariappa and S.Sekar, *On regular generalized b-closed set*, Int. Journal of Math. Analysis, 7(13), (2013), 613624.

[10] K.Mariappa and S.Sekar, *On regular generalized b-continuous map in Topological Spaces*, Kyungpook Mathematical Journal, Vol.54, Issue 3, (2014), 477-483.

[11] O.Njastad, *On some classes of nearly open sets*, Pacific J Math., 15(1965), 961-970.

[12] J.H.Park, Y.B.Park and B.Y.Lee, *On gp-closed sets and gp-continuous functions*, Indian J. Pure Appl. Math., 33(1) (2002), 3-12.

[13] S.Sekar and G. Kumar, *On $g\alpha r$-closed set in Topological Spaces*, International Journal of Pure and Applied Mathematics, vol. 108, no.4, (2016), 791-800.

[14] S.Sekar and K.Mariappa, *On regular generalized b-Closed Map in Topological Spaces*, International Journal of Mathematical Archive (IJMA), Vol.4, Issue 8, (2013), 111-116.

[15] K.C.Rao and K.Joseph, Bull. Pure Appl. Sci., 19(E)(2), (2002), 281.

[16] S.Sekar and G. Kumar, *On $g\alpha r$-closed map in Topological Spaces*, International Journal of Pure and Applied Mathematics.

[17] M.Sheik john, *On w-closed sets in topology*, Acta Ciencia Indica, 4(2000), 389-392.

[18] L.Vinayagamoorthi and N.Nagaveni, *On Generalized αb-closed set*, Proceeding ICMD-Allahabad, Pusbha Publication, 1 (2011).

[19] Veerakumar.M.K.R.S., *Between closed sets and g-closed sets*, Mem.Fac.Sci.Kochi.Univ.Ser.A, Math, 21 (2000), 1-19.

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