International Electronic Journal of Pure and Applied Mathematics

Volume 9 No. 3 2015, 201-203
ISSN: 1314-0744
url: http://www.e.ijpam.eu
doi: http://dx.doi.org/10.12732/iejpam.v9i3.9

NEW GROUP INVARIANT SOLUTIONS OF A VARIABLE-COEFFICIENT KDV EQUATION

Yuanyuan Zhang
College of Science
China Three Gorges University
Yichang, Hubei, 443002, P.R. CHINA

Abstract: In this paper, some new group invariant solutions of a variable-coefficient KdV equation are obtained by employing the Lie point symmetries.

AMS Subject Classification: 35Q53
Key Words: group invariant solutions, variable-coefficient KdV equation

1. Introduction

During the past several years, the study of coupled nonlinear evolution equations (NEEs) has played an important role in explaining many interesting phenomena such as the fluid dynamics, plasma physics and so on[1−4]. Recently, utilizing the two-dimensional equations of an incompressible inviscid fluid and the reductive perturbation method, Hilmi studied the propagation of weakly nonlinear waves in water of variable depth[5]. It is proposed that, in the case of slowly varying depth, the evolution equation is obtained as a variable-coefficient KdV equation

$$6\alpha u_t + 9\alpha^4 u u_x + u_{xxx} - 3\alpha'(t)u = 0 = 0. \tag{1}$$

In this paper, we would like to apply the Lie point symmetries to the variable-coefficient mKPI (vcmKPI) equation to obtain some new group invariant solutions.

Received: February 5, 2015

2. New Group-invariant Solutions of vcKdV Equation

We consider the classical Lie symmetry analysis of Eq.(1). Invariance of it under a Lie group of point transformations with infinitesimal generator

$$V = \xi(x,t,u)\partial_x + \tau(x,t,u)\partial_t + \eta(x,t,u)\partial\partial_u \tag{2}$$

leads to a set of determining equations for the infinitesimals $\xi(x,t,u), \tau(x,t,u)$, and $\eta(x,t,u)$. By using the Maple program GeM developed by Cheviakov[13)], these tedious equations can be easily obtained.

Next, we are going to derive some exact solutions of vcKdV by the symmetry reduction approach. The general method for performing the symmetry reduction using some specific subgroup G_0 of the symmetry group G is to find the invariants of G_0 and rewrite Eq.(1) in terms of these invariants. We present the main results following above several cases.

Case 2.1. The group-invariant solution corresponding to V_1 is $u = u(t)$, where t is the group-invariant. The substitution of this solution into Eq.(1) gives the simple solution $u(x,t) = C\alpha^{\frac{1}{2}}(t)$, where C is a constant and $\alpha(t) > 0$.

The symmetry V_2 gives rise to the group-invariant solution

$$u(x,t) = \frac{2}{3}\frac{U(t) + x\alpha^{\frac{1}{2}}(t)}{\int \alpha^{\frac{7}{2}}(t)dt}, \tag{3}$$

where $U(t)$ satisfies the reduced ordinary differential equation

$$2U'(t)\alpha(t) - \alpha'(t)U(t) = 0. \tag{4}$$

and can be solved as $U(t) = C\alpha^{\frac{1}{2}}(t)$ with C a constant and $\alpha(t) > 0$.

Case 2.2. If $\alpha(t) = t^n (n \neq 0, -\frac{1}{8})$, the group-invariant solution corresponding to V_{31} is

$$u(x,t) = U(\xi)t^{-\frac{10n+2}{3}},\ \xi = xt^{\frac{n-1}{3}}, \tag{5}$$

where $U(\xi)$ satisfies

$$U''' + 2(n-1)\xi U' - (23n+4)U + 9UU' = 0. \tag{6}$$

Case 2.3. If $\alpha(t) = e^{\lambda t}(\lambda \neq 0)$, the group-invariant solution corresponding to V_{32} is

$$u(x,t) = U(\xi)e^{-\frac{10\lambda t}{3}},\ \xi = xe^{\frac{\lambda t}{3}}, \tag{7}$$

where $U(\xi)$ satisfies

$$U''' + 2\lambda\xi U' + 9UU' - 23\lambda U = 0. \tag{8}$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 9, No. 3 (2015)

Case 2.4. The group-invariant solution corresponding to V_{33} is

$$u(x,t) = U(\xi)t^{-\frac{1}{4}},\ \xi = xt^{-\frac{3}{8}}, \tag{9}$$

where $U(\xi)$ satisfies

$$8U''' - 18\xi U' + 72UU' - 9U = 0. \tag{10}$$

Case 2.5. The symmetry V_4 gives rise to the group-invariant solution

$$u(x,t) = \frac{3}{8}xt^{-\frac{5}{8}} + U(\xi)t^{-\frac{5}{8}},\ \xi = xt^{-\frac{9}{1}6}, \tag{11}$$

where $U(\xi)$ satisfies the reduced ordinary differential equation

$$9UU' + U''' = 0, \tag{12}$$

which owns double periodic solution expressed by the Weierstrass elliptic function

$$U(\xi) = -\frac{4}{3}WeierstrassP(\xi + C_1, -\frac{3}{2}C_2, C_3). \tag{13}$$

3. Conclusion

In this paper, some new Group invariable solutions of a variable-coefficient KdV equation are obtained from the symmetry reductions. The method can also be used to solve other variable-coefficient nonlinear partial differential equations.

References

[1] R. Hirota, Exact solutions of the Korteweg-de Vries equation for multiple collisions of solitons, *Phys. Rev. Lett.*, **27** (1971), 1192-1194.

[2] N.C. Freeman and J.J.C. Nimmo, Soliton solutions of the KdV and KP equations: the Wronskian technique, *Phys. Lett. A*, **95** (1983), 1-3.

[3] J.J.C. Nimmo and N.C. Freeman, Rational solutions of the KdV equation in Wronskian form, *Phys. Lett. A*, **96** (1983), 443-446.

[4] V.B. Matveev, Generalized Wronskian formula for solutions of the KdV equations: first applications, *Phys. Lett. A*, **166** (1992), 205-208.

[5] R.S. Johnson, Solitary wave, soliton and shelf evolution equation over variable depth, *J. Fluid Mech*, **276** (1994), 125-138.

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 9, No. 3 (2015)