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**Abstract:** The purpose of this paper is to give a new type of open function and closed function called Quasi  $g\zeta^*$ -open function and Quasi  $g\zeta^*$ -closed function. Also, we obtain its characterizations and its basic properties.

**Key Words:**  $g\zeta^*$ -open set,  $g\zeta^*$ -closed set, quasi  $g\zeta^*$ -open function, quasi  $g\zeta^*$ -closed function

## 1. Introduction

Levine [5] offered a new and useful notion in General Topology that is the notion of a generalized closed set. This notion has been studied extensively in recent years by many topologists. The investigation of generalized closed sets has led to several new and interesting concepts, e.g. new covering properties and new separation axioms weaker than  $T_1$ . Some of these separation axioms have been found to be useful in computer science and digital topology.

After the introduction of generalized closed sets there are many research papers which deal with different types of generalized closed sets. Devi et al.[1] and Maki et al.[2] introduced semi-generalised closed sets (briefly  $sg$ -closed), generalised semi-closed sets (briefly  $gs$ -closed), generalised  $\alpha$ -closed (briefly  $g\alpha$ -closed) sets and  $\alpha$ -generalised closed (briefly  $\alpha g$ -closed) sets respectively. From this set V.Kokilavani, M.Myvizhi and M.Vivek Prabu [3] introduced  $g\zeta^*$ -Closed Sets in Topological spaces, In this paper we have introduced Quasi  $g\zeta^*$ -open and Quasi  $g\zeta^*$ -closed function in topological spaces and studied some of its properties.

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## 2. Preliminaries

Throughout this paper, spaces  $X$  and  $Y$  always mean topological spaces. Let  $X$  be a topological space and  $A$  be a subset of  $X$ . The closure of  $A$  and the interior of  $A$  are denoted by  $cl(A)$  and  $int(A)$ , respectively. A subset  $A$  of a space  $X$  is called  $\alpha$ -open set [4] if  $A \subseteq int(cl(int(A)))$  and the complement of a  $\alpha$ -open set is called  $\alpha$ -closed set.

We recall the following definition used in sequel.

**Definition 2.1.** A subset  $A$  of a space  $(X, \tau)$  is called:

(i) a generalized closed set [5] (briefly  $g$ -closed) if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .

(ii) a generalized  $\#$ - $\alpha$ -closed set [4] (briefly  $g^\# \alpha$ -closed) if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$ .

(iii) a  $\#$  generalized  $\alpha$ -closed set [7] (briefly  $\# g \alpha$ -closed) if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is a  $g^\# \alpha$ -open in  $(X, \tau)$ .

(iv) a generalized  $\zeta^*$ -closed set [3] (briefly  $g\zeta^*$ -closed) if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\# g \alpha$ -open set in  $(X, \tau)$ .

The complement of above mentioned closed sets are their respective open sets.

**Definition 2.2.** A map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called:

(i) a  $g\zeta^*$ -continuous [3] if  $f^{-1}(V)$  is  $g\zeta^*$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

(ii) a  $g\zeta^*$ -irresolute [3] if  $f^{-1}(V)$  is  $g\zeta^*$ -closed in  $(X, \tau)$  for every  $g\zeta^*$ -closed set  $V$  of  $(Y, \sigma)$ .

## 3. QUASI $g\zeta^*$ -Open Functions and QUASI $g\zeta^*$ -Closed Functions

**Definition 3.1.** A function  $f : X \rightarrow Y$  is said to be Quasi  $g\zeta^*$ -open if the image of every  $g\zeta^*$ -open set in  $X$  is open in  $Y$ .

**Example 3.2.** Let  $X = Y = \{a, b, c\}$  with  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$  be topologies on  $(X, \tau)$  and  $(Y, \sigma)$ , respectively. Here  $g\zeta^*$ -open set in  $(X, \tau)$  is  $\{X, \phi, \{a\}, \{b\}, \{a, b\}\}$  and open set in  $(Y, \sigma)$  is  $\{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ . Define a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = a$ ,  $f(b) = b$  and  $f(c) = c$ , then  $f$  is quasi  $g\zeta^*$ -open function. Since the image of each  $g\zeta^*$ -open set in  $(X, \tau)$  is open set in  $(Y, \sigma)$ .

**Definition 3.3.** A function  $f : X \rightarrow Y$  is said to be  $g\zeta^*$ -open if the image of every open set in  $X$  is  $g\zeta^*$ -open in  $Y$ .

**Theorem 3.4** A function  $f : X \rightarrow Y$  is said to be Quasi  $g\zeta^*$ -open if and only if for every subset  $U$  of  $X$ ,  $f(g\zeta^* - \text{int}(U)) \subset \text{int}(f(U))$ .

*Proof.* Let  $f$  be a quasi  $g\zeta^*$ -open function. Now, we have  $\text{int}(U) \subset U$  and  $g\zeta^* - \text{int}(U)$  is a  $g\zeta^*$ -open set. Hence, we obtain that  $f(g\zeta^* - \text{int}(U)) \subset (f(U))$ . As  $f(g\zeta^* - \text{int}(U))$  is open,  $f(g\zeta^* - \text{int}(U)) \subset \text{int}(f(U))$ .

Conversely, assume that  $U$  is a  $g\zeta^*$ -open set in  $X$ . Then  $f(U) = f(g\zeta^* - \text{int}(U)) \subset \text{int}(f(U))$  but  $\text{int}(f(U)) \subset f(U)$ . Consequently  $f(U) = \text{int}(f(U))$  and hence  $f$  is Quasi  $g\zeta^*$ -open.

**Lemma 3.5.** If a function  $f : X \rightarrow Y$  is quasi  $g\zeta^*$ -open, then  $g\zeta^* - \text{int}(f^{-1}(G)) \subset f^{-1}(\text{int}(G))$  for every subset  $G$  of  $Y$ .

*Proof.* Let  $G$  be any arbitrary subset of  $Y$ . Then,  $g\zeta^* - \text{int}(f^{-1}(G))$  is a  $g\zeta^*$ -open set in  $X$  and  $f$  is quasi  $g\zeta^*$ -open, then

$$f(g\zeta^* - \text{int}(f^{-1}(G))) \subset \text{int}(f(f^{-1}(G))) \subset \text{int}(G).$$

Thus  $g\zeta^* - \text{int}(f^{-1}(G)) \subset f^{-1}(\text{int}(G))$ .

**Definition 3.6.** A subset  $S$  is said to be an  $g\zeta^*$ -neighbourhood of a point  $x$  of  $X$  if there exists a  $g\zeta^*$ -open set  $U$  such that  $x \in U \subset S$ .

**Lemma 3.7.** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are two functions and  $g \circ f : X \rightarrow Z$  is Quasi  $g\zeta^*$ -open. If  $g$  is continuous injective, then  $f$  is Quasi  $g\zeta^*$ -open.

*Proof.* Let  $U$  be a  $g\zeta^*$ -open set in  $X$ , then  $(g \circ f)(U)$  is open in  $Z$ . Since  $g \circ f$  is quasi  $g\zeta^*$ -open. Again  $g$  is an injective continuous function,  $f(U) = g^{-1}(g \circ f)(U)$  is open in  $Y$ . This shows that  $f$  is quasi  $g\zeta^*$ -open.

**Theorem 3.8.** If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are two Quasi  $g\zeta^*$ -open functions, then  $g \circ f : X \rightarrow Z$  is a Quasi  $g\zeta^*$ -open function.

*Proof.* Let  $F$  be any  $g\zeta^*$ -open set in  $X$ , since  $f$  is Quasi  $g\zeta^*$ -open function,  $f(F)$  is an open set in  $Y$ , we know that every open set is  $g\zeta^*$ -open [3],  $f(F)$  is a  $g\zeta^*$ -open set. Since  $g$  is a Quasi  $g\zeta^*$ -open function,  $g(f(F))$  is open in  $Z$ . That is  $(g \circ f)(F) = g(f(F))$  is open set in  $Z$  and hence  $g \circ f$  is a Quasi  $g\zeta^*$ -open function.

**Definition 3.9.** A function  $f : X \rightarrow Y$  is called  $(g\zeta^*)^*$ -open function if the image of every  $g\zeta^*$ -open subset of  $X$  is  $g\zeta^*$ -open in  $Y$ .

**Theorem 3.10.** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be any two functions. Then:

(i) If  $f$  is  $g\zeta^*$ -open and  $g$  is quasi  $g\zeta^*$ -open, then  $g \circ f$  is open function.

(ii) If  $f$  is a quasi  $g\zeta^*$ -open function and  $g$  is  $g\zeta^*$ -open function, then  $g \circ f$  is Quasi  $(g\zeta^*)^*$ -open function.

(iii) If  $f$  is a quasi  $(g\zeta^*)^*$ -open function and  $g$  is quasi  $g\zeta^*$ -open function, then  $g \circ f$  is quasi  $g\zeta^*$  open function.

*Proof.* (i) Let  $F$  be any open set in  $X$ , since  $f$  is  $g\zeta^*$ -open function,  $f(F)$  is a  $g\zeta^*$ -open set in  $Y$ . Since  $g$  is a Quasi  $g\zeta^*$ -open function,  $g(f(F))$  is open set in  $Z$ . That is  $(g \circ f)(F) = g(f(F))$  is open set in  $Z$  and hence  $g \circ f$  is a open function.

(ii) Let  $F$  be any  $g\zeta^*$ -open set in  $X$ , since  $f$  is Quasi  $g\zeta^*$ -open function,  $f(F)$  is a open set in  $Y$ . Since  $g$  is a  $g\zeta^*$ -open function,  $g(f(F))$  is  $g\zeta^*$ -open set in  $Z$ . That is  $(g \circ f)(F) = g(f(F))$  is  $g\zeta^*$ -open set in  $Z$  and hence  $g \circ f$  is a Quasi  $(g\zeta^*)^*$ -open function.

(iii) Let  $F$  be any  $g\zeta^*$ -open set in  $X$ , since  $f$  is Quasi  $(g\zeta^*)^*$ -open function,  $f(F)$  is a  $g\zeta^*$ -open set in  $Y$ . Since  $g$  is a Quasi  $g\zeta^*$ -open function,  $g(f(F))$  is open set in  $Z$ . That is  $(g \circ f)(F) = g(f(F))$  is open set in  $Z$  and hence  $g \circ f$  is a Quasi  $g\zeta^*$ -open function.

**Definition 3.11.** A function  $f : X \rightarrow Y$  is said to be Quasi  $g\zeta^*$ -closed if the image of each  $g\zeta^*$ -closed set in  $X$  is closed in  $Y$ .

**Example 3.12.** Let  $X = Y = \{a, b, c\}$  with  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$  be topologies on  $(X, \tau)$  and  $(Y, \sigma)$ , respectively. Here  $g\zeta^*$ -closed set in  $(X, \tau)$  is  $\{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$  and closed set in  $(Y, \sigma)$  is  $\{X, \phi, \{a\}, \{c\}, \{b, c\}, \{a, c\}\}$ . Define a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = a, f(b) = b$  and  $f(c) = c$ , then  $f$  is quasi  $g\zeta^*$ -closed function. Since the image of each  $g\zeta^*$ -closed set in  $(X, \tau)$  is closed set in  $(Y, \sigma)$ .

**Definition 3.13** A function  $f : X \rightarrow Y$  is said to be  $g\zeta^*$ -closed if the image of every closed set in  $X$  is  $g\zeta^*$ -closed in  $Y$ .

**Lemma 3.14.** A function  $f : X \rightarrow Y$  is said to be Quasi  $g\zeta^*$ -closed function, then  $f^{-1}(\text{int}(A)) \subset g\zeta^* - \text{int}(f^{-1}(A))$  for every subset  $A$  of  $Y$ .

*Proof.* This proof is similar to the proof of Lemma 3.5.

**Definition 3.15.** A function  $f : X \rightarrow Y$  is called  $(g\zeta^*)^*$ -closed function if the image of every  $g\zeta^*$ -closed subset of  $X$  is  $g\zeta^*$ -closed in  $Y$ .

**Theorem 3.16.** *If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are two Quasi  $g\zeta^*$ -closed functions, then  $g \circ f : X \rightarrow Z$  is a Quasi  $g\zeta^*$ -closed function.*

*Proof.* Let  $F$  be any  $g\zeta^*$ -closed set in  $X$ , since  $f$  is Quasi  $g\zeta^*$ -closed function,  $f(F)$  is a closed set in  $Y$ , we know that every closed set is  $g\zeta^*$ -closed set [3],  $f(F)$  is an  $g\zeta^*$ -closed set. Since  $g$  is a Quasi  $g\zeta^*$ -closed function,  $g(f(F))$  is closed in  $Z$ . That is  $(g \circ f)(F) = g(f(F))$  is closed set in  $Z$  and hence  $g \circ f$  is a Quasi  $g\zeta^*$ -closed function.

**Theorem 3.17.** *Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be any two functions. Then:*

(i) *If  $f$  is  $g\zeta^*$ -closed function and  $g$  is Quasi  $g\zeta^*$ -closed function, then  $g \circ f$  is closed function.*

(ii) *If  $f$  is Quasi  $g\zeta^*$ -closed function and  $g$  is  $g\zeta^*$ -closed function, then  $g \circ f$  is  $(g\zeta^*)^*$ -closed function.*

(iii) *If  $f$  is  $(g\zeta^*)^*$ -closed function and  $g$  is Quasi  $g\zeta^*$ -closed function, then  $g \circ f$  is Quasi  $g\zeta^*$ -closed function.*

*Proof.* (i) Let  $F$  be any closed set in  $X$ , since  $f$  is  $g\zeta^*$ -closed function,  $f(F)$  is a  $g\zeta^*$ -closed set in  $Y$ . Since  $g$  is a Quasi  $g\zeta^*$ -closed function,  $g(f(F))$  is closed set in  $Z$ . That is  $(g \circ f)(F) = g(f(F))$  is closed set in  $Z$  and hence  $g \circ f$  is a closed function.

(ii) Let  $F$  be any  $g\zeta^*$ -closed set in  $X$ , since  $f$  is Quasi  $g\zeta^*$ -closed function,  $f(F)$  is a closed set in  $Y$ . Since  $g$  is a  $g\zeta^*$ -closed function,  $g(f(F))$  is  $g\zeta^*$ -closed set in  $Z$ . That is  $(g \circ f)(F) = g(f(F))$  is  $g\zeta^*$ -closed set in  $Z$  and hence  $g \circ f$  is a  $(g\zeta^*)^*$ -closed function.

(iii) Let  $F$  be any  $g\zeta^*$ -closed set in  $X$ , since  $f$  is  $(g\zeta^*)^*$ -closed function,  $f(F)$  is a  $g\zeta^*$ -closed set in  $Y$ . Since  $g$  is a Quasi  $g\zeta^*$ -closed function,  $g(f(F))$  is closed set in  $Z$ . That is  $(g \circ f)(F) = g(f(F))$  is closed set in  $Z$  and hence  $g \circ f$  is a Quasi  $g\zeta^*$ -closed function.

**Theorem 3.18.** *Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be any two functions such that  $g \circ f : X \rightarrow Z$  is Quasi  $g\zeta^*$ -closed.*

(i) *If  $f$  is  $g\zeta^*$ -irresolute surjective, then  $g$  is closed.*

(ii) *If  $g$  is  $g\zeta^*$ -continuous injective, then  $f$  is  $(g\zeta^*)^*$ -closed.*

*Proof.* (i) Suppose that  $F$  is an arbitrary closed set in  $Y$ . As  $f$  is  $g\zeta^*$ -irresolute,  $f^{-1}(F)$  is  $g\zeta^*$ -closed in  $X$ . Since  $g \circ f$  is Quasi  $g\zeta^*$ -closed and  $f$  is surjective,  $(g \circ f(f^{-1}(F))) = g(F)$ , which is closed in  $Z$ . This implies that  $g$  is a closed function.

(ii) Suppose  $F$  is any  $g\zeta^*$ -closed set in  $X$ . Since  $g \circ f$  is Quasi  $g\zeta^*$ -closed,  $(g \circ f)(F)$  is closed in  $Z$ . Again  $g$  is a  $g\zeta^*$ -continuous injective function,  $g^{-1}(g \circ f)(F) = f(F)$ , which is  $g\zeta^*$ -closed in  $Y$ . This shows that  $f$  is  $(g\zeta^*)^*$ -closed.

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