

CONNECTEDNESS AND MIXED CONNECTEDNESS ON T_0 -ALEXANDROFF SPACES

Hisham Mahdi

Department of Mathematics

Islamic University of Gaza

P.O. Box 108, Gaza, PALESTINE

Abstract: In this paper, connectedness and its related properties are studied on T_0 -Alexandroff spaces. The concept of posets is implemented in characterizing path-connectedness, hyperconnectedness, extremally disconnectedness, and totally disconnectedness. Special relationships between some types of mixed generalized connectedness, such as α – S -connectedness, τ – P -connectedness, and others, are proved on upper bounded T_0 -Alexandroff spaces.

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1. Introduction

An *Alexandroff space* [14] (briefly A -space) (or *smallest neighborhood space*) X is a topological space in which the arbitrary intersection of open sets is open. In this space, each element x possesses a smallest open neighborhood $V(x)$ which is the intersection of all open sets containing x . For every T_0 A -space (X, τ) , there is a corresponding poset (X, \leq_τ) in one to one and onto way, where each one of them is completely determined by the other. If (X, τ) is a T_0 A -space, we define the corresponding partial order \leq_τ , called (*Alexandroff*) *specialization order*, by: $a \leq_\tau b$ iff $a \in \overline{\{b\}}$ iff $b \in V(a)$. On the other hand, if (X, \leq) is a poset, then the collection $\mathcal{B} = \{\uparrow x : x \in P\}$ forms a base for a T_0 A -space on X , denoted by τ_\leq . So throughout this paper, we consider $(X, \tau(\leq))$ to be a T_0 A -space (X, τ) together

with its corresponding poset (X, \leq) . A poset (X, \leq) satisfies the *ascending chain condition* (briefly *ACC*), if for any increasing sequence $x_1 \leq x_2 \leq x_3 \leq \dots$ in X , there exists $k \in \mathbb{N}$ such that $x_k = x_{k+1} = \dots$. A T_0 A -space whose corresponding poset satisfies the *ACC* is called *Artinian T_0 A -space*. An upper bounded T_0 A -space is introduced as a generalization of an Artinian T_0 -space. A space is *upper bounded T_0 A -space* (briefly *UB T_0 A -space*) if every chain of points in the corresponding poset (X, \leq) is bounded above [5]. We give a detail study of these spaces in [8]. A space is *lower bounded (LB)* [5] if every chain of points is bounded below. Given a poset (X, \leq) , the set of all maximal elements is denoted by $M(X)$ (or simply by M) and the set of all minimal elements is denoted by $m(X)$ (or simply by m). Moreover, for each $x \in X$, we define \hat{x} to be the set of all maximal elements greater than or equal to x and \check{x} the set of all minimal elements less than or equal to x . If X is a *UB T_0 A -space*, then $M \neq \emptyset$ and $\hat{x} \neq \emptyset \quad \forall x \in X$. Similarly in an *LB T_0 A -space*, $m \neq \emptyset$ and $\check{x} \neq \emptyset \quad \forall x \in X$. In previous studies (see [9], [8]), we formulate the results in the language of poset and order instead of the language of topology. This technique proves to be an easier approach. For instance, by using this technique, we see that a set A is open (closed) iff A is up (down) set in the corresponding poset. For each $x \in X$, $V(x) = \uparrow x$. And X is submaximal iff each element in the corresponding poset is either maximal or minimal. If P and Q are disjoint ordered sets then the *disjoint union* $P \cup Q$ is the ordered set formed by defining $x \leq y$ in $P \cup Q$ iff either $x \leq y$ in P or $x \leq y$ in Q . In the disjoint union, the elements of P are incomparable with the elements of Q .

2. Preliminaries

A subfamily m_x of a power set $\mathcal{P}(X)$ of a non-empty set X is said to be a *minimal structure* on X if $\emptyset, X \in m_x$ and $\bigcup A_\alpha \in m_x$ whenever $A_\alpha \in m_x$. The sets in m_x are called m_x -open sets, and the sets whose complements in m_x are called m_x -closed sets. It is clearly that an arbitrary union (resp. arbitrary intersection) of m_x -open (resp. m_x -closed) sets is m_x -open (resp. m_x -closed) set. For $A \subseteq X$, we define m_x interior and m_x closure of A as follows:

$$m_x - Int(A) = \bigcup \{U : U \subseteq A \text{ and } U \in m_x\}, \text{ and}$$

$$m_x - Cl(A) = \bigcap \{F : A \subseteq F \text{ and } X \setminus F \in m_x\}.$$

It is obvious that $m_x - Int(A)$ is the largest m_x -open set inside A and $m_x - Cl(A)$ is the smallest m_x -closed set containing A .

Let (X, τ) be a topological space. A subset A of X is said to be α -open [13] (resp. *preopen* [2], *semi-open* [12], *b-open* [3] (equiv. γ -open [1]), β -open [11] (equiv. *semi-preopen* [4])) if $A \subseteq \overline{A}^\circ$ (resp. $A \subseteq \overline{A}$, $A \subseteq \overline{A}^\circ$, $A \subseteq \overline{A} \cup \overline{A}^\circ$,

$A \subseteq \overline{\overline{A}^o}$). A set F is called j -closed for $j \in \{\alpha, \text{semi}, \text{pre}, b, \beta\}$ if $X \setminus F$ is j -open. A set F is called j -clopen for $j \in \{\alpha, \text{semi}, \text{pre}, b, \beta\}$ if F is both j -open and j -closed. The family of all α -open (resp. preopen, preclosed, semi-open, semi-closed, b -open, b -closed, β -open, β -closed) is denoted by τ_α (resp. $PO(X)$, $PC(X)$, $SO(X)$, $SC(X)$, $BO(X)$, $BC(X)$, $\beta O(X)$, $\beta C(X)$). We have the following facts: The collection τ_α forms a topology on X [13]. $\tau_\alpha = PO(X) \cap SO(X)$ [15]. $PO(X) \cup SO(X) \subseteq BO(X) \subseteq \beta O(X)$ [3]. $\tau \subseteq \tau_\alpha$ (resp. $\tau \subseteq JO(X)$ for $J \in \{S, P, B, \beta\}$). For $j \in \{\text{semi}, \text{pre}, b, \beta\}$, the union (intersection) of any family of j -open (j -closed) sets is j -open (j -closed). Thus for $J \in \{S, P, B, \beta\}$, $JO(X)$ is a minimal structure on X . If a minimal structure $m_x = \tau_\alpha$ (resp. $PO(X)$, $SO(X)$, $BO(X)$, $\beta O(X)$), then $m_x - \text{Int}(A)$ is denoted by $\text{Int}_\alpha(A)$ (resp. $p\text{Int}(A)$, $s\text{Int}(A)$, $b\text{Int}(A)$, $\beta\text{Int}(A)$). Similarly $m_x - \text{Cl}(A)$ is denoted by $\text{Cl}_\alpha(A)$ (resp. $p\text{Cl}(A)$, $s\text{Cl}(A)$, $b\text{Cl}(A)$, $\beta\text{Cl}(A)$). In [8], we prove that if X is a $UB T_0$ A -space, then $PO(X) = \tau_\alpha$ and $PO(X) \subseteq SO(X)$.

3. Connectedness on T_0 -Alexandroff Spaces

A topological space (X, τ) is said to be *connected* if X cannot be expressed as a union of two non-empty and disjoint open subsets of X . X is said to be *disconnected* if it is not connected. In this case, X contains a nontrivial clopen set. X is said to be *path connected* (resp. *arc-connected*) if for each two points a and b in X , there is a continuous function (resp. a continuous, open, and $1-1$ function) $f : [0, 1] \rightarrow X$ such that $f(0) = a$ and $f(1) = b$. Such a function is called a *path* (resp. an *arc*) from a to b . Every arc-connected space is path connected and every path connected space is connected, but not conversely in general. X is *locally connected* if each $x \in X$ has a neighborhood base consisting of open connected sets. X is *locally path connected* if each $x \in X$ has a neighborhood base consisting of path connected sets. For $x \in X$, the largest connected subset C_x of X that containing x is called the *component* of x . It is the union of all connected subsets of X containing x . The components of X are closed sets. The *quasi-component* C'_x of x is the intersection of all clopen sets containing x . It is always true that $C_x \subseteq C'_x$, and if X is locally connected then $C_x = C'_x$. X is *hyperconnected* if every non-empty open subset of X is dense. If X is not hyperconnected, then it is *hyperdisconnected*. X is *extremally disconnected* if the closure of every open set is open. And X is *totally disconnected* if the only non-empty connected subsets of X are the one-point sets.

Theorem 3.1. (see [7]) *Let X be a T_0 -Alexandroff space. The following statements are equivalent.*

1. X is path-connected.
2. X is connected.

3. For every $a, b \in X$, there exist $a_0, \dots, a_{n+1} \in X$ such that $a = a_0, a_{n+1} = b$ and $V(a_i) \cap V(a_j) \neq \emptyset$ if $|i - j| \leq 1$.
4. For every $a, b \in X$, there exist $a_0, \dots, a_{m+1} \in X$ such that $a = a_0, a_{m+1} = b$ and $\overline{V(a_i)} \cap \overline{V(a_j)} \neq \emptyset$ if $|i - j| \leq 1$.
5. For every $a, b \in X$, there exist $a_0, \dots, a_{k+1} \in X$ such that $a = a_0, a_{k+1} = b$ and $\overline{\{a_i\}} \cap \overline{\{a_j\}} \neq \emptyset$ if $|i - j| \leq 1$.

In a T_0 A -space (X, τ) with corresponding poset (X, \leq) , the translation of condition (3) of the above theorem in the language of poset is that there is a finite sequence of connected line segments in the Hasse diagram of the poset X connecting a to b . Moreover if (X, τ_d) is the dual space of (X, τ) ($:=$ the T_0 A -space with corresponding poset that has reverse order), then (X, τ) is connected iff (X, τ_d) is connected.

Theorem 3.2. *Let $(X, \tau(\leq))$ be a T_0 A -space, then X is connected iff the corresponding poset (X, \leq) cannot be represented as a disjoint union of two partially ordered sets.*

Proof. If X is disconnected, then there exist two non-empty disjoint clopen subsets P and Q such that $X = P \cup Q$. Let \leq_1, \leq_2 be the induced orders on P and Q respectively. Then X is the disjoint union of P and Q . Conversely, if (X, \leq) is a disjoint union of two partially ordered sets (P, \leq_1) and (Q, \leq_2) , then by the way of the formulation of \leq , P is both up and down set in (X, \leq) . Hence P is non-trivial clopen set in $(X, \tau(\leq))$. \square

Theorem 3.3. *Let X be a $UB T_0$ A -space with a set of maximal elements M . Then X is connected iff for any proper subset C of M , $\overline{C} \cap \overline{M \setminus C} \neq \emptyset$.*

Proof. If $\overline{C} \cap \overline{M \setminus C} = \emptyset$, then $P = \overline{C}$ is a non-trivial clopen subset in X . Conversely, let $X = P \cup Q$, where P and Q are two non-empty disjoint open subsets in X . Take $C = M \cap P$. So $M \setminus C = M \cap Q$. Moreover, $\overline{C} = P$ and $\overline{M \setminus C} = Q$. \square

Corollary 3.4. *Let X be a T_0 A -space. If X contains a dense subset consisting of a single point, then X is connected.*

Proof. The corresponding poset contains a top element, so X is a $UB T_0$ A -space with no proper subsets of M . \square

Corollary 3.5. *Let X be an $LB T_0$ A -space with a set of minimal elements m . Then X is connected iff for any proper subset D of m , $V(D) \cap V(m \setminus D) \neq \emptyset$.*

Proof. Direct from the fact that the duality of an $LB T_0$ A -space (X, τ) with a set of minimal elements m is a $UB T_0$ A -space (X, τ_d) with a set of maximal elements M_d such that $M_d = m$ and $Cl_d(D) = V(D)$ for any $D \subseteq m$. \square

Corollary 3.6. *Let X be a T_0 A -space. If there exists a singleton set $\{x\}$ in X such that $V(x) = X$, then X is connected.*

Proof. The dual space has a corresponding poset with singleton dense subset. □

Theorem 3.7. *Let $(X, \tau(\leq))$ be a $UB T_0 A$ -space such that $|M| > 1$. If X is connected, then for each $x \in M$, $\downarrow x$ is not open.*

Proof. If there exists $x \in M$ such that $\downarrow x (= \bar{x})$ is open, then $\downarrow x$ is a nontrivial clopen subset in X . □

The converse is not always true as shown in the following example.

Example 3.8. Let $X = \{a, b, c, x, y, z\}$ with the order as shown in the figure below:

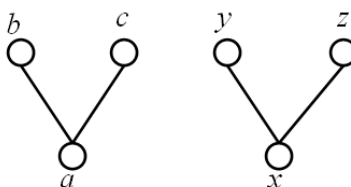


Figure 1: Disconnected $UB T_0 A$ -space

Then for all $r \in M$, $\downarrow r$ is not open, but X is disconnected.

In [7], it was proved that a $T_0 A$ -space is always locally path-connected, and hence a locally connected. So for each $x \in X$, $C_x = C'_x$.

Theorem 3.9. *Let X be a $T_0 A$ -space and $x \in X$. Then $\overline{V(x)} \subseteq C_x$.*

Proof. Obvious □

Theorem 3.10. *Let $(X, \tau(\leq))$ be a T_0 -Alexandroff space. Then*

1. X is totally disconnected iff the specialization order \leq is anti-chain. In this case, τ is discrete.
2. If the corresponding poset (X, \leq) is linearly ordered set, then X is hyperconnected.
3. If X contains a maximum element \top , then X is hyperconnected.
4. It may happens that a hyperconnected T_0 -Alexandroff space is not a linearly ordered and does not contain a maximum element.

Proof. (1) Let x, y be two distinct elements in X such that $x \leq y$. Then by Corollary 3.4, the set $\{x, y\}$ is connected subset of X , so X is not totally disconnected. The converse is obvious.

(2) Let U be a non-empty open set in X . So U is an up set in the corresponding poset. Pick $x \in U \subseteq \overline{U}$, and let $y \in X$. Then either $x \leq y$ or $x \geq y$. If $x \leq y$, then as U up set, $y \in U \subseteq \overline{U}$. If $x \geq y$, then as \overline{U} down set, $y \in \overline{U}$. Hence U is dense.

(3) Any open set U must contains the maximum element \top . Hence $\downarrow \top \subseteq \overline{U}$. Therefore $\overline{U} = X$.

(4) Part (1) of the following example gives a hyperconnected T_0 A -space with corresponding poset which is not linear and does not contain a maximum element. □

Example 3.11. (1) Let $X = \mathbb{N}$ be the set of natural numbers with a non linear order with no maximum described in the figure below, and $\tau(\leq)$ the corresponding T_0 A -topology on X .

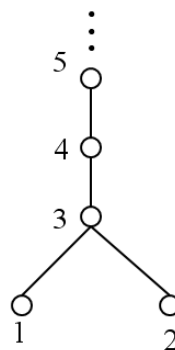


Figure 2: Hyperconnected space with no maximum

If U is an open set in X , then U is up set in the corresponding poset. So we can choose $x \in U \subseteq \overline{U}$ such that $x \geq 5$. Let $y \in X$. Then either $y \leq x$ or $y \geq x$. If $y \leq x$, then as \overline{U} down set, we get $y \in \overline{U}$. If $y \geq x$, then as U up set, we get $y \in U \subseteq \overline{U}$. Therefore $\overline{U} = X$, and hence X is hyperconnected.

(2) In general, a hyperconnected space is connected. The converse is not always true. The dual of the T_0 A -space in part (1) is a connected space which is not hyperconnected.

Theorem 3.12. *Let X be a $UB T_0$ A -space, then X is hyperconnected iff X contains an open and dense subset of X consisting of one point. In the corresponding poset, X contains a maximum element \top .*

Proof. If $|M| \geq 2$, then there exist two elements $x_1 \neq x_2$ in M . Hence $\{x_1\}$ is open subset of X which is not dense. \square

Theorem 3.13. [8] *In a $UB T_0 A$ -space X , the following are equivalent:*

- (1) X is extremally disconnected.
- (2) $PO(X) = SO(X)$.
- (3) For all $x \in X$, $|\hat{x}| = 1$.

Theorem 3.14. *Let X be a $T_0 A$ -space. If X is arc-connected, then for each pair x, y of points of X , there exists a connected subset $B_{x,y}$ of X containing x, y such that $|B_{x,y}| = \mathbf{c}$, where \mathbf{c} is the cardinality of the set $[0, 1]$.*

Proof. Let x, y be two distinct points in X and $p : [0, 1] \rightarrow X$ the arc between x and y such that $p(0) = x$ and $p(1) = y$. Then $B_{x,y} = p[0, 1]$ is connected subset of X containing x and y (a continuous image of the connected space is connected). Moreover, $|B_{x,y}| = |[0, 1]|$. \square

4. Types of Mixed Generalized Connectedness

Let X be a non-empty set and m_1, m_2 two minimal structures on X . Then X is said to be $m_1 - m_2$ -connected, if X cannot be expressed as a union of two non-empty subsets $S_1, S_2 \in X$ such that $S_1 \in m_1$ and $S_2 \in m_2$. It is obvious that the notion $m_1 - m_2$ -connectedness is equivalent to the notion $m_2 - m_1$ -connectedness. If $m_1 = m_2$, then X is said to be m_1 -connected. For special cases, if (X, τ) is a topological space and if $m_1 = \tau$ (resp. $\tau_\alpha, PO(X), SO(X), BO(X), \beta O(X)$), then m_1 -connected is denoted as connected (resp. α -connected [10], \mathcal{P} -connected [18], \mathcal{S} -connected [16], γ -connected [6], β -connected [17]). The mixed generalized connectedness is introduced in [19]. If $m_1 = \tau$ and $m_2 = \tau_\alpha$ (resp. $m_2 = PO(X), m_2 = SO(X), m_2 = BO(X), m_2 = \beta O(X)$), then $m_1 - m_2$ -connected is denoted as $\tau - \tau_\alpha$ -connected (resp. $\tau - P$ -connected, $\tau - S$ -connected, $\tau - SP$ -connected, $\tau - \beta$ -connected). Similarly, we define $\alpha - P$ -connectedness, $\alpha - S$ -connectedness, $\alpha - SP$ -connectedness, $\alpha - \beta$ -connectedness, $S - P$ -connectedness, $S - SP$ -connectedness, $S - \beta$ -connectedness, $P - SP$ -connectedness, $P - \beta$ -connectedness, and $\beta - SP$ -connectedness (for more information, see [19]).

Theorem 4.1. *Let (X, τ) be a topological space. Then all the following statements are equivalent:*

- 1) X is connected.
- 2) X is α -connected [10].

3) X is $\tau - \tau_\alpha$ -connected [19].

4) X is $\tau - P$ -connected [19].

Corollary 4.2. *Let $(X, \tau(\leq))$ be a $UB T_0 A$ -space. Then X is α -connected iff the corresponding poset cannot be written as a disjoint union of two partially ordered sets.*

Theorem 4.3. [19] *Let (X, τ) be a topological space. Then all the following statements are equivalent:*

- 1) X is \mathcal{S} -connected.
- 2) X is $\tau - SP$ -connected.
- 3) X is $S - SP$ -connected.
- 3) X is $\tau - B$ -connected.
- 5) X is $\tau - S$ -connected.
- 4) X is $S - B$ -connected.
- 5) X is $S - P$ -connected.
- 6) X is $\alpha - S$ -connected.
- 7) X is $\alpha - SP$ -connected.
- 8) X is $\alpha - B$ -connected.
- 9) Every non-empty open set is dense.

Corollary 4.4. *A topological space X is \mathcal{S} -connected iff it is hyperconnected.*

Theorem 4.5. [19] *Let (X, τ) be a topological space. Then all the following statements are equivalent:*

- 1) X is β -connected.
- 2) X is \mathcal{S} -connected and \mathcal{P} -connected.
- 3) X is $B - SP$ -connected.
- 4) X is $P - SP$ -connected.
- 5) X is γ -connected.
- 6) $pCl(A) = X$ for all $A \in PO(X)$.

Theorem 4.6. *Let $(X, \tau(\leq))$ be a $UB T_0$ A -space. Then we have the following:*

- 1) X is \mathcal{P} -connected iff it is α -connected.
- 2) If X is \mathcal{S} -connected, then it is α -connected.

Proof. In $UB T_0$ A -space s , $\tau_\alpha = PO(X) \subseteq SO(X)$. □

Corollary 4.7. *Let $(X, \tau(\leq))$ be a $UB T_0$ A -space. Then X is \mathcal{S} -connected iff it is β -connected.*

Proof. Follows from Theorem 4.5 and Theorem 4.6. □

Theorem 4.8. *Let $(X, \tau(\leq))$ be a $UB T_0$ A -space. Then X is \mathcal{S} -connected iff there exists a singleton dense subset of X .*

Proof. Direct from Theorem 3.12 and Corollary 4.4. □

The following theorem summarizes the results of this section about the mixed generalized connectedness on $UB T_0$ - A -spaces.

Theorem 4.9. *For a $UB T_0$ A -space $(X, \tau(\leq))$, consider the following statements:*

- 1) X is \mathcal{S} -connected.
- 2) X is τ - SP -connected.
- 3) X is S - SP -connected.
- 4) X is τ - B -connected.
- 5) X is τ - S -connected.
- 6) X is S - B -connected.
- 7) X is S - P -connected.
- 8) X is α - S -connected.
- 9) X is α - SP -connected.
- 10) X is α - B -connected.
- 11) X is hyperconnected.
- 12) X is β -connected.
- 13) X is B - SP -connected.

- 14) X is $P - SP$ -connected.
- 15) X is γ -connected.
- 16) $pCl(A) = X$ for all $A \in PO(X)$.
- 17) Every non-empty open set is dense.
- 18) There exists a singleton dense subset of X .
- 19) The corresponding poset has a maximum element.
- 20) X is connected.
- 21) X is α -connected .
- 22) X is $\tau - \tau_\alpha$ -connected .
- 23) X is $\tau - P$ -connected .
- 24) X is \mathcal{P} -connected.
- 25) The corresponding poset cannot be written as a disjoint union of two partially ordered sets.

Then the statements from (1) to (19) are equivalent and the statements from (20) to (25) are equivalent. Moreover, any statement from (1) to (19) implies any one of those in (20) to (25).

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