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PERIODIC SOLUTION FOR THE ALLEN-CAHN EQUATION THROUGH THE ADOMIAN DECOMPOSITION METHOD

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Abstract: In this paper, we apply the Adomian Decomposition Method (ADM) for finding the approximate solution of the Allen-Cahn equation which arises to describe the motion of anti-phase boundaries in crystalline solids. The equation under consideration include periodic coefficients. The results obtained by this way have been compared with the numerical simulations to show that the Adomian method is a powerful method for the solution of nonlinear differential equations.

AMS Subject Classification: 35J61, 35J40
Key Words: Allen-Cahn equation, nontrivial periodic solutions, Adomian decomposition method

1. Introduction

In recent years a semi-analytical method named Adomian decomposition method proposed by G. Adomian (1923-1990) has been attracting the attention of many mathematicians, physicists and engineers. Many mathematical modeling which explain natural phenomena are usually formulated in terms of nonlinear partial and ordinary differential equations. However, most of the methods developed in mathematics are used for solving linear differential equations. Since then, this method is known as the Adomian decomposition method (ADM) [4], [5]. The technique

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International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 10, No. 2 (2016)

is based on a decomposition of a solution of a nonlinear differential equation in a series of functions. Each term of the series is obtained from a polynomial generated by a power series expansion of an analytic function. The Adomian method is very simple in an abstract formulation but the difficulty arises in calculating the polynomials that becomes a non-trivial task. This method has widely been used to solve equations that come from nonlinear models as well as to solve fractional differential equations [10], [19] and references therein.

In the presente work we will utilize the Adomian decomposition method to solve the Allen-Cahn equation in the case of periodic coefficients. This equation is a nonlinear partial differential equation that, in materials science, is used to describe the motion of anti-phase boundaries in crystalline solids [1]. There are many and varied work on the Allen-Cahn equation for example, see [7], [20], [11], [23], [18], [22], [14] and references therein. Also, recently have been made research on periodic solutions to the Allen-Cahn equation, and these were reported in [17] and [16]. Also, The fractional Allen-Cahn equation with constant coefficients has been resolved in [15].

Our work is divided in three sections. In "The Adomian Decomposition Method (ADM)" section, we present, in a brief and self-contained manner, the ADM. Several references are given to delve deeper into the subject and to study its mathematical foundation that is beyond the scope of the present work. In "The Allen-Cahn Equation" section, we give a brief introduction to the model described by the Allen-Cahn equation and we will establish that ADM can be use to solve this equation in its nonlinear version. Besides, we will show through an example, the quality and precision of our method by comparison with numerical simulations computed using the Mathematica package. Finally, in the "Conclusion" section, we summarise our findings and present our final conclusions.

2. The Adomian Decomposition Method (ADM)

The ADM is a method to solve ordinary and nonlinear differential equations. Using this method is possible to express analytic solutions in terms of a series [5]. In a nutshell, the method identifies and separates the linear and nonlinear parts of a differential equation. Inverting and applying the highest order differential operator that is contained in the linear part of the equation, it is possible to express the solution in terms of the rest of the equation affected by the inverse operator. At this point, the solution is proposed by means of a series with terms that will be determined and that give rise to the Adomian Polynomials [21]. The nonlinear part can also be expressed in terms of these polynomials. The initial (or the border conditions) and the terms that contain the independent variables will be considered as the initial approximation. In this way and by means of a recurrence relations, it is possible to find the terms of the series that give the approximate solution of the differential equation.

We will consider a general nonlinear partial differential equation in the following form

$$\begin{cases} L_t u(x,t) + Ru(x,t) + Nu(x,t) = g(x,t), \\ u_0(x,t) = f(x). \end{cases} \tag{2.1}$$

where $L = \frac{\partial}{\partial t}$, R is a linear operator that includes partial derivatives with respect to x, N is a nonlinear operator and g is a non-homogeneous term that is u-independent.

Solving for $Lu(x,t)$, we have

$$Lu(x,t) = g(x,t) - Ru(x,t) - Nu(x,t). \tag{2.2}$$

Appling the inverse operator $L^{-1} = \int_0^t (\cdot) d\hat{t}$ to both sides of (2.2) and using the given the initial conditions we obtain

$$u(x,t) = f(x) + L^{-1}(g(x,t)) - L^{-1}\big[Ru(x,t) + Nu(x,t)\big]. \tag{2.3}$$

The ADM method proposes a series solution $u(x,t)$ given by,

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{2.4}$$

The nonlinear term $Nu(x,t)$ is given by

$$Nu(x,t) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n) \tag{2.5}$$

where $\{A_n\}_{n=0}^{\infty}$ is the so-called Adomian polynomials sequence established in [21] and [6] and, in general, give us term to term:

$A_0 = N(u_0)$

$A_1 = u_1 N'(u_0)$

$A_2 = u_2 N'(u_0) + \frac{1}{2} u_1^2 N''(u_0)$

$A_3 = u_3 N'(u_0) + u_1 u_2 N''(u_0) + \frac{1}{3!} u_1^3 N^{(3)}(u_0)$

$A_4 = u_4 N'(u_0) + (\frac{1}{2} u_2^2 + u_1 u_3) N''(u_0) + \frac{1}{2!} u_1^2 u_2 N^{(3)}(u_0) + \frac{1}{4!} u_1^4 N^{(4)}(u_0)$

$\vdots$

Other polynomials can be generated in a similar way. For more details, see [21] and [6] and references therein. Some other approaches to obtain Adomian's polynomials can be found in [12] and [13].

Using (2.4) and (2.5) into equation (2.3), we obtain,

$$\sum_{n=0}^{\infty} u_n(x,t) = f(x) + L^{-1}(g(x,t)) - L^{-1}\Big[R\Big(\sum_{n=0}^{\infty} u_n(x,t)\Big) + \sum_{n=0}^{\infty} A_n(u_0, \ldots, u_n)\Big], \tag{2.6}$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 10, No. 2 (2016)

From the above considerations, the decomposition method defines the components un $u_n(x,t)$ for $n \geq 0$, by the following recursive relationships

$$\begin{cases} u_0(x,t) = f(x) + L^{-1}(g(x,t)), \\ u_{n+1}(x,t) = -L^{-1}\big[Ru_n(x,t) + A_n(u_0, u_1, \dots, u_n)\big]. \end{cases} \tag{2.7}$$

Finally, the approximate solution for $u(x,t)$ is obtained by truncating the series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{2.8}$$

The decomposition procedure of Adomian will be easily set, without linearising the problem. In this approach, the solution is found in the form of a convergent series with easily computed components; in many cases, the convergence of this series is very fast and only a few terms are needed in order to have an idea of how the solutions behave. Convergence conditions of this series are examined by several authors, mainly in [8], [9], [2] and [3].

3. The Allen-Cahn Equation

The Allen-Cahn equation is a simple mathematical model for certain phase separation processes. It also serves as a prototypical example for semilinear parabolic partial differential equations. The presence of a small parameter that defines the thickness of interfaces separating different phases makes the analysis challenging. The Allen-Cahn equation in $\mathbb{R}^n$ is given by

$$\frac{\partial u}{\partial t} = \epsilon^2 \Delta u - f(u), \quad x \in \Omega, \quad t \in \mathbb{R}^+, \tag{3.1}$$

with the initial condition

$$u(x,0) = u_0(x), \quad x \in \partial\Omega, \tag{3.2}$$

and subjects to the periodic or homogeneous Neumann-Dirichlet boundary conditions, where Ω is a bounded domain in $\mathbb{R}^n$, u represents the concentration of one of the two metallic components of the alloy, and the parameter $\epsilon > 0$ represents the inter-facial width.

Without lose of generality, we consider the commonly used double well potential which gives

$$f(u) = u^3 - u$$

Roughly speaking, the Allen-Cahn equation (3.1) describes regions with $u \approx -1$ and $u \approx 1$ that grow and decay at the expense of one another [1]. Recently in [16] is reported the existence of classical nontrivial periodic solutions for the following time-periodic Allen-Cahn equation subject to time-periodic condition:

$$\frac{\partial u}{\partial t} - \Delta u = m(t)(u^3 - u), \tag{3.3}$$

where $m(t)$ is a positive periodic function.

In the following section we will develop an algorithm using the method described in Section 2 in order to solve the nonlinear Allen-Cahn equation (3.3) subject to a time-periodic initial condition without resort to any truncation or linearization.

3.1. General Solution of the Allen-Cahn Equation Through ADM

Comparing (3.1) with equation (2.2) for the case $n = 1$ we have that $g(x,t) = 0$, L and R becomes:

$$L_t u = \frac{\partial}{\partial t} u, \quad Ru = -[\frac{\partial^2}{\partial x^2} + 1]u, \tag{3.4}$$

while the nonlinear term is given by

$$Nu = u^3. \tag{3.5}$$

By using now equation (2.7) through the ADM we obtain recursively

$$\begin{cases} u_0(x,t) = f(x), \\ u_{n+1}(x,t) = -L^{-1}\Big[Ru_n(x,t) + A_n(u_0, u_1, \ldots, u_n)\Big], \quad n = 0, 1, 2, \ldots \end{cases} \tag{3.6}$$

Also the nonlinear term $Nu = u^3$ is decomposed as

$$Nu = u^3 = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n) \tag{3.7}$$

where $\{A_n\}_{n=0}^{\infty}$ is the so-called Adomian polynomials sequence, the terms will be calculated according to [6] and [21] and these are:

$$\begin{aligned}
(u_0 + u_1 + u_2 + u_3 + u_4 + u_5 + \cdots)^3 &= u_0^3 + 3u_0^2u_1 + 3u_0^2u_2 + 3u_0^2u_3 + 3u_0^2u_4 \\
&+3u_0^2u_5 + 3u_0u_1^2 + 6u_0u_1u_2 + 6u_0u_1u_3 + 6u_0u_1u_4 + 6u_0u_1u_5 + 3u_0u_2^2 + 6u_0u_2u_3 \\
&+6u_0u_2u_4 + 6u_0u_2u_5 + 3u_0u_3^2 + 6u_0u_3u_4 + 6u_0u_3u_5 + 3u_0u_4^2 + 6u_0u_4u_5 + 3u_0u_5^2 \\
&+u_1^3 + 3u_1^2u_2 + 3u_1^2u_3 + 3u_1^2u_4 + 3u_1^2u_5 + 3u_1u_2^2 + 6u_1u_2u_3 + 6u_1u_2u_4 + 6u_1u_2u_5 \\
&+3u_1u_3^2 + 6u_1u_3u_4 + 6u_1u_3u_5 + 3u_1u_4^2 + 6u_1u_4u_5 + 3u_1u_5^2 + u_2^3 + 3u_2^2u_3 + 3u_2^2u_4 \\
&+3u_2^2u_5 + 3u_2u_3^2 + 6u_2u_3u_4 + 6u_2u_3u_5 + 3u_2u_4^2 + 6u_2u_4u_5 + 3u_2u_5^2 + u_3^3 + 3u_3^2u_4 \\
&+3u_3^2u_5 + 3u_3u_4^2 + 6u_3u_4u_5 + 3u_3u_5^2 + u_4^3 + 3u_4^2u_5 + 3u_4u_5^2 + u_5^3 + \cdots
\end{aligned} \tag{3.8}$$

The above expression can be rearranged by grouping terms in which the sum of subscripts of u_n be the same. This procedure gives the Adomian polynomials for $N(u)$. The first few polynomials are given by
$A_0 = u_0^3,$
$A_1 = 3u_0^2 u_1,$
$A_2 = 3u_0^2 u_2 + 3u_0 u_1^2,$
$A_3 = 3u_0^2 u_3 + 6u_0 u_1 u_2 + u_1^3,$
$A_4 = 3u_0^2 u_4 + 6u_0 u_1 u_3 + +3u_0 u_2^2 + 3u_1^2 u_2,$
$A_5 = 3u_0^2 u_5 + 6u_0 u_1 u_4 + 6u_0 u_2 u_3 + 3u_1^2 u_3 + 3u_1 u_2^2,$
$\vdots$
Using the expressions obtained above for equation (3.3), we will illustrate, with one example, the efectiveness of ADM to solve the nonlinear Allen-Cahn equation subject to a time-periodic initial condition.

Example

Using Adomian decomposition method, we solve the Allen-Cahn equation subject to periodic initial condition $u(x, 0) = \sin(\pi x)$.

In this example we have

$$u_t = u_{xx} + m(t)(u^3 - u), \quad m(t) = 11 + 10\cos t.$$

In Adomian's operator notation, we have

$$Ru = u_{xx} - (11 + 10\cos t)u, \quad Nu = (11 + 10\cos t)u^3$$

Next we consider the solution as a series $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$ and the nonlinearity $N(u) = \sum_{n=0}^{\infty} A_n$, and upon substitution, we obtain

$$\sum_{n=0}^{\infty} u_n(x,t) = u(x,0) + L^{-1}\Big[R\Big(\sum_{n=0}^{\infty} u_n(x,t)\Big) + \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n)\Big],$$

where $L^{-1} = \int_0^t (\cdot) d\hat{t}$.

The components of the series solution are given by the recursion scheme:
$u_0(x,t) = \sin \pi x$
$u_{k+1}(x,t) = L^{-1}(Ru_k) + L^{-1}(A_k), \quad k \geq 0.$
The first few components are as follows:

$$A_0 = u_0^3 = \sin^3 \pi x$$

$$\begin{aligned} u_1(x,t) &= \int_0^t [-(\pi^2 + 11 + 10\cos\hat{t})\sin \pi x] d\hat{t} + \int_0^t [(11 + 10\cos\hat{t})\sin^3 \pi x] d\hat{t} \\ &= t\sin^3 \pi x - (\pi^2 + 11)t \sin \pi x - 10 \sin t \sin \pi x, \end{aligned}$$

$$A_1 = 3u_0^2 u_1 = 3t\sin^5 \pi x - (3\pi^2 t + 33t + 30\sin t)\sin^3 \pi x,$$

$$u_2(x,t) = \int_0^t \Big[(\pi^4 + +19\pi^2 + 121)\hat{t}\sin\pi x - (3\pi^2+44)\hat{t}\sin^3\pi x\Big]d\hat{t}$$

$$+\int_0^t \Big[3\hat{t}\sin^5\pi x + 110\sin\hat{t}\sin\pi x + (110+10\pi^2)\hat{t}\cos\hat{t}\sin\pi x + 100\sin\hat{t}\cos\hat{t}\sin\pi x\Big]d\hat{t}$$

$$+\int_0^t \Big[6\pi^2\hat{t}\sin\pi x\cos^2\pi x - 10\hat{t}\cos\hat{t}\sin^3\pi x - 30\sin\hat{t}\sin^3\pi x\Big]d\hat{t}$$

$$= \frac{t^2}{2}\Big[(\pi^4 + 19\pi^2 + 121)\sin\pi x - (3\pi^2+44)\sin^3\pi x + 3\sin^5\pi x + 6\pi^2\sin\pi x\cos^2\pi x\Big]$$

$$+t\sin t\Big[(10\pi^2+110)\sin\pi x - 10\sin^3\pi x\Big] + 20\cos t\sin^3\pi x + 50\sin^2 t\sin\pi x - 5\sin^3\pi x,$$

$$A_2 = 2u_0^2 u_2 + 3u_0 u_1^2 = 3\sin\pi x\Big(t\sin^3\pi x - 10\sin\pi x\sin t - t(\pi^2+11)\sin\pi x\Big)^2$$

$$= t^2\Big[(\pi^4 + 19\pi^2 + 121)\sin^3\pi x - (3\pi^2+44)\sin^5\pi x + 3\sin^7\pi x + 6\pi^2\sin^3\pi x\cos^2\pi x\Big]$$

$$+2t\sin t\Big[(10\pi^2+110)\sin^3\pi x - 10\sin^5\pi x\Big] + 40\cos t\sin^5\pi x$$

$$+100\sin^2 t\sin^3\pi x - 40\sin^5\pi x,$$

$$u_3(x,t) = 2t - 5x + \frac{25}{4}\sin(t+\pi x) - \frac{25}{4}\sin(t-\pi x) + \frac{25}{8}\sin(t-3\pi x)$$

$$-\frac{25}{8}\sin(t+3\pi x) + \frac{75}{8}\sin(t-5\pi x) - \frac{75}{8}\sin(t+5\pi x) - \frac{25}{16}\sin(2t+\pi x)$$

$$+\frac{25}{16}\sin(2t-\pi x) - \frac{25}{32}\sin(2t-3\pi x) + \frac{25}{32}\sin(2t+3\pi x) - \frac{75}{32}\sin(2t-5\pi x)$$

$$+\frac{75}{32}\sin(2t+5\pi x) - \frac{75}{8}\sin\pi x + \frac{75}{16}\sin 3\pi x + \frac{225}{16}\sin 5\pi x + \frac{15}{2}\pi^2\cos(t+\pi x)$$

$$-\frac{15}{2}\pi^2\cos(t-\pi x) - \frac{15}{2}\pi^2\cos(t-3\pi x) + \frac{15}{2}\pi^2\cos(t+3\pi x) + \frac{15}{2}x^2\cos(t-2\pi x)$$

$$+\frac{15}{2}x^2\cos(t+2\pi x) + 15x\cos 2\pi x - \frac{55}{4}t\sin\pi x + \frac{55}{8}t\sin 3\pi x + \frac{165}{8}t\sin 5\pi x$$

$$-5x^2\cos t - \frac{15}{2}x\cos(t-2\pi x) - \frac{15}{2}x\cos(t+2\pi x) + \frac{55}{8}t\sin(t+\pi x) - \frac{55}{8}t\sin(t-\pi x)$$

$$+\frac{55}{16}t\sin(t-3\pi x) - \frac{55}{16}t\sin(t+3\pi x) + \frac{165}{16}t\sin(t-5\pi x) - \frac{165}{16}t\sin(t+5\pi x)$$

$$-15x^2\cos 2\pi x - \frac{121}{16}t^2\sin\pi x + \frac{121}{32}t^2\sin 3\pi x + \frac{363}{32}t^2\sin 5\pi x - \frac{11}{2}tx + 5x\cos t + 5x^2$$

$$+\frac{25}{4}\pi^2 t\sin(t+\pi x) - \frac{25}{4}\pi^2 t\sin(t-\pi x) + \frac{15}{4}\pi^2 t\sin(t-3\pi x) - \frac{15}{4}\pi^2 t\sin(t+3\pi x)$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 10, No. 2 (2016)

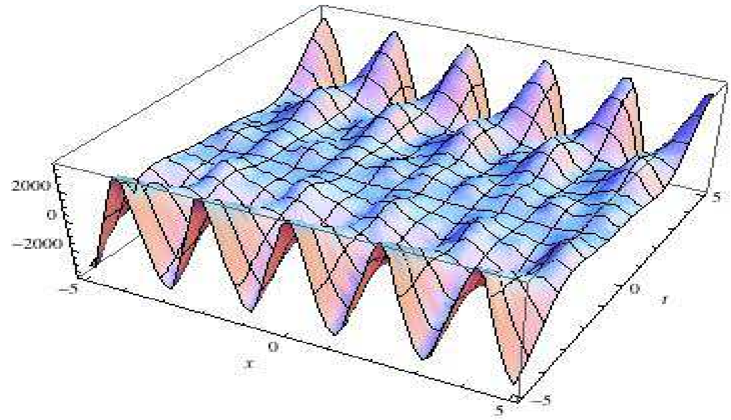

Figure 1: $u \approx u_0 + u_1$

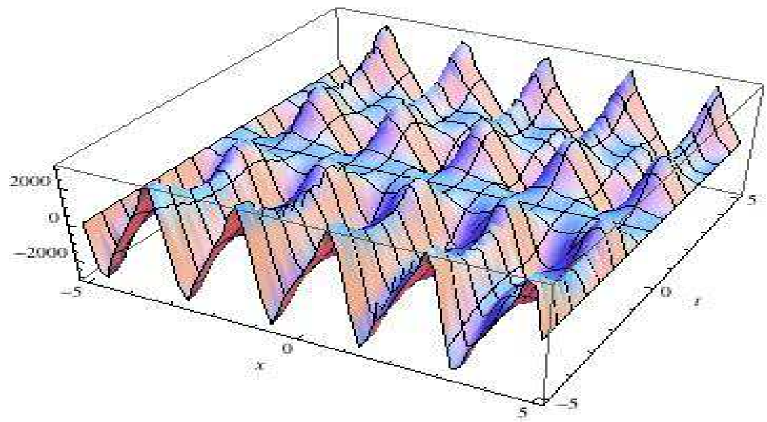

Figure 2: $u \approx u_0 + u_1 + u_2$

$$-\frac{11}{2}\pi^2 t^2 \sin \pi x + \frac{1}{2}\pi^4 t^2 \sin \pi x + \frac{33}{2}\pi^2 t^2 \sin 3\pi x + \frac{33}{2} tx \cos 2\pi x + \frac{5}{2}\pi^2 t \sin \pi x$$
$$+\frac{45}{2}\pi^2 t \sin 3\pi x - \frac{33}{2} tx^2 \cos 2\pi x - \frac{1}{6}\pi^6 t^3 \sin \pi x - \frac{3025}{32}\pi^2 t^3 \sin 5\pi x + \frac{11}{2} tx^2.$$

The components u_j, $j = 0, 1, 2, 3$ the approach to u are computed using the Mathematica package.

In the figures 1, 2 and 3 we show the graphs of the successive approximations made through ADM and in tables 1, 2, 3 and 4 we do the comparison of the solution found with ADM (up to four terms) *versus* found by means of numerical simulations. Moreover, the results are consistent with the theoretical results found recently in [16]. All the numerical work and the graphics was accomplished with the Mathematica

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 10, No. 2 (2016)

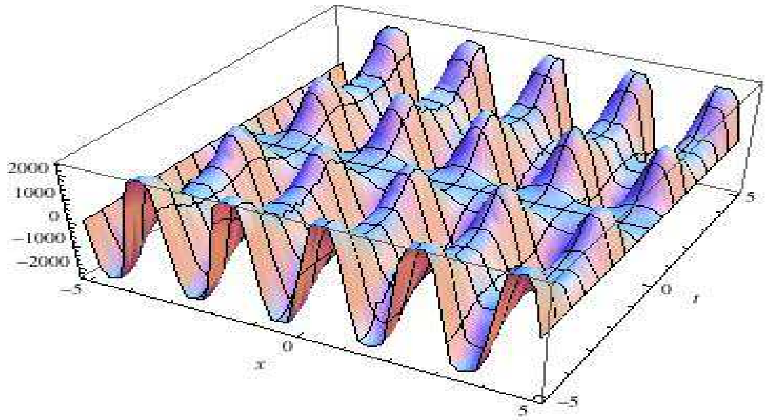

Figure 3: $u \approx u_0 + u_1 + u_2 + u_3$

$t = 1.0$			
x	u_{ADM}	$u_{Numeric}$	Error
0.5	239.02687715	239.02687714999	2×10^{-12}
1.0	$3.93778493 \times 10^{-14}$	$3.93778493 \times 10^{-13}$	3×10^{-13}
1.5	-239.02687715	-239.02687714999	2×10^{-12}
2.0	$-7.87556986 \times 10^{-14}$	$-7.87556986 \times 10^{-13}$	7×10^{-13}
2.5	239.02687715	239.02687714999	2×10^{-12}
3.0	$1.18133548 \times 10^{-13}$	$1.18133548 \times 10^{-12}$	1×10^{-12}
3.5	-239.02687715	-239.02687714999	2×10^{-12}
4.0	$-1.57511397 \times 10^{-13}$	$-1.57511397 \times 10^{-12}$	1×10^{-12}
4.5	239.02687715	239.02687714999	2×10^{-12}
5.0	$1.96889246 \times 10^{-13}$	$1.96889246 \times 10^{-12}$	1×10^{-12}

Table 1: Table for $t = 1.0$

software package.

4. Conclusions

We therefore, conclude that the Laplace-Adomian decomposition method is a notable non-sophisticated powerful tool that produces high quality approximate solutions for nonlinear partial differential equations using simple calculations and that attains converge with only few terms.

$t = 2.0$			
x	u_{ADM}	$u_{Numeric}$	Error
0.5	618.1417099	618.14179098997	3×10^{-11}
1.0	$1.13437401 \times 10^{-13}$	$1.13437401 \times 10^{-12}$	1×10^{-12}
1.5	-618.14179099	-618.14179098997	3×10^{-11}
2.0	$-2.26874803 \times 10^{-13}$	$-2.26874803 \times 10^{-12}$	2×10^{-12}
2.5	618.1417099	618.14179098997	3×10^{-11}
3.0	$3.40312204 \times 10^{-13}$	$3.40312204 \times 10^{-12}$	3×10^{-12}
3.5	-618.14179099	-618.14179098997	3×10^{-11}
4.0	$-4.53749605 \times 10^{-13}$	$-4.53749605 \times 10^{-12}$	4×10^{-12}
4.5	618.1417099	618.14179098997	3×10^{-11}
5.0	$5.67187007 \times 10^{-13}$	$5.67187007 \times 10^{-12}$	5×10^{-12}

Table 2: Table for $t = 2.0$

$t = 3.0$			
x	u_{ADM}	$u_{Numeric}$	Error
0.5	641.42209603	641.4220960298	2×10^{-10}
1.0	$1.56220421 \times 10^{-13}$	$1.56220421 \times 10^{-11}$	1×10^{-11}
1.5	-641.42209603	-641.4220960298	2×10^{-10}
2.0	$-3.12440843 \times 10^{-13}$	$-3.12440843 \times 10^{-11}$	3×10^{-11}
2.5	641.42209603	641.4220960298	2×10^{-10}
3.0	$4.68661264 \times 10^{-13}$	$4.68661264 \times 10^{-11}$	4×10^{-11}
3.5	-641.42209603	-641.4220960298	2×10^{-10}
4.0	$-6.24881685 \times 10^{-13}$	$-6.24881685 \times 10^{-11}$	6×10^{-11}
4.5	641.42209603	641.4220960298	2×10^{-10}
5.0	$7.81102107 \times 10^{-13}$	$7.81102107 \times 10^{-11}$	7×10^{-11}

Table 3: Table for $t = 3.0$

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$t = 4.0$			
x	u_{ADM}	$u_{Numeric}$	Error
0.5	489.61630221	489.616302206	4×10^{-9}
1.0	$1.88958234 \times 10^{-13}$	$1.88958234 \times 10^{-10}$	1×10^{-10}
1.5	-489.61630221	-489.616302206	4×10^{-9}
2.0	$-3.77916468 \times 10^{-13}$	$-3.77916468 \times 10^{-10}$	3×10^{-10}
2.5	489.61630221	489.616302206	4×10^{-9}
3.0	$5.66874702 \times 10^{-13}$	$5.66874702 \times 10^{-10}$	5×10^{-10}
3.5	-489.61630221	-489.616302206	4×10^{-9}
4.0	$-7.55832937 \times 10^{-13}$	$-7.55832937 \times 10^{-10}$	7×10^{-10}
4.5	489.61630221	489.616302206	4×10^{-9}
5.0	$9.44791171 \times 10^{-13}$	$9.44791171 \times 10^{-10}$	9×10^{-10}

Table 4: Table for $t = 4.0$

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