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CONJECTURE ON RATIONALIZATION OF DENOMINATOR

Yoshihiro Mochimaru

Tokyo Institute of Technology
57-121, Yamaguchi, Tokorozawa
Saitama 359-1145, JAPAN

Abstract: Rationalization of denominator involving the sum of five square roots of different prime numbers is shown, with a special case given by a reduced mere polynomial of primes.

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Key Words: rationalization, prime numbers

1. Introduction

Prime numbers are extending to infinity and further distribution is not yet sought analytically, thus estimation involving prime numbers is hard in many cases. Many characteristics found regarding primes are presented in Ref.[1].

In this paper, rationalization of denominator involving the sum of five square root of different prime numbers is presented.

2. Analysis

Let the function $p(i)$ for an integer $i(1 \leq i \leq 5)$ be

$$p(i) = i + 1$$

Let the function $q(i, j)$ for integers $i, j(1 \leq i < j \leq 5)$ be

$$q(i, j) = 1 + j - (i^2 - 9i)/2$$

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Let the function $r(i,j,k)$ for integers $i,j,k (1 \le i < j < k \le 5)$ be

$$r(i,j,k) = 11 + k - (j^2 - 9j)/2 + (i-3)(i-4)(i-5)/6$$

Let the function $s(i,j,k,\ell)$ for integers i,j,k,ℓ $(1 \le i < j < k < \ell \le 5)$ be

$$s(i,j,k,\ell) = 17 + (i+j+k+\ell)$$

Non-zero components of 32×32 square matrix A for five parametric constants $\alpha_i (i = 1, \cdots, 5)$ are defined as follows:
For an integer i $(1 \le i \le 5)$

$$a^1{}_{p(i)} = \alpha_i$$

For an integer i $(1 \le i \le 5)$

$$a^{p(i)}{}_1 = 1$$

$$a^{p(i)}{}_{q(j,i)} = \alpha_j \text{ for any integer } j \text{ such that } 1 \le j < i$$

$$a^{p(i)}{}_{q(i,j)} = \alpha_j \text{ for any integer } j \text{ such that } i < j \le 5$$

For integers i,j such that $1 \le i < j \le 5$

$$a^{q(i,j)}{}_{p(i)} = 1$$

$$a^{q(i,j)}{}_{p(j)} = 1$$

$$a^{q(i,j)}{}_{r(k,i,j)} = \alpha_k \text{ for any integer } k \text{ such that } 1 \le k < i$$

$$a^{q(i,j)}{}_{r(i,k,j)} = \alpha_k \text{ for any integer } k \text{ such that } i < k < j$$

$$a^{q(i,j)}{}_{r(i,j,k)} = \alpha_k \text{for any integer } k \text{ such that } j < k \le 5$$

For integers i,j,k such that $1 \le i < j < k \le 5$

$$a^{r(i,j,k)}{}_{q(i,j)} = 1$$

$$a^{r(i,j,k)}{}_{q(i,k)} = 1$$

$$a^{r(i,j,k)}{}_{q(j,k)} = 1$$

$$a^{r(i,j,k)}{}_m = \alpha_\ell \;,\; m = 17 + (i+j+k+\ell) \text{ for any integer } \ell (1 \le \ell \le 5)$$
$$\text{such that } (\ell - i)(\ell - j)(\ell - k) \ne 0$$

For integers i, j, k, ℓ such that $1 \leq i < j < k < \ell \leq 5$

$$a^{s(i,j,k,\ell)}{}_{r(i,j,k)} = 1$$

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$$a^{s(i,j,k,\ell)}{}_{32} = \alpha_m \ , \ m = \frac{5!}{i \cdot j \cdot k \cdot \ell}$$

For an integer i $(1 \leq i \leq 5)$

$$a^{32}{}_{26+i} = 1$$

In case α_i's are primes different from each other, the following rationalization is assumed:

$$\begin{aligned} &\frac{1}{\sqrt{\alpha_1} + \sqrt{\alpha_2} + \sqrt{\alpha_3} + \sqrt{\alpha_4} + \sqrt{\alpha_5}} \\ = \quad & x_0 + \sum_i x_i \sqrt{\alpha_i} + \sum_{i<j} x_{i,j} \sqrt{\alpha_i \alpha_j} \\ & + \sum_{i<j<k} x_{i,j,\ k} \sqrt{\alpha_i \alpha_j \alpha_k} + \sum_{i<j<k<\ell} x_{i,j,\ k,\ell} \sqrt{\alpha_i \alpha_j \alpha_k \alpha_\ell} \\ & + x_{1,2,3,4,5} \sqrt{\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5} \end{aligned}$$

For this matrix A

$$|\det A| > \left[\frac{1}{2} (\alpha_i)_{\max} \right]^{16} \ (> 0)$$

(conjecture for any combination of primes). Thus

$$x_0 = A^1{}_1 / |A|$$

$$x_i = (-1)^{1+p(i)} A^1{}_{p(i)} / |A|$$

$$x_{i,j} = (-1)^{1+q(i,j)} A^1{}_{q(i,j)} / |A|$$

$$x_{i,j,\ k} = (-1)^{1+r(i,j,\ k)} A^1{}_{r(i,j,\ k)} / |A|$$

$$x_{i,j,\ k,\ell} = (-1)^{1+s(i,j,\ k,\ell)} A^1{}_{s(i,j,\ k,\ell)} / |A|$$

$$x_{1,2,3,4,5} = -A^1{}_{32} / |A|$$

where $|A| \equiv \det A$, $A^i{}_j$: cofactor at $a^i{}_j$. Under this circumstance, all $a^i{}_j$'s are integers, so that $A^i{}_j$ and $|A|$ are also integers, which shows that $x_0, x_i, x_{i,j}, x_{i,j,\ k}$, $x_{i,j,\ k,\ell}, x_{1,2,3,4,5}$ are all rational. Eventually $x_0 = x_{i,j} = x_{i,j,\ k,\ell} = 0$.

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3. Discussion

In case that $\alpha_1 + \alpha_2 + \alpha_3 = \alpha_4 + \alpha_5$ (α_i's are primes and different from each other), ${A^1}_i, (1 \le i \le 32)$ and $|A|$ have common divisor(s). The final common irreducible denominator for $x_i, x_{i,j,\ k}, x_{1,2,3,4,5}$ is some divisor of

$$\left|2\left[w^2 - 64(\alpha_1\alpha_2\alpha_3)^2\alpha_4\alpha_5\right]\right|$$

$$\equiv f(\alpha_1, \alpha_2, \alpha_3; \alpha_4, \alpha_5)$$

where

$$w \equiv [\alpha_4\alpha_5 - (\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1)]^2 - 4\alpha_1\alpha_2\alpha_3(\alpha_1 + \alpha_2 + \alpha_3)$$

Consequently $f \sim$ order of $[(\alpha_i)_{\max}]^8$. Naturally f is not an indeterminate form derived from $|A|\,(= 0)$, but one of the divisor of $|A|\,(\neq 0)$. As an example $f(2, 7, 47; 13, 43) = 2068650000$, for which irreducible common denominator = 6895500 and

$$x_1 = \frac{2540322}{6895500} = \frac{423387}{114925}$$

$$x_2 = \frac{1202957}{6895500}$$

$$x_3 = -\frac{284187}{6895500} = -\frac{94729}{2298500}$$

$$x_4 = -\frac{766069}{6895500}$$

$$x_5 = \frac{388601}{6895500}$$

$$x_{1,2,3} = -\frac{82411}{6895500}$$

$$x_{1,2,4} = -\frac{280257}{6895500} = -\frac{93419}{2298500}$$

$$x_{1,2,5} = \frac{87753}{6895500} = \frac{29251}{2298500}$$

$$x_{1,3,4} = \frac{62087}{6895500}$$

$$x_{1,3,5} = -\frac{57823}{6895500}$$

$$x_{1,4,5} = -\frac{63501}{6895500} = -\frac{21167}{2298500}$$

$$x_{2,3,4} = \frac{34497}{6895500} = \frac{11499}{2298500}$$

$$x_{2,3,5} = -\frac{27963}{6895500} = -\frac{9321}{2298500}$$
$$x_{2,4,5} = -\frac{34831}{6895500}$$
$$x_{3,4,5} = \frac{15921}{6895500} = \frac{5307}{2298500}$$
$$x_{1,2,3,4,5} = \frac{6188}{6895500} = \frac{3094}{3447750}$$

4. Conclusion

Rationalization of denominator for the sum of five square roots of different primes is shown, with a special reduction case.

References

[1] R. Crandall, C. Pomerance, *Prime Numbers, A Computational Perspective*, Springer-Verlag, New York (2005).

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