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# FOUR WAY ROUGH-FUZZY QUANTIFIERS

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**Abstract:** Using rough sets, in 2005, G.Ganesan et. al., have approximated the fuzzy inputs through the concept of thresholds. In 2008, they have studied the rough approximated connectives on fuzzy predicates. In this paper, we study the rough approximated quantifiers on fuzzy predicates.

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**Key Words:** rough sets, fuzzy predicates, rough approximated quantifiers

## 1. Introduction

Consider the following information system with the attributes $P_1, P_2, \ldots P_m$ and the records $a_1, a_2 \ldots a_n$ with logical entities [i.e. true or false, or partially true etc].

| | $P_1$ | $P_2$ | $\ldots$ | $P_i$ | $\ldots$ | $P_m$ |
|---|---|---|---|---|---|---|
| $a_1$ | $P_1(a_1)$ | $P_2(a_1)$ | $\ldots$ | $P_i(a_1)$ | $\ldots$ | $P_m(a_1)$ |
| $a_2$ | $P_1(a_2)$ | $P_2(a_2)$ | $\ldots$ | $P_i(a_2)$ | $\ldots$ | $P_m(a_2)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $a_j$ | $P_1(a_j)$ | $P_2(a_j)$ | $\ldots$ | $P_i(a_j)$ | $\ldots$ | $P_m(a_j)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $a_n$ | $P_1(a_n)$ | $P_2(a_n)$ | $\ldots$ | $P_i(a_n)$ | $\ldots$ | $P_m(a_n)$ |

Here, each entity $P_i(a_j)$ may be viewed as the truth value of the predicate $P_i$ under the argument $a_j$. This idea has lead G.Ganesan et. al., to concentrate on deriving logical inferences in information system with logical values.

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As Pawlak's RS Model [7] involves in computing minimal features or reducts in information systems, in 2005, G.Ganesan et. al., discussed the concepts of thresholds in Rough-Fuzzy Computing. Later, in 2008, G.Ganesan et. al., introduced the rough connectives between the fuzzy predicates [2] and [6]. In 2014, G.Ganesan et. al., demonstrated the implication rules and Equivalent forms in R-F Predicates [5].

This paper is organized into four sections. Section two deals with the basic concepts of fuzzy predicates; rough connectives along with the implication and equivalence rules of R-F Predicate Calculus. In third section, we introduce the concepts of rough quantifiers and demonstrated the inference theory with a few examples.

## 2. Mathematical Aspects

For any two fuzzy predicates $P$ and $Q$ [with arguments $a$ and $b$], the fuzzy conjunction ($\wedge$), fuzzy disjunction ($\vee$), fuzzy negation (neg), fuzzy implication ($\rightarrow$) and fuzzy bi implication ($\leftrightarrow$) are defined as follows.

| | |
|---|---|
| Fuzzy Conjunction ($\wedge$) | :$P(a) \wedge Q(b) = \min(P(a), Q(b))$ |
| Fuzzy Disjunction ($\vee$) | :$P(a) \vee Q(b) = \max(P(a), Q(b))$ |
| Fuzzy Negation (neg) | :$neg(P(a)) \quad = 1 - P(a)$ |
| Fuzzy implication ($\rightarrow$) | :P(a)$\rightarrow$Q(b) $= \max(1 - P(a), Q(b))$ |
| Fuzzy bi-implication ($\leftrightarrow$) | :P(a)$\leftrightarrow$Q(b) $= \min(P(a) \rightarrow Q(b), Q(b) \rightarrow P(a))$ |

### 2.1. Rough-Fuzzy Connectives

The lower and upper rough approximations of a given predicate $P$ with argument $a$ and a threshold $\alpha$ are defined by

$$P_\alpha(a) = \cup\{[x] \in X : [x] \subseteq P(a)[\alpha]\},$$

and $P^\alpha(a) = \cup\{[x] \in X : [x] \cap P(a)[\alpha] \neq \Phi\}$ respectively, where $[\alpha]$ represents strong $\alpha$ cut [1].

Let $\tau$ denote negation in usual predicate calculus.

The unary and binary rough connectives, for the given fuzzy predicates $P_i(x)$ and $P_j(y)$ are defined [2], [3] and [5] as follows:

(a) **Rough conjunction** $(\underset{\alpha}{\wedge}, \overset{\alpha}{\wedge}) \Rightarrow P_i(x) \underset{\alpha}{\wedge} P_j(y) = P_{i,\alpha}(x) \wedge P_{j,\alpha}(y)$ and $P_i(x) \overset{\alpha}{\wedge} P_j(y) = P_i^\alpha(x) \wedge P_j^\alpha(y)$ respectively.

(b) **Rough disjunction** $(\underset{\alpha}{\vee}, \overset{\alpha}{\vee}) \Rightarrow P_i(x) \underset{\alpha}{\vee} P_j(y) = P_{i,\alpha}(x) \vee P_{j,\alpha}(y)$ and $P_i(x) \overset{\alpha}{\vee} P_j(y) = P_i^\alpha(x) \vee P_j^\alpha(y)$ respectively.

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(c) **Rough implication** $(\underset{\alpha}{\longrightarrow}, \overset{\alpha}{\longrightarrow}) \Rightarrow P_i(x) \underset{\alpha}{\longrightarrow} P_j(y) = P_i^{1-\alpha}(x) \longrightarrow P_{j,\alpha}(y)$ and $P_i(x) \overset{\alpha}{\longrightarrow} P_j(y) = P_{i,1-\alpha}(x) \longrightarrow P_j^{\alpha}(y)$ respectively.

(d) **Rough bi-implication** $(\underset{\alpha}{\longleftrightarrow}, \overset{\alpha}{\longleftrightarrow}) \Rightarrow P_i(x) \underset{\alpha}{\longleftrightarrow} P_j(y) = [P_i(x) \underset{\alpha}{\longrightarrow} P_j(y)] \wedge [P_j(y) \underset{\alpha}{\longrightarrow} P_i(x)]$ and $P_i(x) \overset{\alpha}{\longleftrightarrow} P_j(y) = [P_i(x) \overset{\alpha}{\longrightarrow} P_j(y)] \wedge [P_j(y) \overset{\alpha}{\longrightarrow} P_i(x)]$ respectively.

(e) **Rough negation** $(\tau_\alpha, \tau^\alpha) \Rightarrow \tau_\alpha P_i(x) = (negP_i(x))_\alpha = \tau(P_i^{1-\alpha}(x))$ and $\tau^\alpha P_i(x) = (negP_i(x))^\alpha = \tau(P_{i,1-\alpha}(x))$ respectively.

The Equivalence rules are defined as follows and are shown in Table 1.

Further, the connectives defined in 2.1 satisfy the following properties

(i) $(negP_i(x))_\alpha = \tau(P_i^{1-\alpha}(x))$

(ii) $(negP_i(x))^\alpha = \tau(P_{i,1-\alpha}(x))$

The implication rules are given in Table 2.

## 3. Rough Fuzzy Quantifiers

In this section, the quantifiers are introduced similar to the quantifiers are used as discussed in [6]. Here, we introduce four types of quantifiers namely, lower universal quantifier, upper universal quantifier, lower existential quantifier and upper existential quantifier.

### 3.1. R-F Quantifiers

For any fuzzy predicate $P(x)$ and a threshold $\alpha$,

(a) The **lower and upper universal quantifiers**, denoted by $(\underset{\alpha}{\forall}\, x)$ and $(\overset{\alpha}{\forall}\, x)$, are defined as $(\underset{\alpha}{\forall}\, x)\ P(x) = (\forall\, x)\ P_\alpha(x)$ and $(\overset{\alpha}{\forall}\, x)\ P(x) = (\forall\, x)\ P^\alpha(x)$ respectively.

(b) The **lower and upper existential quantifiers**, denoted by $(\underset{\alpha}{\exists}\, x)$ and $(\overset{\alpha}{\exists}\, x)$, are defined as $(\underset{\alpha}{\exists}\, x)\ P(x) = (\exists\, x)\ P_\alpha(x)$ and $(\overset{\alpha}{\exists}\, x)\ P(x) = (\exists\, x)\ P^\alpha(x)$ respectively.

The theory of inferences related to rules in Table 2 are not discussed here. In Section 3, we introduce the four way rough fuzzy quantifiers along with appropriate demonstrations.

The four quantifiers in 3.1 are said to follow the following axioms.

For any fuzzy predicates $P(x)$, $Q(x)$, and a threshold $\alpha$,

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| | | | | |
|---|---|---|---|---|
| Double Negation Laws | | | | |
| $LE_1$ | $\tau(\tau(P_\alpha(x))) = P_\alpha(x)$ | | $UE_1$ | $\tau(\tau(P^\alpha(x))) = P^\alpha(x)$ |
| Idempotent Laws | | | | |
| $LE_2$ | $(P(x) \underset{\alpha}{\wedge} P(x)) = P_\alpha(x)$ | | $UE_2$ | $(P(x) \overset{\alpha}{\wedge} P(x)) = P^\alpha(x)$ |
| $LE_3$ | $(P(x) \underset{\alpha}{\vee} P(x)) = P_\alpha(x)$ | | $UE_3$ | $(P(x) \overset{\alpha}{\vee} P(x)) = P^\alpha(x)$ |
| Commutative Laws | | | | |
| $LE_4$ | $P(x) \underset{\alpha}{\wedge} Q(y) = Q(y) \underset{\alpha}{\wedge} P(x)$ | | $UE_4$ | $P(x) \overset{\alpha}{\wedge} Q(y) = Q(y) \overset{\alpha}{\wedge} P(x)$ |
| $LE_5$ | $P(x) \underset{\alpha}{\vee} Q(y) = Q(y) \underset{\alpha}{\vee} P(x)$ | | $UE_5$ | $P(x) \overset{\alpha}{\vee} Q(y) = Q(y) \overset{\alpha}{\vee} P(x)$ |
| Associative Laws | | | | |
| $LE_6$ | $(P(x) \underset{\alpha}{\wedge} Q(y)) \underset{\alpha}{\wedge} R(z) =$ <br> $P(x) \underset{\alpha}{\wedge} (Q(y) \underset{\alpha}{\wedge} R(z))$ | | $UE_6$ | $(P(x) \overset{\alpha}{\wedge} (Q(y)) \overset{\alpha}{\wedge} R(z) =$ <br> $P(x) \overset{\alpha}{\wedge} (Q(y) \overset{\alpha}{\wedge} R(z))$ |
| $LE_7$ | $(P(x) \underset{\alpha}{\vee} Q(y)) \underset{\alpha}{\vee} R(z) =$ <br> $P(x) \underset{\alpha}{\vee} (Q(y) \underset{\alpha}{\vee} R(z))$ | | $UE_7$ | $(P(x) \overset{\alpha}{\vee} (Q(y)) \overset{\alpha}{\vee} R(z) =$ <br> $P(x) \overset{\alpha}{\vee} (Q(y) \overset{\alpha}{\vee} R(z))$ |
| Distributive Laws | | | | |
| $LE_8$ | $P(x) \underset{\alpha}{\wedge} (Q(y) \underset{\alpha}{\vee} R(z)) =$ <br> $(P(x) \underset{\alpha}{\wedge} Q(y)) \underset{\alpha}{\vee} (P(x) \underset{\alpha}{\wedge} R(z))$ | | $UE_8$ | $P(x) \overset{\alpha}{\wedge} (Q(y) \overset{\alpha}{\vee} R(z)) =$ <br> $(P(x) \overset{\alpha}{\wedge} Q(y)) \overset{\alpha}{\vee} (P(x) \overset{\alpha}{\wedge} R(z))$ |
| $LE_9$ | $P(x) \underset{\alpha}{\vee} (Q(y) \underset{\alpha}{\wedge} R(z)) =$ <br> $(P(x) \underset{\alpha}{\vee} Q(y)) \underset{\alpha}{\wedge} (P(x) \underset{\alpha}{\vee} R(z))$ | | $UE_9$ | $P(x) \overset{\alpha}{\vee} (Q(y) \overset{\alpha}{\wedge} R(z)) =$ <br> $(P(x) \overset{\alpha}{\vee} Q(y)) \overset{\alpha}{\wedge} (P(x) \overset{\alpha}{\vee} R(z))$ |
| Laws of Contradiction | | | | |
| $LE_{10}$ | $P(x) \underset{\alpha}{\wedge} (negP(x)) = .f.$ | | $UE_{10}$ | $P(x) \overset{\alpha}{\wedge} (negP(x)) = .f.$ |
| Laws of Excluded Middle | | | | |
| $LE_{11}$ | $P(x) \underset{\alpha}{\vee} (negP(x)) = .t.$ | | $UE_{11}$ | $P(x) \overset{\alpha}{\vee} (negP(x)) = .t.$ |
| Domination Laws | | | | |
| $LE_{12}$ | $P(x) \underset{\alpha}{\wedge} .f. = .f.$ | | $UE_{12}$ | $P(x) \overset{\alpha}{\wedge} .f. = .f.$ |
| $LE_{13}$ | $P(x) \underset{\alpha}{\vee} .t. = .t.$ | | $UE_{13}$ | $P(x) \overset{\alpha}{\vee} .t. = .t.$ |
| Identity | | | | |
| $LE_{14}$ | $P(x) \underset{\alpha}{\wedge} .t. = P_\alpha(x)$ | | $UE_{14}$ | $P(x) \overset{\alpha}{\wedge} .t. = P^\alpha(x)$ |
| $LE_{15}$ | $P(x) \underset{\alpha}{\vee} .f. = P_\alpha(x)$ | | $UE_{15}$ | $P(x) \overset{\alpha}{\vee} .f. = P^\alpha(x)$ |
| Contrapositive | | | | |
| $LE_{16}$ | $P(x) \underset{\alpha}{\longrightarrow} Q(y) =$ <br> $((negQ(y)) \underset{\alpha}{\longrightarrow} ((negP(x)))$ | | $UE_{16}$ | $P(x) \xrightarrow{\alpha} Q(y) =$ <br> $((negQ(y)) \xrightarrow{\alpha} ((negP(x)))$ |
| $LE_{17}$ | $(P(x) \underset{\alpha}{\longrightarrow} Q(y)) = (negP(x) \underset{\alpha}{\vee} Q(y))$ | | $UE_{17}$ | $(P(x) \xrightarrow{\alpha} Q(y)) = (negP(x) \overset{\alpha}{\vee} Q(y))$ |
| $LE_{18}$ | $P(x) \underset{\alpha}{\longleftrightarrow} Q(y) =$ <br> $(P(x) \underset{\alpha}{\wedge} Q(y)) \vee (negP(x) \underset{\alpha}{\wedge} negQ(y))$ | | $UE_{18}$ | $P(x) \overset{\alpha}{\longleftrightarrow} Q(y) =$ <br> $(P(x) \overset{\alpha}{\wedge} Q(y)) \vee (negP(x) \overset{\alpha}{\wedge} negQ(y))$ |

(a) The lower and upper universal quantifiers for the conjunction of $P(x)$ and $Q(x)$

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| | | |
|---|---|---|
| $ME_1$ | $\tau(P(x) \underset{\alpha}{\wedge} Q(y)) = (negP(x) \overset{1-\alpha}{\vee} negQ(y))$ | De Morgan's Laws |
| $ME_2$ | $\tau(P(x) \overset{\alpha}{\wedge} Q(y)) = (negP(x) \underset{1-\alpha}{\vee} negQ(y))$ | |
| $ME_3$ | $\tau(P(x) \underset{\alpha}{\vee} Q(y)) = (negP(x) \overset{1-\alpha}{\wedge} negQ(y))$ | |
| $ME_4$ | $\tau(P(x) \overset{\alpha}{\vee} Q(y)) = (negP(x) \underset{1-\alpha}{\wedge} negQ(y))$ | |
| $ME_5$ | $\tau(P(x) \underset{\alpha}{\longrightarrow} Q(y)) = (P(x) \overset{1-\alpha}{\wedge} negQ(y))$ | |
| $ME_6$ | $\tau(P(x) \overset{\alpha}{\longrightarrow} Q(y)) = (P(x) \underset{1-\alpha}{\wedge} negQ(y))$ | |
| $ME_7$ | $P(x) \underset{\alpha}{\longrightarrow} (Q(y) \underset{\alpha}{\longrightarrow} R(z)) = (P(x) \overset{1-\alpha}{\wedge} Q(y)) \underset{\alpha}{\longrightarrow} R(z)$ | |
| $ME_8$ | $P(x) \overset{\alpha}{\longrightarrow} (Q(y) \overset{\alpha}{\longrightarrow} R(z)) = (P(x) \underset{1-\alpha}{\wedge} Q(y)) \overset{\alpha}{\longrightarrow} R(z)$ | |
| $ME_9$ | $P(x) \underset{\alpha}{\longleftrightarrow} negQ(y) = \tau(P(x) \overset{1-\alpha}{\longleftrightarrow} Q(y))$ | |
| $ME_{10}$ | $P(x) \overset{\alpha}{\longleftrightarrow} negQ(y) = \tau(P(x) \underset{1-\alpha}{\longleftrightarrow} Q(y))$ | |

Table 1

are defined as $(\underset{\alpha}{\forall}\, x)\ (P(x) \wedge Q(x)) = (\forall\, x)\ (P(x) \underset{\alpha}{\wedge} Q(x)) = (\forall\, x)\ (P_\alpha(x) \wedge Q_\alpha(x))$ and $(\overset{\alpha}{\forall}\, x)\ (P(x) \wedge Q(x)) = (\forall\, x)\ (P(x) \overset{\alpha}{\wedge} Q(x)) = (\forall\, x)\ (P^\alpha(x) \wedge Q^\alpha(x))$ respectively.

(b) The lower and upper universal quantifiers for the disjunction of $P(x)$ and $Q(x)$ are defined as $(\underset{\alpha}{\forall}\, x)\ (P(x) \vee Q(x)) = (\forall\, x)\ (P(x) \underset{\alpha}{\vee} Q(x)) = (\forall\, x)\ (P_\alpha(x) \vee Q_\alpha(x))$ and $(\overset{\alpha}{\forall}\, x)\ (P(x) \vee Q(x)) = (\forall\, x)\ (P(x) \overset{\alpha}{\vee} Q(x)) = (\forall\, x)\ (P^\alpha(x) \vee Q^\alpha(x))$ respectively.

(c) The lower and upper existential quantifiers for the conjunction of $P(x)$ and $Q(x)$ are defined as $(\underset{\alpha}{\exists}\, x)\ (P(x) \wedge Q(x)) = (\exists\, x)\ (P(x) \underset{\alpha}{\wedge} Q(x)) = (\exists\, x)\ (P_\alpha(x) \wedge Q_\alpha(x))$ and $(\overset{\alpha}{\exists}\, x)\ (P(x) \wedge Q(x)) = (\exists\, x)\ (P(x) \overset{\alpha}{\wedge} Q(x)) = (\exists\, x)\ (P^\alpha(x) \wedge Q^\alpha(x))$ respectively.

(d) The lower and upper existential quantifiers for the disjunction of $P(x)$ and $Q(x)$ are defined as $(\underset{\alpha}{\exists}\, x)\ (P(x) \vee Q(x)) = (\exists\, x)\ (P(x) \underset{\alpha}{\vee} Q(x)) = (\exists\, x)\ (P_\alpha(x) \vee Q_\alpha(x))$ and $(\overset{\alpha}{\exists}\, x)\ (P(x) \vee Q(x)) = (\exists\, x)\ (P(x) \overset{\alpha}{\vee} Q(x)) = (\exists\, x)\ (P^\alpha(x) \vee Q^\alpha(x))$ respectively.

(e) The lower and upper universal quantifiers for the implication of $P(x)$ and $Q(x)$ are defined as $(\underset{\alpha}{\forall}\, x)\ (P(x) \longrightarrow Q(x)) = (\forall\, x)\ (P(x) \underset{\alpha}{\longrightarrow} Q(x)) = (\forall\, x)$

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| | **Lower Implications** | | **Upper Implications** |
|---|---|---|---|
| $LI_1$ | $\frac{(P(x)\underset{\alpha}{\wedge}Q(y))}{\therefore P_\alpha(x)}$ | $UI_1$ | $\frac{(P(x)\overset{\alpha}{\wedge}Q(y))}{\therefore P^\alpha(x)}$ |
| $LI_2$ | $\frac{(P(x)\underset{\alpha}{\wedge}Q(y))}{\therefore Q_\alpha(x)}$ | $UI_2$ | $\frac{(P(x)\overset{\alpha}{\wedge}Q(y))}{\therefore Q^\alpha(y)}$ |
| $LI_3$ | $\frac{P_\alpha(x)}{\therefore (P(x)\underset{\alpha}{\vee}P(x))}$ | $UI_3$ | $\frac{P^\alpha(x)}{\therefore (P(x)\overset{\alpha}{\vee}Q(y))}$ |
| $LI_4$ | $\frac{Q_\alpha(y)}{\therefore (P(x)\underset{\alpha}{\vee}Q(y))}$ | $UI_3$ | $\frac{Q^\alpha(y)}{\therefore (P(x)\overset{\alpha}{\vee}Q(y))}$ |
| $LI_5$ | $\frac{\tau(P_\alpha(x))}{\therefore (P(x)\underset{\alpha}{\longrightarrow}Q(y))}$ | $UI_5$ | $\frac{\tau(P^\alpha(x))}{\therefore (P(x)\overset{\alpha}{\longrightarrow}Q(y))}$ |
| $LI_6$ | $\frac{Q_\alpha(y)}{\therefore (P(x)\underset{\alpha}{\longrightarrow}Q(y))}$ | $UI_6$ | $\frac{Q^\alpha(y)}{\therefore (P(x)\overset{\alpha}{\longrightarrow}Q(y))}$ |
| $LI_7$ | $\frac{\tau(P(x)\underset{\alpha}{\longrightarrow}Q(y))}{\therefore P_\alpha(x)}$ | $UI_7$ | $\frac{\tau(P(x)\overset{\alpha}{\longrightarrow}Q(y))}{\therefore P^\alpha(x)}$ |
| $LI_8$ | $\frac{\tau(P(x)\underset{\alpha}{\longrightarrow}Q(y))}{\therefore \tau(Q_\alpha(y))}$ | $UI_8$ | $\frac{\tau(P(x)\overset{\alpha}{\longrightarrow}Q(y))}{\therefore \tau(Q^\alpha(y))}$ |
| $LI_9$ | $\frac{(P(x)\underset{\alpha}{\vee}Q(y)),\tau(P_\alpha(x))}{\therefore Q_\alpha(y)}$ | $UI_9$ | $\frac{(P(x)\overset{\alpha}{\vee}Q(y)),\tau(P^\alpha(x))}{\therefore Q^\alpha(y)}$ |
| $LI_{10}$ | $\frac{P_\alpha(x),(P(x)\underset{\alpha}{\longrightarrow}Q(y))}{\therefore Q_\alpha(y)}$ | $UI_{10}$ | $\frac{P^\alpha(x),(P(x)\overset{\alpha}{\longrightarrow}Q(y))}{\therefore Q^\alpha(y)}$ |
| $LI_{11}$ | $\frac{(negQ(y))_\alpha,(P(x)\underset{\alpha}{\longrightarrow}Q(y))}{\therefore (negP(x))_\alpha}$ | $UI_{11}$ | $\frac{(negQ(y))^\alpha,(P(x)\overset{\alpha}{\longrightarrow}Q(y))}{\therefore (negP(x))^\alpha}$ |
| $LI_{12}$ | $\frac{\tau(Q_\alpha(y)),(P(x)\underset{\alpha}{\longrightarrow}Q(y))}{\therefore \tau(P_\alpha(x))}$ | $UI_{12}$ | $\frac{\tau(Q^\alpha(y)),(P(x)\overset{\alpha}{\longrightarrow}Q(y))}{\therefore \tau(P^\alpha(x))}$ |
| $LI_{13}$ | $\frac{(P(x)\underset{\alpha}{\longrightarrow}Q(y)),(Q(y)\underset{\alpha}{\longrightarrow}R(z))}{\therefore (P(x)\underset{\alpha}{\longrightarrow}R(z))}$ | $UI_{13}$ | $\frac{(P(x)\overset{\alpha}{\longrightarrow}Q(y)),(Q(y)\overset{\alpha}{\longrightarrow}R(z))}{\therefore (P(x)\overset{\alpha}{\longrightarrow}R(z))}$ |
| $LI_{14}$ | $\frac{(P(x)\underset{\alpha}{\vee}Q(y)),(P(x)\underset{\alpha}{\longrightarrow}R(z)),(Q(y)\underset{\alpha}{\longrightarrow}R(z))}{\therefore R_\alpha(z)}$ | $UI_{14}$ | $\frac{(P(x)\overset{\alpha}{\vee}Q(y)),(P(x)\overset{\alpha}{\longrightarrow}R(z)),(Q(y)\overset{\alpha}{\longrightarrow}R(z))}{\therefore R^\alpha(z)}$ |

Table 2

$(negP(x) \underset{\alpha}{\vee} Q(x))$ and $(\overset{\alpha}{\forall}\ x)\ (P(x) \longrightarrow Q(x)) = (\forall\ x)\ (P(x) \overset{\alpha}{\longrightarrow} Q(x)) = (\forall\ x)\ (negP(x) \overset{\alpha}{\vee} Q(x))$ respectively.

(f) The lower and upper existential quantifiers for the implication of $P(x)$ and $Q(x)$ are defined as $(\underset{\alpha}{\exists}\ x)\ (P(x) \longrightarrow Q(x)) = (\exists\ x)\ (P(x) \underset{\alpha}{\longrightarrow} Q(x)) = (\exists\ x)\ (negP(x) \underset{\alpha}{\vee} Q(x))$ and $(\overset{\alpha}{\exists}\ x)\ (P(x) \longrightarrow Q(x)) = (\exists\ x)\ (P(x) \overset{\alpha}{\longrightarrow} Q(x)) = (\exists\ x)\ (negP(x) \overset{\alpha}{\vee} Q(x))$ respectively.

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### 3.2. Negating R-F Quantifiers

In this section the negation of lower and upper approximated quantifiers using is a threshold $\alpha$ are introduced similar to the negation of quantifiers [6].

**Definition 3.2.1.** Consider the finite universe of discourse A $= \{a_1, a_2, \dots, a_n\}$ and let $P(x)$ be any fuzzy predicate and $\alpha$ is a threshold.

Then

(a) the lower universal quantifier $(\underset{\alpha}{\forall}\ x)\ P(x)$ is same as the conjunction of $P_\alpha(a_1), P_\alpha(a_2), \dots, P_\alpha(a_n)$
That is, $(\underset{\alpha}{\forall}\ x)\ P(x) = (\forall\ x)\ P_\alpha(x) = P_\alpha(a_1) \wedge P_\alpha(a_2) \wedge \dots \wedge P_\alpha(a_n)$

(b) the upper universal quantifier $(\overset{\alpha}{\forall}\ x)\ P(x)$ is same as the conjunction of $P^\alpha(a_1), P^\alpha(a_2), \dots, P^\alpha(a_n)$
That is, $(\overset{\alpha}{\forall}\ x)\ P(x) = (\forall\ x)\ P^\alpha(x) = P^\alpha(a_1) \wedge P^\alpha(a_2) \wedge \dots \wedge P^\alpha(a_n)$

(c) the lower existential quantifier $(\underset{\alpha}{\exists}\ x)\ P(x)$ is same as the disjunction of $P_\alpha(a_1), P_\alpha(a_2), \dots, P_\alpha(a_n)$
That is, $(\underset{\alpha}{\exists}\ x)\ P(x) = (\exists\ x)\ P_\alpha(x) = P_\alpha(a_1) \vee P_\alpha(a_2) \vee \dots \vee P_\alpha(a_n)$

(d) the upper existential quantifier $(\overset{\alpha}{\exists}\ x)\ P(x)$ is same as the disjunction of $P^\alpha(a_1), P^\alpha(a_2), \dots, P^\alpha(a_n)$
That is, $(\overset{\alpha}{\exists}\ x)\ P(x) = (\exists\ x)\ P^\alpha(x) = P^\alpha(a_1) \vee P^\alpha(a_2) \vee \dots \vee P^\alpha(a_n)$

By using De Morgans laws and the above definitions the following equivalences on negation of quantifiers for both lower and upper approximations are proved as given below:

**Properties 3.2.2.**

(a)
$$\begin{aligned} \tau((\underset{\alpha}{\forall}x)P(x)) &= \tau((\forall x)(P_\alpha(x))) \\ &= \tau(P_\alpha(a_1) \wedge P_\alpha(a_2) \wedge \dots \wedge P_\alpha(a_n)) \\ &= (negP(a_1))^{1-\alpha} \vee (negP(a_2))^{1-\alpha} \vee \cdots \vee (negP(a_n))^{1-\alpha} \\ &= (\overset{1-\alpha}{\exists}\ x)(negP(x)) \end{aligned}$$

Therfore $\tau((\underset{\alpha}{\forall}x)P(x)) = (\overset{1-\alpha}{\exists}\ x)(negP(x))$.

In similar way the following hold true:

(b) $\tau((\overset{\alpha}{\forall}x)P(x)) = (\underset{1-\alpha}{\exists}\ x)(negP(x))$,

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| | | | |
|---|---|---|---|
| $LE_{19}$ | $\tau((\underset{\alpha}{\forall} x)P(x)) = (\overset{1-\alpha}{\exists} x)(negP(x))$ | $UE_{19}$ | $\tau((\overset{\alpha}{\forall} x)P(x)) = (\underset{1-\alpha}{\exists} x)(negP(x))$ |
| $LE_{20}$ | $\tau((\underset{\alpha}{\exists} x)P(x)) = (\overset{1-\alpha}{\forall} x)(negP(x))$ | $UE_{20}$ | $\tau((\overset{\alpha}{\exists} x)P(x)) = (\underset{1-\alpha}{\forall} x)(negP(x))$ |

Table 3

(c) $\tau((\underset{\alpha}{\exists} x)P(x)) = (\overset{1-\alpha}{\forall} x)(negP(x))$,

(d) $\tau((\overset{\alpha}{\exists} x)P(x)) = (\underset{1-\alpha}{\forall} x)(negP(x))$.

All the equivalences in 3.2.2. are listed in Table 3.

### 3.3. Rules of Inference in R-F Quantifiers

Here, some important rules of inference for predicates involving quantifiers for both lower and upper approximations are described. In this section the two way *rules of specification* and *rules of generalization* is introduced similar to as similar in [6], which are useful to eliminate or include the appropriate quantifiers during the derivation process.

The lower and upper rules of specification as well as the lower and upper rules of generalization are defined as:

**Definition 3.3.1.** For a given fuzzy predicate $P$ and a threshold $\alpha$

(1) the lower and upper Universal Specification is defined as

Rule **USL:** For the given premise $(\overset{\alpha}{\forall} x)\ P(x)$, to conclude that $P_\alpha(y)$ is true.

Rule **USU:** For the given premise $(\overset{\alpha}{\forall} x)\ P(x)$, to conclude that $P^\alpha(y)$ is true.

(2) the lower and upper Universal Generalization is defined as

Rule **UGL:** For the given premise $P_\alpha(x)$ is true for all elements $x$ in the domain, to conclude that $(\underset{\alpha}{\forall} y)\ P(y)$ is true.

Rule **UGU:** For the given premise $P^\alpha(x)$ is true for all elements $x$ in the domain, to conclude that $(\overset{\alpha}{\forall} y)\ P(y)$ is true.

(3) the lower and upper Existential Specification is defined as

Rule **ESL:** For the given premise $(\underset{\alpha}{\exists} x)\ P(x)$, to conclude that $P_\alpha(y)$ is true.

Rule **ESU:** For the given premise $(\overset{\alpha}{\exists} x)\ P(x)$, to conclude that $P^\alpha(y)$ is true.

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(4) the lower and upper Existential Generalization is defined as

Rule **EGL:** For the given premise $P_\alpha(x)$ is true for at least one element in the domain, to conclude that $(\underset{\alpha}{\exists}\, y)\ P(y)$ is true.

Rule **EGU:** For the given premise $P^\alpha(x)$ is true for at least one element in the domain, to conclude that $(\overset{\alpha}{\exists}\, y)\ P(y)$ is true.

The rules of specification and rules of generalization for both lower and upper approximations using a threshold $\alpha$ are described in the definition 3.3.1. are listed in Table 4.

**Example 3.3.2.** $B^\alpha(x)$ follows from the premises $(\overset{\alpha}{\forall}\, x)\ (A(x) \longrightarrow B(x))$ and $A^\alpha(x)$.

*Solution.*

| | | |
|---|---|---|
| (1) | $(\overset{\alpha}{\forall}\, x)\ (A(x) \longrightarrow B(x))$ | Rule **P**, |
| (2) | $(A(x) \overset{\alpha}{\longrightarrow} B(x))$ | Rule **USU**, (1) |
| (3) | $A^\alpha(x)$ | Rule **P** |
| (4) | $B^\alpha(x)$ | Rule **T**, (2), (3) and $UI_{10}$. |

Hence $B^\alpha(x)$ is followed from the given premises.

Now, the following example describes indirect method of proof.

**Example 3.3.3.** $(\underset{\alpha}{\forall}\, x)\ (P(x) \vee Q(x)) \Rightarrow (\underset{\alpha}{\forall}\, x)\ P(x) \vee (\underset{\alpha}{\exists}\, x)\ Q(x)$.

*Solution.* To prove this by using Proof by Contradiction (Indirect Method of Proof) for this assume $\tau((\underset{\alpha}{\forall} x)P(x) \vee (\underset{\alpha}{\exists} x)Q(x))$ as an additional premise.

| | | |
|---|---|---|
| (1) | $\tau((\underset{\alpha}{\forall} x)P(x) \vee (\underset{\alpha}{\exists} x)Q(x))$ | Rule **P** (assumed) |
| (2) | $\tau((\underset{\alpha}{\forall} x)P(x)) \wedge \tau((\underset{\alpha}{\exists} x)Q(x))$ | Rule **T**, (1), and $E_{17}$ |
| (3) | $\tau((\underset{\alpha}{\forall} x)P(x))$ | Rule **T**, (2), and $I_1$ |
| (4) | $(\overset{1-\alpha}{\exists}\, x)(negP(x))$ | Rule **T**, (3), and $LE_{19}$ |
| (5) | $\tau((\underset{\alpha}{\exists} x)Q(x))$ | Rule **T**, (2), and $I_2$ |
| (6) | $(\overset{1-\alpha}{\forall}\, x)(negQ(x))$ | Rule **T**, (5), and $LE_{20}$ |
| (7) | $(negP(y))^{1-\alpha}$ | Rule **ESU**, (4) |
| (8) | $(negQ(y))^{1-\alpha}$ | Rule **USU**, (6) |
| (9) | $(negP(y))^{1-\alpha} \wedge (negQ(y))^{1-\alpha}$ | Rule **T**, (7), (8) |
| (10) | $\tau(P(y) \underset{\alpha}{\vee} Q(y))$ | Rule **T**, (9), and $ME_1$ |
| (11) | $(\underset{\alpha}{\forall}\, x)\ (P(x) \vee Q(x))$ | Rule **P** |
| (12) | $(\forall\, x)\ (P(x) \underset{\alpha}{\vee} Q(x))$ | Rule **T**, (11) |
| (13) | $(P(y) \underset{\alpha}{\vee} Q(y))$ | Rule **USL**, (12) |
| (14) | $\tau(P(y) \underset{\alpha}{\vee} Q(y)) \wedge (P(y) \underset{\alpha}{\vee} Q(y))$ | Rule **T**, (10), (13) |

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| | | | |
|---|---|---|---|
| **USL** | $(\underset{\alpha}{\forall}\ x)\ P(x) \Rightarrow P_\alpha(y)$ | **USU** | $(\overset{\alpha}{\forall}\ x)\ P(x) \Rightarrow P^\alpha(y)$ |
| **UGL** | $P_\alpha(x) \Rightarrow (\underset{\alpha}{\forall}\ y)\ P(y)$ | **UGU** | $P^\alpha(x) \Rightarrow (\overset{\alpha}{\forall}\ y)\ P(y)$ |
| **ESL** | $(\underset{\alpha}{\exists}\ x)\ P(x) \Rightarrow P_\alpha(y)$ | **EGU** | $(\overset{\alpha}{\exists}\ x)\ P(x) \Rightarrow P^\alpha(y)$ |
| **EGL** | $P_\alpha(x) \Rightarrow (\underset{\alpha}{\exists}\ y)\ P(y)$ | **EGU** | $P^\alpha(x) \Rightarrow (\overset{\alpha}{\exists}\ y)\ P(y)$ |

Table 4

This is a contradiction, therefore $(\underset{\alpha}{\forall}\ x)\ (P(x) \vee Q(x)) \Rightarrow (\underset{\alpha}{\forall}\ x)\ P(x) \vee (\underset{\alpha}{\exists}\ x)\ Q(x)$. Similarly, we can prove the following:

(a) $B_\alpha(x)$ follows from the premises $(\underset{\alpha}{\forall}\ x)\ (A(x) \longrightarrow B(x))$ and $A_\alpha(x)$.

(b) $(\underset{\alpha}{\forall}\ x)\ (A(x) \longrightarrow C(x))$ follows from the premises $(\underset{\alpha}{\forall}\ x)\ (A(x) \longrightarrow B(x))$, $(\underset{\alpha}{\forall}\ x)\ (B(x) \longrightarrow C(x))$.

(c) $(\overset{\alpha}{\forall}\ x)\ (A(x) \longrightarrow C(x))$ follows from the premises $(\overset{\alpha}{\forall}\ x)\ (A(x) \longrightarrow B(x))$, $(\overset{\alpha}{\forall}\ x)\ (B(x) \longrightarrow C(x))$.

(d) $(\underset{\alpha}{\exists}\ x)\ B(x)$ follows from the premises $(\underset{\alpha}{\forall}\ x)\ (A(x) \longrightarrow B(x))$,$(\underset{\alpha}{\exists}\ x)\ A(x)$.

(e) $(\overset{\alpha}{\exists}\ x)\ B(x)$ follows from the premises $(\overset{\alpha}{\forall}\ x)\ (A(x) \longrightarrow B(x))$,$(\overset{\alpha}{\exists}\ x)\ A(x)$.

(f) $(\underset{\alpha}{\exists}\ x)\ (Q(x) \wedge R(x))$ follows from the premises $(\underset{\alpha}{\forall}\ x)\ (P(x) \longrightarrow Q(x))$, $(\underset{\alpha}{\exists}\ x)\ (P(x) \wedge R(x))$.

(g) $(\overset{\alpha}{\exists}\ x)\ (Q(x) \wedge R(x))$ follows from the premises $(\overset{\alpha}{\forall}\ x)\ (P(x) \longrightarrow Q(x))$, $(\overset{\alpha}{\exists}\ x)\ (P(x) \wedge R(x))$.

(h) $(\underset{\alpha}{\exists}\ x)\ (P(x) \wedge negB(x))$ follows from the premises $(\underset{\alpha}{\exists}\ x)\ (C(x) \wedge negB(x))$, $(\underset{\alpha}{\forall}\ x)\ (C(x) \longrightarrow P(x))$.

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(i) $(\overset{\alpha}{\exists}\, x)\ (P(x) \wedge negB(x))$ follows from the premises $(\overset{\alpha}{\exists}\, x)\ (C(x) \wedge negB(x))$, $(\overset{\alpha}{\forall}\, x)\ (C(x) \longrightarrow P(x))$.

(j) $(\overset{1-\alpha}{\forall}\, x)\ (P(x) \longrightarrow negS(x))$ follows from the premises $(\underset{\alpha}{\exists}\, y)\ (Q(y) \wedge negR(y))$, $(\underset{\alpha}{\exists}\, x)\ (P(x) \wedge S(x)) \longrightarrow (\underset{\alpha}{\forall}\, y)\ (Q(y) \longrightarrow R(y))$.

(k) $(\underset{1-\alpha}{\forall}\, x)\ (P(x) \longrightarrow negS(x))$ follows from the premises $(\overset{\alpha}{\exists}\, y)\ (Q(y) \wedge negR(y))$, $(\overset{\alpha}{\exists}\, x)\ (P(x) \wedge S(x)) \longrightarrow (\overset{\alpha}{\forall}\, y)\ (Q(y) \longrightarrow R(y))$.

## 4. Conclusion

In this paper, we have introduced a Naïve Approach on defining rough quantifiers in Predicate Calculus which may lead various applications in deriving hybrid [rough-fuzzy] rule based systems.

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