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DIFFUSION OF LIQUID INDUSTRIAL MEDIA IN RUBBER-METAL PRODUCTS

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Abstract: In this short paper we investigate kinetics of the diffusion to a liquid penetrating in thin ribbon samples of elastomer compositions. By the weight-sorption method and some practical measuring we construct diffusion curves corresponding to the equilibrium saturation. The diffusion coefficient is determined to the test couples polymer-liquid using for this purpose the Fick's law. Next taking into account the equilibrium concentration for the rubber-metal cylinder damper we obtain the diffusion curves. These resulting curves serve to investigate the influence of the cyclic compressive loading on the kinetics of the diffusion.

AMS Subject Classification: 82D15
Key Words: diffusion, elastomer, rubber-metal dampers, weight-sorption method, Fick' second law

1. Introduction

We watch during last decades that in many areas of the science and technologies the elastomers are used instead of some traditional materials. It turns out that it is an indispensable constructional material applicable especially nowadays in the modern engineering structures undergone any cyclic loads. Then the elastomers made by rubber-metal material (plate, damper pad, membrane, etc) neutralizes and in a great

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extent suppress the detrimental, and also harmful effect due to vibrations arisen in the above mentioned object. These rubber-metal elements keep both the stability and geometric configuration of the engineering constructions. Note that they work in liquid, gas environment, under vibrations, and unfavorable temperature in most cases.

The kinetics of the diffusion of liquid (in industrial environment) in thin ribbon samples, and the matrix of elastomeric compositions used for the manufacture of technical rubber-metal products was studied in [1, 2, 9, 10]. By the known weight-sorption method, and with the aid of some analytical scale we discuss the behavior of the diffusion curves to an equilibrium saturation. Thus taking into account the Fick's law we obtain the diffusion coefficients for test couples polymer-liquid.

Purpose of the Present Investigation

In the present work in relevance with some further investigation our aim is to construct the sorption curves to the massive gum-metal engineering products (cylindrical dampers) at known equilibrium concentrations and diffusion coefficients for tape samples as for this purpose use cylindrical coordinates. The reason to investigate this as we have already noted above is the fact that rubber dampers with solid cylindrical elastomeric body having a total mass exceeding many times the upper limit of the classical analytical scale. Thus we combine the weight-sorption method with measuring by the analytical scale.

It is important to note that being familiar with the sorption curves, we are able first to select experimental samples with required to our experiment gradations for concentration of liquid in the elastomer, and second to investigate the influence of the cyclic loading on the diffusion's kinetics.

In the second section we give some known results and main formulae.

The third section is devoted to our experimental results as well as discuss some consequence.

Finally we show the behavior of the sorption curves which approximate with a proper precision the experimental results shown on Fig. 2.

2. Preliminaries

In this section we trace the existing results, and at the same time remind some basic elements of the theory.

Theoretical Formulation

Depending on the shape of the exploring body, and according to the initial and boundary conditions there exist well known classical solutions of Fick's equation, [5, 7, 8].

The Fick's law in Cartesian coordinates has the form

$$\frac{\partial C}{\partial t} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right),$$

where $D = \text{const} > 0$ is the diffusion constant. It is known that

$$\begin{aligned} \text{grad}C \equiv& \nabla C \equiv \left(\frac{\partial C}{\partial x}, \frac{\partial C}{\partial y}, \frac{\partial C}{\partial z}\right)^T, \\ \text{div}\vec{W} \equiv& \nabla \vec{W} \equiv \frac{\partial W_1}{\partial x} + \frac{\partial W_2}{\partial y} + \frac{\partial W_3}{\partial z}, \end{aligned}$$

where $C = C(x, y, z, t)$ and $\vec{W} = (W_1,\ W_2,\ W_3)^T$ are sufficiently smooth functions. Letting $W_1 = \frac{\partial C}{\partial x}$, $W_2 = \frac{\partial C}{\partial y}$, $W_3 = \frac{\partial C}{\partial z}$ in the diffusion equation obtain

$$\frac{\partial C}{\partial t} = D\ \text{div}(\text{grad}C).$$

In cylindrical coordinates, [5, 7],

$$\begin{aligned} x &= r\cos\varphi, \\ y &= r\sin\varphi, \\ z &= z, \end{aligned}$$

where r is the polar radius, φ the polar angle, z is the height, get the equation

$$\frac{\partial C}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{dC}{dr}\right).$$

Actually the multiplier D in the real process is not constant, and it is a function depending on many independent variables, but we assume that D changes a little bit in our case thus one may take that it is a real constant for convenience. In the engineering practice we restrict our consideration on the case when the variation of the concentration C of the penetrating fluid w.r.t. the time t is governed by the basic equation of diffusion in cylindrical variables,

$$\frac{\partial C}{\partial t} = D\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{dC}{dr}\right)\right], \tag{1}$$

where D is assumed to be a real constant. We note that the above stated equation (1) describes the radial flow penetrating in a cylinder. Next we set in (1)

$$C = u(r)\exp(-D\alpha^2 t),$$

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thus obtain the known Bessel differential equation of zero order,

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + \alpha^2 u = 0. \tag{2}$$

We note that the solutions of (2) are represented by the Bessel functions satisfying the corresponding initial and boundary conditions on a cylinder with radius $r = a$. In our case, the diffusing medium has also cylindrical shape with the same radius a as on its surface. The boundary concentration $C = C_1 = \text{const} > 0$, and the very beginning the medium is free of diffusing liquid, i.e. the initial and boundary conditions are:

$$\begin{aligned} C(r,0) = 0, \quad t = 0, \quad (0 < r < a), \\ C(0,t) = C_1 > 0, \quad t > 0, \\ C(a,t) = C_1 > 0, \quad t > 0. \end{aligned}$$

In this case, we represent the solution by Bessel functions (see e.g., [6]) of first order, and rank zero denoted by $J_0(s)$, and its derivative J' w.r.t. s (s), i.e.

$$C = C_1\left(1 + \frac{2}{a}\sum_{n=1}^{\infty}\frac{1}{\alpha_n}\frac{J_0(\alpha_n r)}{J_0'(\alpha_n a)}\exp(-D\alpha_n^2 t)\right), \tag{3}$$

where $\alpha_n s$ $(n = 1, 2, ...)$ are the roots of the equations

$$J_0 = 0 \quad (n = 1, 2, \ldots), \tag{4}$$

respectively. The roots of the Bessel functions are tabulated, [5, 6, 7]. It is known that the series in (3), for $r = 0$, has a finite sum, i.e. it is convergent. It is shown (see e.g., [4]) that the first four summands are sufficient to calculate approximately the variable C. For this purpose take the exact values of the roots α_i $(i = 1, 2, 3, 4)$,

$$\alpha_1 = \frac{2,4050}{a}, \quad \alpha_2 = \frac{5,5200}{a}, \quad \alpha_3 = \frac{8,6540}{a}, \quad \alpha_4 = \frac{11,7915}{a}, \tag{5}$$

(see e.g. [4]-[8]).

In the asymptotic case for a large period of time one may prove that the formula (3) reduces to one summand (see e.g. [5, 7, 8]). It is known that the functions $J_0(x)$ and $J_0'(x)$ can be represented by the series

$$J_0(x) = 1 - (x/2)^2 + \frac{(x/2)^4}{1^2 2^2} - \frac{(x/2)^6}{1^2 2^2 3^2} + \cdots,$$

$$J_0'(x) = -\frac{1}{2}x + \frac{\left(\frac{1}{2}x\right)^3}{1^2 2} - \frac{\left(\frac{1}{2}x\right)^5}{1^2 2^2 3} + \cdots,$$

(see e.g. [5, 6]). In this case, having in mind the weight-sorption method we have the average concentration

$$\overline{C}(t) = \frac{2}{\pi a^2} \int_0^a C(t, r) r dr. \tag{6}$$

The amount of the substance Q, which penetrates (diffuses) into the cylindrical body on the unit length can be calculated by the equality

$$Q(t) = 2\pi \int_0^a C(t, r) r dr = \frac{2}{a^2} \int_0^a C(t, r) r dr.$$

After substituting the function (3) in (6) and taking into account the experimental value C_1 we get

$$\overline{C}(t) = C_1 \left(1 - \frac{4}{a^2} \sum_{k=1}^{\infty} \frac{1}{\alpha_k^2} \exp(-D\alpha_k^2 t) \right), \tag{7}$$

and

$$Q(t) = \pi a^2 C_1 \left(1 - \frac{4}{a^2} \sum_{k=1}^{\infty} \frac{1}{\alpha_k^2} \exp(-D\alpha_k^2 t) \right).$$

Obviously, here exists the equality $Q(t) = \pi a^2 \overline{C}(t)$.

3. Experimental Data

Here we prepare for our experiment samples having two different sizes. Their general form is shown on Figure 1.

The crude patterns of the elastic elements are made of sheet material. The fixating of the vulcanizate to the metal body is accomplished by isocyanate glues ("Chemosis 211" and "Chemosis 220"), [3]. Furthermore, knowing the diffusion coefficient D we find $\overline{C}(t)$ from (7) as an approximate concentration instead of (3) and replace the first four summands of (3) and (7), using for this purpose $(\alpha_i)_{i=1}^4$ from (5)

$$\frac{Q}{\pi a^2} = \overline{C} = C_1 \Big(1 - \frac{4}{2,405^2} \exp\Big\{ -\frac{D(2,405^2)t}{a^2} \Big\} - \frac{4}{(5,520)^2} \exp\Big\{ -\frac{D(5,520^2)t}{a^2} \Big\}$$
$$- \frac{4}{(8,654)^2} \exp\Big\{ -\frac{D(8,654^2)t}{a^2} \Big\} - \frac{4}{(11,7915)^2} \exp\Big\{ -\frac{D(11,7915^2)t}{a^2} \Big\} \Big).$$

To obtain the solution we use the software "MathCad Prime 3.0". Thus we watch on the Figure 2 .

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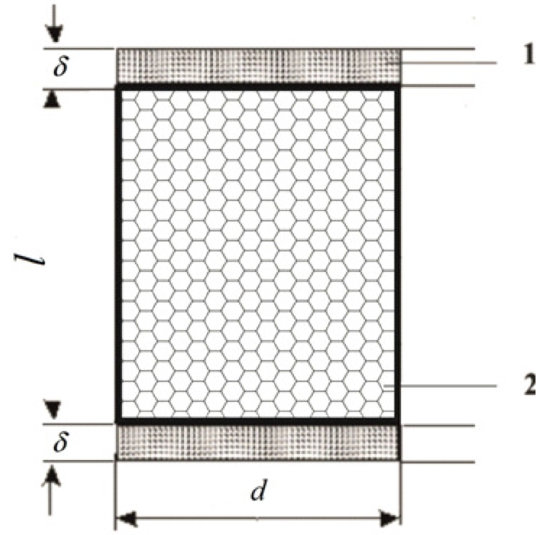

Figure 1: Cylindrical rubber vibration dampers: 1) metal; 2) elastomer

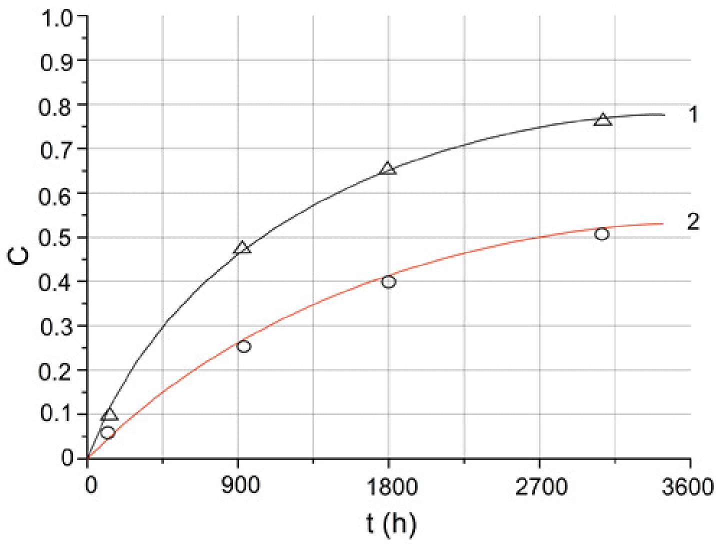

Figure 2: Graph the concentration-time for two couples: 1) PU with oil; 2) PU with water

A graphic dependence of the concentration C w.r.t. the time t i.e., it is a graphical mapping $C = C(t)$ for two of the tasting pairs of elastomer-liquid. For the rest of couples the corresponding programs are constructed similarly. Now may conclude that the sorption curves approximate in a proper precision the experimental results (shown in dots) on Fig. 2.

These curves are important for the control on the samples through the further testing of:

- the influence of cyclic loading on the kinetics of the diffusion;
- the short time vibration creep;
- the generation of proper heat under cyclic loading, that is the (heat production).

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