International Electronic Journal of Pure and Applied Mathematics

Volume 9 No. 1 2015, 1-15
ISSN: 1314-0744
url: http://www.e.ijpam.eu
doi: http://dx.doi.org/10.12732/iejpam.v9i1.1

A MATHEMATICAL MODEL FOR THE RELATIVE ASSESSMENT OF SURFACE AND DEEP LEARNING

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Abstract: In this paper we establish a mathematical model for the assessment of the human cognition, in particular, the student ability to learn in different practical cases. The method allows us to obtain quantitative and qualitative estimates to a group of undergraduates for their acquired knowledge. We illustrate the theory by a practical example. A qualitative analysis to the obtained data is presented.

AMS Subject Classification: 90A05, 92G20, 92G99, 90A19
Key Words: assessment, mathematical model, estimate, statistic, test theory

1. Introduction

Effective practices of assessment play a great role in the assimilation of knowledge,

Received: September 11, 2014

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[21], which is acknowledged by the known Black and Wiliam study, taking into account 250 sources, [3]. The cognition of results and their analysis leads to acquisition of certain skills, [27]. The creation of knowledge, skills and competences, which can use in the practice is the public expectation of the university education during 21 century, [8]. It is well known that the engineering specialties require a deeper approach and critical investigation of new facts and ideas, as well as their inclusion in the existing cognitive structures and putting the links between sets of ideas into effect, [1], [6], [28]. The laboratory work allows the following important results to be assessed:

- technical and manipulative skills to the use of laboratory equipment, tools, materials, software, etc.;
- understanding of abstract concepts and special terms;
- skills for scientific study and problem' solving;
- analysing and interpreting the set of real data, [9].

Frequently, the undergraduates are passive receivers of any result, moreover it turns out that focusing themselves it gets too late. Very often they accept the information indifferently as the unrelated facts and terms being only touched, registered and in such an insulation, as well they look for complete formulae to direct solving the problems under consideration. The experience to work by more complicated principles, being incomprehensible, may lead to the feeling of dissatisfactory and cursory possession of knowledge, thus the undergraduates get attempt to use solely a mechanical memory.

In the contemporary education the induction of both passive and reproductive forms plays an essential role to the teaching practice and assessment, which contradict to the education goals. In the most of cases one emphasizes on the facts, and at the same time there exists a very strong link with a proper theoretical frame. The assessment of the results cannot be realized only in the context of a unique exam at the end of the academic term. An aspect of interest is Dunning-Kruger effect of illusive competitiveness by which one cannot identify the proper mistakes, [26].

On the other hand the passageway to highest education is obstructed due to the following reasons:

- the increasing number of undergraduates stemming from different social and cultural groups leads to different levels of qualification, [30];
- underscored decreasing of the fundamental knowledge.

The international study known as TIMSS 2006 and PISA 2006-2011, as well as the results of the national study of the achievements of Bulgarian high school students (both ninth and tenth class) by “Chemistry and ecology of environment”

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in accordance with DOI and DZI data show that 80 % of school boys and girls do not achieve the standard knowledge, [30]. A good deal of them form misconceptions on the fundamentals of Chemistry that has a stable negative effect and it is at the same time a stone wall for the progress of learning, and education at all, [5]-[7], [15], [23], [25]. This conclusion spurs one to use a preliminary assessment by which to be estimated the preliminary cognition of the undergraduates, their levels of skills, identification of the preliminary delusions, profiles of interest, and preferable style of learning. This is so called cognitive diagnostic assessment, [4]. By this assessment get a profile of both strong and weak sides of the undergraduate, as well as of those which are under risk. The forming feedback give assistance to the lecturer as well the undergraduates with teaching and learning process. Realizing during the first weeks of the academic term, this feedback gives information in the assistance of teacher's planning and leads to differential approach and monitoring the eventual progress. In other words actually it is a better predictor to the future exam's results, [15]. Some methods of assessment in laboratory work for students knowledge as well analysis of results and discussion of interest can be found in [16].

The investigation of the process of digestion of new knowledge as well as the control and the assessment by modern electronic appliance are very important directions in the contemporary pedagogy, which is an elaborate compound of different approaches and different points of view. During the last decades a lot of methods based on the contemporary computer science and digital technologies took place in the investigation of the process of education, control and assessment of knowledge. So the reader may find interesting ideas to these directions in [10]-[13].

In the present paper we propose a mathematical method of assessment applicable to students and similar of technical point of view to those established in [18]-[22], as for this purpose use certain mathematical formulae from classical mathematics, [29], [14]. We discuss this method as a different approach for assessment that seems more effective for determined group of units in particular undergraduates, and defers a little bit for instance from the statistic method of Gompertz and Desirability Function Approach. Desirability Function "converts satisfaction of measured values into values of 0 to 1", [17]. In the latter Kim and Lin have proposed a nonlinear Desirability Function based on an exponential function (see e.g., [24]).

The paper has the following structure:

1) Introductory notes as we have already seen above.

2) The goals of the present paper and the structure of the diagnostic control as well the explanation and argumentation to the effective practices of assessment and also the open problems; touchstones and eventual criteria for assessment are discussed.

3) A simple method for assessment is established, that is, with a light modification it is applicable for an active selfassessment to the undergraduates; the scale of

assessment is shown as well.

4) An experiment of interest is considered, that is, in the field of Chemistry and at the same time illustrates the above stated method; some descriptors for the certain clusters are introduced as well.

5) The verification data inscribed in a certain table format is discussed.

6) Concluding remarks.

2. Goals and Structure of the Diagnostic Control

The aim of this study is:

A. To create accessible and arguing approach for qualitative assessment (by point's method) of cursory and deep knowledge in general fields as Mathematics, Physics and Chemistry being learnt by future engineers. The accent comes down on the determining of their effectiveness.

B. Under above stated verification to identify more frequently admitted misconceptions and weaknesses.

C. Establishing preferable manners of learning.

In the structure of the diagnostic control we offer certain ideas which can be seen in [3], [8], [16], [25], [30]. Creating control questions for the future engineers we have used the approach of constructivism and the corresponding SOLO taxonomy to the learning results, [28] in the following five stages:

1) Prestructural - the task is complete without understanding through elementary way. The feature here is that the undergraduate feels comfortably. Symbolically this means "I have no idea".

2) Uninstructural - the answer focuses on a unique fact or a part of the received information that means "I know only one thing among all we learn". This is a typical instance of cursory knowledge.

3) Multistructural - quantitative acquiring of knowledge, that is, the undergraduates may remember some facts and particular information, but do not make certain links between them, that means "I know many things in connection with all that we learn".

4) Relational - the learning get qualitatively with creation of links between existing facts and phenomena. This stage puts in appearance as an example of deep learning, that means "All things that I know may link and are connected with my learning process".

5) Extended abstract - it is also a deep learning by which the undergraduates remind their ideas from the above stated relational stage and can make some predictions, generalizations, abstracts, as well to create a conception in a new unknown situation, that means “I am the expert”.

Furthermore, we illustrate the theoretical constructions, as consider a concrete example for assessment of knowledge of the undergraduates. Symbolically the set of questions including first, second and third stage denote by A, and those of fourth and fifth by B. We call them task A and task B for short, respectively. The questions in task B are attributes of A. Task A forms three clusters: (i) general knowledge by Chemistry; (ii) the needed mathematical skills, specific knowledge by the corresponding field; (iii) needed laboratory skills. We note that the attributes B corresponding to A are in hierarchy connection.

The knowledge test of the undergraduates goes in two stages: (i) test; (ii) discussion with undergraduates on the admitted weaknesses; (iii) test - here no need of any preliminary preparation, and the duration is about 20-25 min.

The questions in task B completely overlap the content of A, but in higher level. Here it is required the undergraduates to understand the cause and effect connection with the considered matter and discussed phenomena. We emphasize that the undergraduates must possess certain critical skills to interpret the results, to establish correlations between quantities and methodical factors which influence on the experimental working. Also the undergraduates must possess skills for change of the experimental plan in the case of nonstandard results and to make intelligent conclusions. It is necessarily the problems to be resolved, in order to be formulated some hypotheses and to verify them. Thus the structural knowledge ensure the unity of the form and intension. For instance the answers of the undergraduate are right, but it turns out that there is not any information to the task B, i.e. it seems the knowledge is insufficiently effective.

3. Relative Assessment Method

Here we consider a simple method for assessment which passing through a little modification can be applied for selfassessment of undergraduates and students, i. e. everyone could verify himself the moment knowledge on the corresponding field, as at the same time one can be directed to the corresponding sources in order to remind one’s knowledge and to learn new knowledge. The more general methods of taking decisions as well methods for assessment of different objects and situations are presented in [5], [7], [14], [18], [23], [29].

Next describe the method, [14], [18]-[22], [29], as for this purpose list two groups of questions directed to the undergraduates concerning the task A and B:

1) The definition of the of rating scale for the tasks A and B; introduce in our

case 10 and 5 points rating systems, respectively; we note the estimation of B supplement the estimation to A.

2) Analysis of the qualitative results in four levels, i.e., the rating scale is stratified preliminarily; we receive the information for the proper skills of the undergraduates which are distributed in different levels.

The estimation scale. To construct the estimation scale introduce some supplementary quantities:

$$\max\{k_i\} = 10\ p. \ - \ \text{maximal number of points for task A;} \tag{1}$$

$$\max\{l_i\} = 5\ p.\ (i = 1, 2, \ldots, n) \ - \ \text{maximal number of points for task B;} \tag{2}$$

$$\alpha_i = 0.5k_i \quad (i = 1, 2, \ldots, n). \tag{3}$$

Here the natural number n is the number of the undergraduates taking part in the experiment. Taking into account the theory of the convex sets, [14], [29], and the fact that the acquired points for A and B are

$$0 \le k_i \le 10, \quad 0 \le l_i \le 5 \quad (i = 1, 2, \ldots, n),$$

respectively, obtain the statement:

Proposition 1.

$$C_i = (1 - \lambda_i)\alpha_i + \lambda_i k_i, \tag{4}$$

where C_i $(i = 1, 2, \ldots, n)$ are values of the estimates to n undergraduates, and the coefficients

$$\lambda_i = l_i / \max\{l_i\} = l_i/5. \tag{5}$$

In addition to the formulae (4) and (5) we introduce restricting condition: if the points $l_i = 0$, i.e. $\lambda_i = 0$, then there is a lowest estimation $C_i = \alpha_i$. The values of α_i $(i = 1, 2, \ldots, n)$ (3) depends on the level of difficulty of the task A, i.e. it depends on the level of training of the undergraduates with the task A; in the accordance with some authors, [1], [3], [9], [16], [25], [27], the level of difficulty for the matter, about the scale from 0 to 1, measures by weight coefficient varying from 0.4 to 0.7 - this level of difficulty in these limits is a most proper for practice, [6], [26], [28].

In the present experiment we suppose in (3) that the multiplier to be 0.5, i.e., $\alpha_i = 0.5k_i$ $(i = 1, 2, \ldots, n)$.

The motivation for construction of the equality (5) stems from the fact, that the questions in the task A concern the basic topics which are learnt in the contemporary university courses and contain basic knowledges, and the topics in the task B require deeper understanding of the questions, therefore they only correct the basic estimation obtain to the questions A, as at the same time they perturb in positive sense the scale of the basic estimation for A. Thus for instance for maximal number

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of points obtained to A, and maximal number of points to B ($l_i = 5, \quad \lambda_i = 1$), one obtains maximal number $C_i = 10$ points, and if $l_i = 1$ ($\lambda_i = 1/5$), then there is $C_i = 6$ points; if $l_i = 0$, then $\lambda_i = 0$ and $C_i = \alpha_i = 5$. To make easier such a calculation we take into account the above stated quantities and the equalities (1), (2) which allow us to use the formulae:

Corollary (for the quantities C_i).

$$C_i = 0.5(1 + l_i/\max\{l_i\}k_i \quad (i = 1, 2, \dots, n). \tag{6}$$

Taking into consideration the formula (6) we conclude that the ratio of the numbers C_i and k_i leads to more exact information for the real knowledge of the undergraduates at the moment. Introduce the relative dimensionless quantity P_i, that we call it coefficient of relative level of knowledge; it must be in the interval $0 \leq P_i \leq 1$ ($i = 1, 2, \dots, n$). Define this quantity by the equality,

$$P_i = 1 - (k_i - C_i)/k_i \quad (i = 1, 2, \dots, n). \tag{7}$$

Letting C_i from (6) in (7) obtain the statement:

Proposition 2.

$$P_i = 0.5(1 + l_i/\max\{l_i\}) \quad (i = 1, 2, \dots, n), \tag{7'}$$

which is equivalent to (7).

The coefficient P_i shows the relative closeness of the number of earned points k_i (to task A) nearby the real estimate C_i, which forms by the number of the points l_i concerning the task B and that influence to C_i . In the case $P_i = 1$ we have the extreme instance when the undergraduate is perfect, and the decreasing of coefficient P_i shows that one's knowledge take lower level which can be different in comparison with the estimate k_i. Next introduce the coefficient of effectiveness (CE for short), $v_i(A)$, $w_i(B)$ for our tasks A and B, respectively. By these CE we get more exact information about the moment knowledge for each one of the undergraduates on the topics under consideration. In the following we define CE:

Definition 1. (CE for tasks A and B)

- CE for task A

$$v_i(A) = k_i/\max\{k_i\} \quad (i = 1, 2, \dots, n),$$

- CE for task B

$$w_i(B) = l_i/\max\{l_i\} \quad (i = 1, 2, \dots, n).$$

Now we define the generalized coefficient of effectiveness (GCE for short).

Definition 2. (of GCE)

$$S_i = v_i(A)w_i(B) \quad (i = 1, 2, \dots, n). \tag{8}$$

4. Experimental

Here we consider an experiment in the field of Chemistry, illustrating the offered above stated method. Consider a laboratory exercise by Analytic Chemistry with second year undergraduates. The check of the knowledges accomplishes in two stages: 1) test; 2) discussion with undergraduates for admitted weaknesses and mistakes; here we do not need of preliminary preparation, and the period of time is about 20-25 min. The selection of topics as well the concrete questions we have taken use of the programs in the higher school as well some proper examples from California Chemistry Diagnostic Test, [4], [6], [15], [26]. This contains a text, visual and graph information. The questions are of complete type with set of possible answers, as well open questions and questions with supplementation. At the very beginning of this investigation we identify the preferable styles of learning following the question list of Honey and Mumford [15]. We introduce descriptors for the above stated clusters in accordance with the results of the learning process, that is:

A1 - General knowledge on Chemistry, accids, alcals, chemical stability, stechiometry and corresponding chemical laws; the conception for quantity of matter is of a special interest;

A2 - Basic mathematical knowledge for quadratic equations, linear equations and elementary trigonomtric functions, as well exponents, logarithmic functions and comparison of exponent with equal and different powers;

A3 - Specific requirements on Analytical Chemistry concern the cited knowledge by Chemistry in the context of the quantitative analysis and the current teaching matter as well the needed laboratory skills: technical and manipulative skills to applying laboratory equipment and reactives, understanding of the laboratory procedures and methods, cognitions of the necessary requirements in order to guarantee the quality of the analytical process and identifying the variables so that a certain problem to B to be successfully solved.

Test data. The number of undergraduates is $n = 17$, taking numbers from 1 to 17. Note that k_i concerns the task A, and l_i makes B, respectively. The general result to task A: Denote by D_i the general estimate, that is the sum of the numbers $C_i(A1)$, $C_i(A2)$, $C_i(A3)$ obtained for A1 - A3; thus we calculate the numbers

$$D_i = \sum_{j=1}^{3} C_i(AJ) \qquad (i = 1, 2, \dots, n).$$

Next write the result in the following Table 4. Generalize the effectiveness (in %), defining the quantity $E = (D_i/30)100$ [%], that is presented by the Table 5. To make a precise the assessment we calculate the effectiveness as taking into account the *Definition 1*:

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- CE to task A

$$v_i(A) = k_i/\max\{k_i\} \quad (i = 1, 2, \ldots, 17),$$

- CE to task B

$$w_i(B) = l_i/\max\{l_i\} \quad (i = 1, 2, \ldots, 17).$$

Next taking into account the earned points of tasks A and B as well as *Definition 2*(GCE) and (8) obtain

$$S_i = v_i(A)w_i(B)(i = 1, 2, \ldots, 17).$$

Since for A1 - A3 we have maximal number of points 30, then obtain CE for the points earned to the task A , Table 6. The CE to the points earned to the task B are written in Table 7. Next calculate GCE by $S_i = v_i(A)w_i(B)$, and the obtained result write in Table 8. All obtained results sort according to the obtained estimates. Analysis of the obtained results and subsequent discussion. Next we use three criteria that put in an appearance as touchstones for our further consideration:

1) Absolute value of the obtained points from Table 1, 2 and 3

$$D_i(A1) = 81.4, \quad D_i(A2) = 62.3, \quad D_i(A3) = 87.3.$$

It seems that the greatest number of points is realized in A3. This result could be explained by the current knowledges learnt through the lectures and seminars as well their realization directly in the laboratory exercises. The lowest results are through A2, where there is lack of basic mathematical skills applicable to simple calculation. By this criterion it seems difficult to classify the knowledges. Therefore it may use for current estimate.

2) Defining of the assimilation of knowledges measured by percentage as a ratio between the realized result over maximal existing one, that is Table 5. The reader can look at Table 5, where we have shown the results of percentage acceptance of the knowledges concerning for 17 undergraduates.

3) Differentiating of clusters A and B by introducing coefficients of efficiency, Table 5 and 6, respectively. Then the GCE is given in Table 8, concerning each one of the undergraduates.

Results. We defer four intervals of effectiveness using for this purpose the GCE and Si as it follows:

- weak level (WL): $0.11 \leq S_i \leq 0.21$;
- middle level (ML): $0.11 \leq S_i \leq 0.21$;
- good level (GL): $0.22 \leq S_i \leq 0.32$;

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- perfect level (PL): $0.33 \leq S_i$.

In the Table 8 one can read that the undergraduates distribute as follows:

WL - 5 undergraduates; ML - 5 undergraduates; GL - 4 undergraduates; PL - 3 undergraduates.

Having in mind the acceptance of knowledges from A and B on the first look is not disturbing, that is: 29 % WL; 29 % ML; 24 % GL; 18 % PL. The potential wanting of knowledge determinate by the coefficient of relative level of knowledge P_i , introduced in (7). The analysis of the styles of learning shows preference to the type "activist 53.82 %, and "pragmatic 17.65 %" on the account of "thinker 17.65 %", and "theoretic 5.88 %.

Frequently an incorrectness being watched and some mistakes at realizing the questions from A3 connecting with attribute B:

- wanting of cause-effect understanding at the choice of proper answer - 41.1 %;
- demonstration of better theoretical and practical knowledges in comparison with solving of problems - 70.5 %;
- fragment preparation at solving of numerical tasks - 51.6 %;
- difficulties in the case of generalization of data and forming of conclusions - 60.7 %;
- difficulties in the case of planning oneself the experiment - 80.5 %;
- difficulties at the correlation between variables and with the choice or change in the analysis conditions - 75.6 %;
- nonforming of hypotheses and conceptions;
- difficulties with representing of the results of the experimental work in written form and their estimation, save in the cases when exist complete forms to fill - 61.2 %.

About A1 we have concentrated on some questions connected with the application of knowledges in the analytical practice. The already ascertained wrong conceptions about the chemical balance (CB) are the following: (i) CB is a static state; (ii) the reactions through two directions are independent each other; (iii) the balance constant depends on concentrations. Some difficulties encounter at calculation of the balance constant and its use for analytical purposes, as well the prediction of the change of the direction of the reaction under the change of the conditions. Of special interest to the presentation of results of the analysis is the conception of the quantity of matter which make difficulties 30 % of all undergraduates.

The recommended measures for an exit from the set of contradicting estimates go in two directions:

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1) Application of effective strategies for both teaching and learning. Active learning, [5], learning together, [7], and forming of equips require different styles of learning. By proper tasks every undergraduate will develop these styles which either does not possess or possesses in insufficient extent.

2) Implication of the undergraduates in situation in which their wrong knowledges do not work.

3) Engagement with tasks of interest which are actual and concerning the practice to the corresponding field.

4) Information technologies put in an appearance as a part of "living" undergraduates, and they can be included in the learning program by different ways:

- virtual learning laboratory;

- on line connection with the lecturer and colleagues.

  The on line questions before exercise would annul the disturbance due to the ingoing the laboratory.

- creating and using WIKI. It is necessary every undergraduate to prepare in written and also oral form presentation of proper topic. To make easier for the undergraduates we offer some criteria for working out and estimation, selfassessment and by the lecturer of written document for already complete work.

5) Improving the procedure on the assessment.

It is known that in accordance with the classical system of examination we estimate basically declarative knowledges, [2]. The alternative methods of estimation as the selfassessment and the partner estimation take special place in the forming estimate, [3], [16], [27].

Conclusion. Save the restrictions of this investigation due to the small sampling as a pilot investigation it gives us a realistic information to the assessment of the cursory and deep knowledges, which is acknowledged by the long time watching on the key skills of the undergraduates and the methods of selfassessment and partner assessment as well their correlation along with that of the lecturer. At the same time we do not pretend that the method stated above is a new one or is perfect. Note that we have established a different approach for assessment that seems more effective for determined group of undergraduates, and defers a little bit from Gompertz method or for instance the Desirability Function Approach.

No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$A\|k_i$	6	5	8	3	4	7	5	9	4	5	7	6	8	4	6	4	3
α_i	3	2.5	4	1.5	2	3.5	2.5	4.5	2	2.5	3.5	3	4	2	3	2	1.5
$B\|li$	3	5	4	2	2	3	4	5	4	3	4	3	4	3	4	3	4
λ_i	0.6	1	0.8	0.4	0.4	0.6	0.8	1	0.8	0.6	0.8	0.6	0.8	0.6	0.8	0.6	0.8
C_i	4.8	5	7.2	2.1	2.8	5.6	4.5	9	3.6	4	6.3	4.8	7.2	3.2	5.4	3.2	2.7

Table 1: Cluster A1

No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$A\|k_i$	5	7	9	3	3	6	5	8	3	7	6	6	8	3	6	3	3
α_i	2.5	3.5	4.5	1.5	1.5	3	2.5	4	1.5	3.5	3	3	4	1.5	3	1.5	1.5
$B\|li$	4	3	5	2	2	4	3	5	2	3	4	4	4	2	3	2	2
λ_i	0.8	0.6	1	0.4	0.4	0.8	0.6	1	0.4	0.6	0.8	0.8	0.8	0.4	0.6	0.4	0.4
C_i	4.5	5.6	4.5	2.1	2.1	5.4	4	8	2.1	5.6	5.4	5.4	7.2	2.1	4.8	2.1	2.1

Table 2: Cluster A2

No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$A\|k_i$	7	6	8	4	4	7	6	9	4	6	7	7	9	4	7	4	3
α_i	3.5	3	4	2	2	3.5	3	4.5	2	3	3.5	3.5	4.5	2	3.5	2	1.5
$B\|li$	4	3	5	2	2	4	4	5	2	3	4	3	4	2	3	4	3
λ_i	0.8	0.6	1	0.4	0.4	0.8	0.8	1	0.4	0.6	0.8	0.6	0.8	0.4	0.6	0.8	0.6
C_i	6.3	4.8	8	2.8	2.8	6.3	5.3	9	2.8	4.8	6.3	5.6	8.1	2.8	5.6	3.6	2.4

Table 3: Cluster A3

No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
D_i	15.6	15.4	19.7	7	7.7	17.3	17.9	26	8.5	14.4	18	15.8	22.5	8.1	15.8	8.9	7.7

Table 4

No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$E[\%]$	52	51	65	23	26	57	60	86	28	48	60	53	75	27	53	30	26

Table 5

No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
I	6	5	8	3	4	7	5	9	4	5	7	6	8	4	6	4	3
II	5	7	9	3	3	6	5	8	3	7	6	6	8	3	6	3	3
III	7	6	8	4	4	7	6	9	4	6	7	7	9	4	7	4	3
$v_i(A)$	0.6	0.6	0.83	0.33	0.37	0.66	0.53	0.87	0.37	0.6	0.66	0.63	0.83	0.37	0.63	0.37	0.3

Table 6

Acknowledgments

The present study was partly supported by both the grant No. 11353 (NIS-UCTM) and BG 051 PO 001-3.3.06-0014 to the Operative Program (2012-2015) “Develop-

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No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
I	3	5	4	2	2	3	4	5	4	3	4	3	4	3	4	3	4
II	4	3	5	2	2	4	3	5	2	3	4	4	4	2	3	2	2
III	4	3	5	2	2	4	4	5	2	3	4	3	4	2	3	4	3
$w_i(A)$	0.37	0.37	0.47	0.2	0.2	0.37	0.37	0.5	0.27	0.3	0.4	0.33	0.4	0.23	0.33	0.3	0.3

Table 7

No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
S_i	0.22	0.22	0.39	0.07	0.07	0.24	0.2	0.44	0.1	0.18	0.26	0.21	0.33	0.09	0.2	0.11	0.09

Table 8

ment of Human Resources". The authors thank to Professor Nina Penkova for the valuable notes and support.

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International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 9, No. 1 (2015)