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ON PRE GENERALIZED b-CONTINUOUS MAP IN TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce a new class of pre generalized b-continuous map and study some of their properties as well as inter relationship with other continuous maps.

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1. Introduction

Continuous map was studied for different types of closed sets by various researchers for past many years. In 1996, Andrijevic [3, 4] introduced new type called b-open sets. A.A.Omari and M.S.M. Noorani [2] were introduced and studied b-continuous map and b-closed map.

The aim of this paper is to continue the study of pre generalized b-continuous map, pre generalized b-closed map have been introduced and studied their relations with various generalized closed maps. Through out this paper (X, τ) and (Y, σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

Let $A \subseteq X$, the closure of A and interior of A will be denoted by $cl(A)$ and

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$int(A)$ respectively, union of all b-open sets X contained in A is called b-interior of A and it is denoted by $bint(A)$, the intersection of all b-closed sets of X containing A is called b-closure of A and it is denoted by $bcl(A)$.

2. Preliminaries

Definition 2.1. Let a subset A of a topological space (X, τ), is called

1) a pre-open set [15] if $A \subseteq int(cl(A))$.

2) a semi-open set [11] if $A \subseteq cl(int(A))$.

3) a α -open set [16] if $A \subseteq int(cl(int(A)))$.

4) a b-open set [4] if $A \subseteq cl(int(A)) \cup int(cl(A))$.

5) a generalized closed set (briefly g-closed)[10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

6) a generalized α- closed set (briefly $g\alpha$- closed) [12] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.

7) a generalized b-closed set (briefly gb-closed) [2] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

8) a generalized $\alpha*$-closed set (briefly $g\alpha*$- closed) [13] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.

9) a pre generalized-closed set (briefly pg- closed) [19] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre-open in X.

10) a semi generalized closed set (briefly sg- closed) [6] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

11) a generalized αb-closed set (briefly $g\alpha b$-closed) [20] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.

12) a regular generalized b-closed set (briefly rgb- closed) [14] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

13) a pre generalized b-closed set (briefly pgb- closed) [16] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre open in X.

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3. On Pre Generalized b-Continuous Map

In this section, we introduce pre generalized b-continuous map (pgb-continuous) in topological spaces by using the notions of pgb-closed maps and study some of their properties.

Definition 3.1. Let X and Y be two topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is called pre generalized b-continuous map (briefly, pgb-continuous map) if the image of every closed set in Y is pgb-open in X.

Theorem 3.2. *If a map $f : (X, \tau) \to (Y, \sigma)$ from a topological space X into a topological space Y is continuous, then it is pgb-continuous but not conversely.*

Proof. Let V be an open set in Y. Since f is continuous, then $f^{-1}(V)$ is open in X. As every open set is pgb-open, $f^{-1}(V)$ is sg^*b-open in X. Therefore f is pgb-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.3. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a, f(b) = b, f(c) = c$, then f is pgb-continuous but not continuously as the inverse image of an open set $\{b, c\}$ in Y is $\{b, c\}$ which is not open set in X.

Theorem 3.4. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is b-continuous, then it is pgb-continuous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is b-continuous. Let V be an open set in Y, Since f is b-continuous then $f^{-1}(V)$ is b-open. Hence every b-open is pgb-open in X. Therefore f is pgb-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.5. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$. the f is pgb-continuous but not b-continuous as the inverse image of an open set $\{a, c\}$ in Y is $\{a, b\}$ which is not b-open set in X.

Theorem 3.6. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha$-continuous then it is pgb-continuous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is α-continuous. Let V be an open set in Y, Since f is $g\alpha$-continuous then $f^{-1}(V)$ is $g\alpha$-open. Hence every $g\alpha$-open is pgb-open in X. Therefore f is pgb-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.7. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$, then f is pgb-continuous but not $g\alpha$-continuously as the inverse image of an open set $\{a, b\}$ in Y is $\{a, c\}$ which is not $g\alpha$-open set in X.

Theorem 3.8. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha*$-continuous, then it is pgb-continuous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha*$-continuous. Let V be an open set in Y, Since f is $g\alpha*$-continuous then $f^{-1}(V)$ is $g\alpha*$ open. Hence every $g\alpha*$ open is pgb-open in X. Therefore f is pgb-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.9. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b, f(b) = a, f(c) = c$, then f is pgb-continuous but not $g\alpha*$-continuously as the inverse image of an open set $\{a, b\}$ in Y is $\{a, b\}$ which is not $g\alpha*$-open set in X.

Theorem 3.10. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is g-continuous, then it is pgb-continuous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is g-continuous. Let V be an open set in Y, Since f is g-continuous, then $f^{-1}(V)$ is g-open. Hence every g-open is pgb-open in X. Therefore f is pgb-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.11. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$, then f is pgb-continuous but not g-continuous as the inverse image of an open set $\{a, c\}$ in Y is $\{a, b\}$ which is not g-open set in X.

Theorem 3.12. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha b$- continuous, then it is pgb-continuous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha b$-continuous. Let V be an open set in Y, Since f is $g\alpha b$-continuous then $f^{-1}(V)$ is $g\alpha b$-open. Hence every $g\alpha b$-open is pgb-open in X. Therefore f is pgb-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.13. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = b, f(c) = a$, then f is $g\alpha b$-continuous but not pgb-continuous as the inverse image of an open set $\{a, b\}$ in Y is $\{b, c\}$ which is not $g\alpha b$-open set in X.

Theorem 3.14. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is rgb continuous then it is pgb-continuous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is rgb continuous. Let V be an open set in Y, Since f is rgb continuous then $f^{-1}(V)$ is rgb open. Hence every rgb open is pgb-open in X. Therefore f is pgb continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.15. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$, then f is pgb continuous but not rgb-continuous as the inverse image of an open set $\{a, b\}$ in Y is $\{b, c\}$ which is not rgb-open set in X.

Theorem 3.16. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is pgb-continuous, then it is pg-continuous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is pgb-continuous. Let V be an open set in Y, since f is pgb-continuous then $f^{-1}(V)$ is pgb-open. Hence every pgb-open is pg-open in X. Therefore f is pg-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.17. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$, then f is pg-continuous but not pgb-continuous as the inverse image of an open set $\{a, c\}$ in Y is $\{b, c\}$ is not pgb-open set in X.

Theorem 3.18. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is sg continuous, then it is pgb-continuous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is sg continuous. Let V be an open set in Y, Since f is sg continuous then $f^{-1}(V)$ is sg open. Hence every sg open is pgb open in X. Therefore f is pgb-continuous. □

The converse of above theorem need not be true as seen from the following example.

Example 3.19. Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a, f(b) = c, f(c) = b$, then f is pgb-continuous but not sg-continuous as the inverse image of an open set $\{b, c\}$ in Y is $\{b, c\}$ which is not sg-open set in X.

4. Applications

Theorem 4.1. *If a map $f : (X, \tau) \to (Y, \sigma)$ then*

(i) the following are equivalent

(a) f is pgb-continuous

(b) The inverse image of open set in Y is pgb-open in X.

(ii) If $f : (X, \tau) \to (Y, \sigma)$is pgb-continuous, then $f(b^*(A)) \subset cl(f(A))$ for every subset A of X

Proof. (i) Let us assume that $f : X \to Y$ be pgb -continuous. Let F be open in Y. Then F^c is closed in Y. Since f is pgb-continuous, $f^{-1}(F^c)$ is pgb-closed in X. But $f^{-1}(F^c) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is pgb-closed in X. So $f^{-1}(F)$ is pgb-open in X. Hence (a) $\Rightarrow$ (b)

Conversely, let us assume that the inverse image of each open set in Y is pgb open in X. Let G be closed in Y. Then G^c is open in Y. By assumption $X - f^{-1}(G)$ is open in X. So $f^{-1}(G)$ is pgb-closed in X. Therefore f is pgb-continuous. Hence (b) $\Rightarrow$ (a). We have (a) and (b) are equivalent.

(ii) Let us assume that f is pgb-continuous. Let A be any subset of X. Then $cl(f(A))$ is closed in Y. Since f is pgb-continuous, $f^{-1}(cl(f(A)))$ is pgb- closed in X and it contains A. But $b^*(A)$ is the intersection of all b^* closed sets containing. Therefore $b^*(A) \subset f^{-1}(cl(f(A)))$. So that $f(b^*(A)) \subset cl(f(A))$. □

Theorem 4.2. *Let X and Y be topological spaces, $V \subseteq Y$ and $f : X \to Y$. Then f is pgb-continuous if and only if V is closed in Y implies is pgb closed in X.*

Proof. Necessary part : Let $f : X \to Y$ be pgb-continuous. Let $V \subseteq Y$ be closed. Then $V^C \subseteq Y$ is open in Y. Since $f : X \to Y$ is pgb-continuous, $f^{-1}(V)$ is pgb-open in X. Hence $f^{-1}(V^C) = [f^{-1}(V)]^C$ is pgb-closed in X.

Sufficiency part: Suppose that V is closed in Y implies that $f^{-1}(V)$ is pgb-closed in X. Let A be open in Y. Then A^C is closed in Y. By our assumption, $f^{-1}(A^C) = [f^{-1}(V)]^C$ is pgb-closed in X. Consequently, $f^{-1}(A)$ is pgb-open in X. Hence the proof. □

Theorem 4.3 (PASTING LEMMA for pgb-continuous maps)**.** *Let $X = A \cup B$ be a topological space with topology τ and Y be a topological space with topology σ. Let $f : (A, \tau/A) \to (Y, \sigma)$ and $g : (B, \tau/B) \to (Y, \sigma)$ be pgb-continuous map such that $f(x) = g(x)$ for every $x \in A \cup B$. Suppose that A and B are pgb-closed sets in X, then $\alpha : (X, \tau) \to (Y, \sigma)$ is pgb-continuous.*

Proof. Let F be any closed set in Y. Clearly $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F) = C \cup D$, where $C = f^{-1}(F)$ and $D = g^{-1}(F)$. But C is pgb-closed in A and A is pgb-closed in X. So C is pgb-closed in X. Since we have prove the result, if $B \subseteq A \subseteq X$, B is pgb-closed in A and A is pgb-closed in X, then B is pgb-closed in X. Also $C \cup D$ is pgb-closed in X. Therefore $\alpha^{-1}(F)$ is pgb-closed in X. Hence α is pgb-continuous. □

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