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EFFECT OF ROTATION, A RADIAL MAGNETIC FIELD AND A TEMPERATURE GRADIENT ON THE DIFFUSION IN A BINARY MIXTURE

Pravin Kumar Srivastava

Department of Applied Science and Humanities
Bundekhand Institute of Engineering and Technology
Jhansi, 284128, INDIA

Abstract: The diffusion of a binary mixture of incompressible viscous fluids filling the gap between two co- axial circular cylinders, the inner one rotating and the outer one being at rest, has been discussed. The cylinders are maintained at constant temperatures, the temperature of the outer cylinder being greater than that of the inner one. The separation of the components of the fluid due to (i) the pressure gradient created by the rotation of the cylinder, (ii) an applied radial magnetic field and (iii) the temperature gradient has been studied. The solution of the diffusion equation reveals that the effect of the pressure gradient is to separate the two components of the mixture in a manner so that the heavier and more abundant component is deposited at the outer cylinder. It is observed that the presence of the radial magnetic field reduces this effect of separation and so the application of the magnetic field is not useful for this purpose. It is concluded that the presence of the temperature gradient enhances the separation effect.

AMS Subject Classification: 76R50, 76W05
Key Words: the rotation of the cylinder, temperature gradient, radial magnetic field and separation of components

1. Introduction

Consider a mixture of two components of a fluid, the composition of one of which is described by the concentration C_1 defined as the ratio of mass of that component to the total mass of the fluid in a given volume element. In the flow of such a

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mixture the diffusion of such a species take place by three mechanisms, namely the concentration gradient, the pressure gradient and the temperature gradient. The diffusion flux density **i** for the rarer species under the effect of magnetic field is given by Landau and Lifshitz [1] as

$$\mathbf{i} = \beta\,\mathbf{j} - \rho D[grad\,C_1 + k_T grad\,T + k_p grad\,p] \tag{1}$$

where D is the diffusion or the mass transfer coefficient, $k_T D$ is the thermal diffusion coefficient , $k_p D$ is the barodiffusion coefficient, **j** is the current density and β is constant representing the electrical characteristics of the fluid. The last term is important when there is a considerable pressure gradient in the fluid. Sarma [2] and Srivastava [3] have discussed the barodiffusion in the binary mixture of the viscous fluids due to an axial pressure gradient created by the rotation of a disk under isothermal conditions. Srivastava [4] has discussed the effect of a pressure gradient created by the rotation of circular cylinder under isothermal conditions. Srivastava and Kalita [5] have also discussed the effect of a radial magnetic field on the separation of the binary mixture under isothermal conditions. Hurle and Jakeman [6] have discussed the effect of a temperature gradient on the diffusion of a binary mixture in the absence of pressure gradient.

In this paper we have discussed the diffusion in a binary mixture of incompressible vicious fluids filling the space between two coaxial circular cylinders. The inner cylinder rotates with constant angular velocity and the outer cylinder is at rest. The inner and outer cylinders are maintained at constant temperatures $T_1$and $T_2(T_2 > T_1)$ respectively. Further, the concentration $C_1$of the lighter and rarer component of the mixture is maintained at a constant value C_0 at the outer cylinder while the inner one is considered as impervious. A radial magnetic field $H_r = A/r$,A where is constant, is applied. Thus, in this paper we discuss the diffusion of the rarer component under mechanisms of the concentration gradient, the temperature gradient, the pressure gradient and the applied magnetic field.

2. Mass Transfer Equations

We consider here the case when one of the components is present in a small quantity hence the density; viscosity and the other material constants of the mixture are independent of the distribution of the components. The flow problem of the binary mixture is identical to that of a single fluid but the velocity is understood as the mass average velocity $\mathbf{v} = (\rho_1\mathbf{v}_1 + \rho_2\mathbf{v}_2)/\rho$ and the density $\rho = \rho_1 + \rho_2$, where the subscripts 1 and 2 respectively denote the rarer and more abundant components in the binary mixture. The steady equations for the steady magneto-hydrodynamic flow are

$$(\mathbf{v}.\nabla)\,\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu\,\nabla^2\mathbf{v} + \frac{\mu}{\rho}\mathbf{j}\times\mathbf{H} \tag{2}$$

$$\mathbf{j} = \mu\sigma\left(\mathbf{v} \times \mathbf{H}\right) \tag{3}$$

$$\nabla.\mathbf{v} = 0, \quad \nabla.\mathbf{H} = 0, \quad \nabla.\mathbf{j} = 0 \tag{4}$$

where $\mathbf{H}$ is the magnetic field, $\mathbf{j}$ is the electric current density, ν is the kinematic coefficient of viscosity, μ is the magnetic permeability and $\sigma$is the conductivity of the fluid. Here we have taken the electric field $\mathbf{E} = \mathbf{0}$. The equation governing the temperature is given by

$$k\nabla^2 T + \Phi = 0, \tag{5}$$

where k is the thermal conductivity of the fluid andΦ is the viscous dissipation function. The additional equation for the species conservation is given by

$$\rho\left(\mathbf{v}.\nabla\right)C_1 = -\nabla.\mathbf{i} \tag{6}$$

Substituting$\mathbf{i}$ from Eq.(1) into Eq. (6), we get the following equation for C_1 as

$$\rho\left(\mathbf{v}.\nabla\right)C_1 = \rho D[\nabla^2 C_1 + \nabla.(k_T gradT) + \nabla.(k_p gradp)] \tag{7}$$

Landau and Lifshitz [1] have suggested an expression for $k_p$as

$$k_p = (m_2 - m_1)\left(\frac{C_1}{m_1} - \frac{C_2}{m_2}\right)\frac{C_1 C_2}{p_\infty}, \tag{8}$$

where p_∞ denote the pressure in the working medium, m_1 and m_2 are masses of two kinds of the particles and C_2 is the concentration of the second component given by $C_2 = 1 - C_1$.Assuming C_1 to be small, we get the following equation fork_p

$$k_p = \frac{(m_2 - m_1)C_1}{m_2 p_\infty} \tag{9}$$

The expression fork_T has been suggested by Hurle and Jakeman[6] is

$$k_T = S_T C_1(1 - C_1), \tag{10}$$

where S_T is called Soret coefficient. Again assuming C_1 to be small we can write the following expression for the coefficientk_T as

$$k_T = S_T C_1. \tag{11}$$

3. Boundary Conditions

The boundary conditions on C_1 are different in different cases. At the solid surface of the body insoluble in the fluid, the mass flux of the rarer component of the mixture normal to the surface is zero. This can be written as

$$\rho \mathrm{v}_n C_1 - \rho D \left[\frac{\partial C_1}{\partial n} + k_T \frac{\partial T}{\partial n} + k_p \frac{\partial p}{\partial n} \right] + \beta j_n = 0, \tag{12}$$

at the surface, where v_n and j_n are normal components of the fluid velocity and current density respectively and $\frac{\partial}{\partial n}$denotes the derivative in the direction normal to the surface. The first part in the equation represents the convective flux and the second part, which is in the parenthesis, denotes the diffusion flux. If, however there is diffusion from a body which dissolve in the fluid, equilibrium is rapidly established near its surface and the concentration in the fluid adjoining the body is the saturation constant C_0 . The boundary condition at the surface is

$$C_1 = C_0. \tag{13}$$

4. Formulation of the Problem

We consider the flow of a binary mixture of incompressible viscous fluids filling the space between two coaxial circular cylinders $r = a$ and $r = b(b > a)$. The cylinder$r = a$ rotates with a constant angular velocity and the cylinder $r = b$ is at rest. The cylinders $r = a$ and $r = b$ are maintained at constant temperatures T_1 and T_2 ($T_2 > T_1$) respectively. A radial magnetic field $H_r = A/r$ is applied on the whole system where A is a constant given by $H_0 a$, H_0 is the value of H_r at $r = a$. Further the concentration C_1 of the first component is maintained at a constant value C_0 at$r = b$ and $r = a$ is taken as impervious. Taking the cylindrical polar coordinates (r, θ, z), z-axis coinciding with the axis of the cylinder, the only non-vanishing component of the velocity will be v_θ in the direction of θ. The conditions on the velocity and temperature are given by

$$\mathrm{v}_\theta = a\Omega, \quad T = T_1 \text{ at } r = a;\ \mathrm{v}_\theta = 0\ v = 0,\ T = T_2 \text{ at } r = b. \tag{14}$$

The equation of continuity is identically satisfied in this problem. The equations of motions (2) in this case can be written as

$$\frac{dp}{dr} = \rho \frac{\mathrm{v}_\theta^2}{r} \tag{15}$$

$$\frac{d^2 \mathrm{v}_\theta}{dr^2} + \frac{1}{r}\frac{d\mathrm{v}_\theta}{dr} - \frac{\mathrm{v}_\theta}{r^2} + \frac{j_z}{\rho} H_r = 0 \tag{16}$$

where p is the pressure. Neglecting the effect of induced magnetic field, we get from Eq. (3)

$$j_r = j_\theta = 0 \text{ and } j_z = -\sigma \, \mathrm{v}_\theta H_r. \tag{17}$$

Also Eq. (5) reduces to

$$k\left(\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right) = -\mu\left(\frac{d\mathrm{v}_\theta}{dr} - \frac{\mathrm{v}_\theta}{r}\right)^2 \tag{18}$$

Substituting j_z from Eq. (17) in Eq. (16) and taking $H_r = \frac{A_0}{r}$, we get

$$r^2\frac{d^2\mathrm{v}_\theta}{dr^2} + r\frac{d\mathrm{v}_\theta}{dr} - \left(1 + \frac{\sigma A_0^2}{\rho\nu}\right)\mathrm{v}_\theta = 0 \tag{19}$$

Assuming that Ω and $(T_2 - T_1)$ to be small such that the flow is laminar and stable, the solutions of the Eqs. (15), (18) and (19) are given by

$$\frac{dP}{dy} = \frac{V^2}{y} \tag{20}$$

$$V(y) = \frac{y_1^{2N} - y^{2N}}{y^N(y_1^{2N} - 1)} \tag{21}$$

$$T*(y) = \frac{(\lambda - B)\log y}{\lambda \log y_1} + \frac{B}{\lambda^2}\frac{(y^2 - 1)}{y^2} \tag{22}$$

where $V = \frac{\mathrm{v}_\theta}{a\Omega}$, $P = \frac{p}{\rho a^2\Omega^2}$, $T* = \frac{T-T_1}{T_2-T_1}$, $y = \frac{r}{a}$, $y_1 = \frac{b}{a}$, $B = \frac{\nu a^2\Omega^2\rho}{k}$, $\lambda = \frac{b^2-a^2}{a^2}$, $N^2 = 1 + M^2$, $M^2 = \frac{H_0^2 a^2\sigma}{\rho\nu}$ = square of Hartmann Number

Let C_0 denote the undisturbed concentration of the first component of the fluid at the outer cylinder. It is obvious that C_1, the concentration of the first component at any

point of the fluid will be function of y only i.e.

$$C_1 = C_0 f(y) \tag{23}$$

with the condition that

$$f(y) = 1 \text{ at } y = y_1. \tag{24}$$

Substituting $C_1$from Eq. (23) in Eq. (6), we get

$$\frac{d}{dy}\left(y\frac{df}{dy}\right) + S\left\{\frac{d}{dy}\left(yf\frac{dT*}{dy}\right)\right\} + \beta\left\{\frac{d}{dy}\left(yf\frac{dP}{dy}\right)\right\} = 0 \tag{25}$$

where$\beta = \frac{(m_2-m_1)a^2\Omega^2\rho}{m_2 p_\infty}$ and $S = (T_2 - T_1)S_T$

The non-dimensional number β is called the coefficient of baro-diffusion and is small since Ω as well as $(m_2 - m_1)$ is small. The number S which is known as Soret number or thermal diffusion number is also small since S_T is very small (see, Hurle & Jakeman[6]).

At the inner cylinder $\mathbf{v}.\mathbf{n} = 0$ and $j_n = 0$, hence the boundary condition (12) at the inner cylinder becomes

$$\frac{df}{dy} + Sf\frac{dT*}{dy} + \beta f\frac{dP}{dy} = 0 \text{ at } y = 1. \tag{26}$$

5. Solution of the Diffusion Equations and Discussions

Substituting $\frac{dP}{dy}$ from Eq. (20) and $T*$ from Eq. (22) in Eq. (25), the solution of Eq. (25) under boundary conditions (24) and (26) is given by

$$f(y) = \exp\left[-S\left\{\left(1-\frac{B}{\lambda}\right)\frac{\log y}{\log y_1} + \frac{B}{\lambda^2}\left(1-\frac{1}{y^2}\right) - 1\right\} - \frac{\beta}{2Ny^{2N}(y_1^{2N}-1)^2}\left\{y^{4N} - y_1^{4N} - 4Ny_1^{2N}y^{2N}\log\frac{y}{y_1}\right\}\right]. \tag{27}$$

The value of $f(y)$ at $y = 1$ is given by

$$f(1) = \exp\left\{S + \frac{\beta}{2N(y_1^{2N}-1)^2}\left(y_1^{4N} - 4Ny_1^{2N}\log(y_1) - 1\right)\right\} \tag{28}$$

The value of $N > 1$ and $y_1 > 1$ which gives that the value of the expression

$$\frac{y_1^{4N} - 4Ny_1^{2N}\log(y_1) - 1}{2N(y_1^{2N}-1)^2} > 0$$

Moreover, $\beta > 0$ since $m_2 > m_1$. Hurle and Jakeman [6] have found that $S_T > 0$ for most of the fluids. Here we have assumed that $(T_2 - T_1) > 0$, hence $S > 0$. Under these conditions Eq. (28) gives that $f(1) > 1$ i.e. the concentration of the first component, which is lighter and rarer, at the inner cylinder is more than it is maintained at the outer one. Also we have $C_1 + C_2 = 1$ which shows that the second component of the binary mixture which is heavier gets collected at the outer cylinder. Hence we conclude that the effect of radial pressure gradient as well as the radial temperature gradient is to gather the lighter component of the fluid near the inner cylinder and to throw away the heavier component to the outer cylinder i.e. to affect the separation of two component of the mixture. If we neglect the effects of baro-diffusion and temperature gradient i.e. put $\beta = 0$ and $S = 0$ in Eq. (27), we get $f(y) = 1$, which implies that the concentration C_1 of the lighter and rarer

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$y_1\backslash N$	1.0	1.5	2.0	2.5	3.0
2.0	1.0744	1.0727	1.0709	1.0689	1.0671
3.0	1.0850	1.0802	1.0756	1.0718	1.0687
4.0	1.0911	1.0832	1.0769	1.0723	1.0683
5.0	1.0944	1.0846	1.0774	1.0724	1.0689

Table 1: The values of $f(1)$ for $S = 0.05$ and $\beta = 0.10$

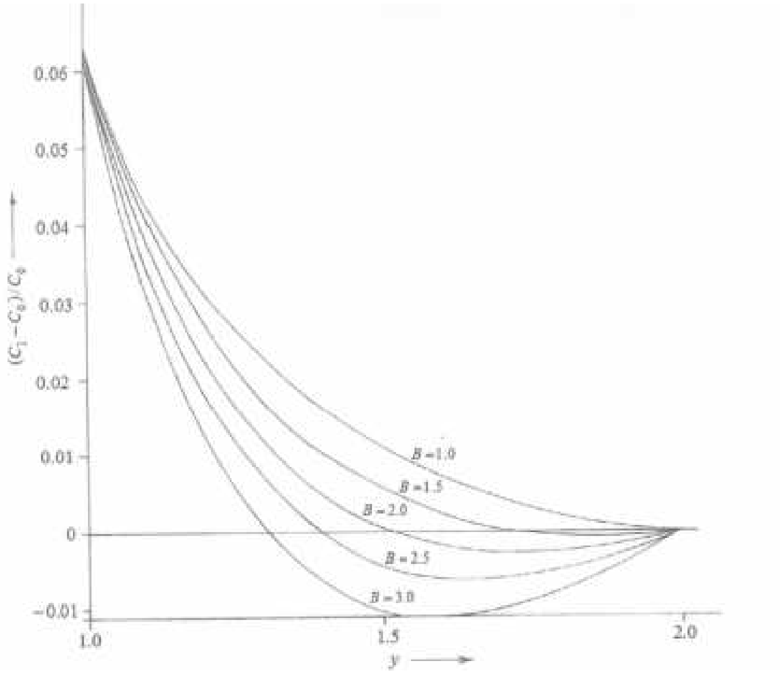
$N\backslash\beta$	0.01	0.02	0.03	0.04	0.05
1.0	1.0535	1.0558	1.0581	1.0604	1.0627
2.0	1.0532	1.0552	1.0571	1.0590	1.0610
3.0	1.0528	1.0544	1.0560	1.0576	1.0592
4.0	1.0525	1.0538	1.0551	1.0563	1.0576
5.0	1.0523	1.0533	1.0544	1.0554	1.0565

Table 2: The Values of $f(1)$ for $S = 0.05$ and $y_1 = 2.0$

component is undisturbed through out the entire region of the flow and separation of two species will not take place.

The expression of $f(1)$ given in Eq. (28) shows that $f(1)$ is proportional toe^Swhich implies that the separation effect increases with increase of the Soret number$S$. Taking $S = 0.05$, $\beta = 0.10$, the values of $f(1)$ are given in Table-1 for $y_1 = 2.0$, 3.0, 4.0, 5.0 and $N = 1.0$, 1.5, 2.0, 2.5, 3.0. The table shows that the separation effects increases with the increase of y_1 , that is, the gap between two cylinders but decreases with increase of N , that is, the strength of radial magnetic field. Also taking $S = 0.05$ and $y_1 = 2.0$, we have given the values of $f(1)$ in Table –2 for the values of $\beta = 0.01$, 0.02, 0.03, 0.04, 0.05 and $N = 1.0$, 2.0, 3.0, 4.0, 5.0. It shows that the effect of separation increases with the increase of β, that is, with the increase in the difference of masses of two components. Also in this case the separation effect decreases with the increase of N Hence we conclude that the effect of temperature gradient and pressure gradient helps the separation of two components of the fluid but the application of radial magnetic field is not useful for this purpose.

Taking $S = 0.05$, $y_1 = 2.0$, $\beta = 0.05$, $N = 1.0$ the graph of $f(y)$ is plotted in Figure 1 against y for $B = 1.0$, 1.5, 2.0, 2.5, 3.0. The graph shows that the separative effect is maximum at the inner rotating cylinder and it decreases with the increase of the distance from the surface of inner rotating cylinder for all values of B. Since $B \sim (\mu/k)$, the graph reveals that the effect of separation decreases with the increase of the ratio of the coefficient of viscosity to the coefficient of thermal conductivity. From this graph it is clear that the most of the lighter component of the mixture is deposited in the narrow region near the rotating cylinder and so boundary layer is



formed near the rotating cylinder, the thickness of which decreases with the increase in the values of B. There are surfaces at $y = 1.75$, 1.52, 1.40 and 1.31 respectively for $B = 1.5$, 2.0, 2.5 and 3.0 between the two cylinders at which the concentration of the lighter component becomes same as that maintained at the outer cylinder and then further decreases. In between these surfaces and outer cylinder the concentration of the heavier component is more. These surfaces of undisturbed concentration shift towards the inner cylinder with the increase in the values of B.

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References

[1] L.D. Landau, E.M. Lifshitz, *Fluid Mechanics*, 2-nd edition, Pergamon Press (1987).

[2] G.S.R. Sarma, Barodiffusion in a binary mixture at a rotating disk, *Z.A.M.P.*, **24** (1973), 789-800.

[3] A.C. Srivastava, Barodiffusion in a binary mixture confined between two disks, *Mathematical Forum*, **2**, No. 2 (1979), 16-21.

[4] A.C. Srivastava, Barodiffusion in a binary mixture due to rotation of a circular cylinder, *J. Math. and Physical Science*, **16**, No. 5 (1982), 463-467.

[5] A.C. Srivastava, C. Kalita, Barodiffusion in a binary mixture of electrically conducting viscous fluids confined between two rotating concentric cylinders, *Mathematical Forum*, **4**, No. 2 (1981), 1-11.

[6] D.T.J. Hurle, Jakeman, Soret-driven thermosolutal convention, *J. Fluid Mech.*, **47** (1971), 667-687.