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HEAT GENERATING HYDROMAGNETIC THIRD GRADE FLUID FLOW THROUGH ISOTHERMAL PLATES WITH OHMIC HEATING EFFECT

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Abstract: This paper studies the combined effect of a transverse magnetic field and internal heat generation on the flow of an incompressible third-grade fluid through parallel plates of uniform temperature. Analytical solutions of the dimensionless nonlinear ordinary differential equations are obtained using regular perturbation method implemented on a symbolic package-Mathematica. Variations of flow parameters on the velocity and temperature profiles are presented and discussed.

Key Words: third grade fluid, Ohmic heating, magnetic field, internal heat

1. Introduction

Studies on non-Newtonian fluid flow through parallel plates have been an area of active research over the years due to its numerous domestic and industrial applications. Common examples of these fluids are found as pharmaceutical mixtures, lubricating

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oil, condensed milk, honey and many more. Till date, there is no general constitutive equation that can adequately describe the complex rheological behaviour of these fluids Of interest in this paper is the third grade fluid model which has the ability to describe the shear thinning/thickening of these fluids. The third grade fluid model has been used extensively in studying the flow behaviour of many non-Newtonian fluids, some of these works can be found in [1-5].

The present study is motivated by the fact that heat is generated within moving non-Newtonian fluids during Ohmic heating by passing alternating electric current through it [6-11] After an exhaustive survey, it is observed that literature lacks studies on third grade fluid flow through isothermal plates taking the effect of Ohmic heating and internal heat generation into consideration. Hence, the purpose of this paper is to investigate all these important effects. Due to nonlinearity of the problem regular perturbation method [12-13] will be used to obtain analytical solution to the problem. To confirm the accuracy of the computation, comparison of the limiting case with previously obtained result in literature is conducted. The rest of the paper is organized as follows Section 2 presents the formulation and non-dimensionalization of the problem. In section 3, the method of solution is described while section 4 deals with the discussion of results based on the physics of the problem. Finally, section 5 concludes the paper.

2. Flow Analysis

Consider the steady flow of an incompressible, electrically conducting third–grade fluid with internal heat generation through infinite parallel isothermal plates distance 2h apart. The fluid is assumed to be electrically conducting and subjected to a constant transverse magnetic field of strengthB_0. It is further assumed that magnetic Reynolds number of the fluid is small so that electric field, induced magnetic and Hall effects may be neglected. Then the fluid governing equations can be written as [14].

$$0 = -\frac{d\hat{p}}{dx} + \mu\frac{d^2u'}{dy'^2} + 6\beta_3\frac{d^2u'}{dy'^2}\left(\frac{du'}{dy'}\right)^2 - \sigma B_0^2 u'. \tag{1}$$

While, the balanced energy equation is given by

$$0 = k\frac{d^2T'}{dy'^2} + \left(\frac{du'}{dy'}\right)^2\left\{\mu + 2\beta_3\left(\frac{du'}{dy'}\right)^2\right\} + \sigma B_0^2 u'^2 + Q_0\left(T - T_0\right)\ . \tag{2}$$

Additional term in the momentum equation is due to magnetic field while the last term in the energy equation represents the effect of internal heat generation [15]. The no-slip condition for the fluid velocity at the walls together with the exchange of heat with the ambient at the walls

$$u' = 0 = T' \quad \text{on} \quad y = a, \tag{3}$$

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and the symmetric condition along the channel centreline is given by

$$\frac{du'}{dy'} = 0 = \frac{dT'}{dy'} \quad \text{on} \quad y = 0. \tag{4}$$

Introducing the following dimensionless parameters

$$y = \frac{y'}{a},\ u = \frac{u'}{U},\ \gamma = \frac{\beta_3 U^2}{\mu a^2},\ Ha^2 = \frac{\sigma_e B_0^2 a^2}{\mu},\ \theta = \frac{T - T_0}{T_1 - T_0},$$
$$G = -\frac{a^2}{\mu U}\frac{dP}{dx},\ Br = \frac{\mu U^2}{k\left(T_1 - T_0\right)},\ \delta = \frac{Q_0 a^2 \left(T_1 - T_0, \right)}{\mu U^2},$$

we obtain the dimensionless equations with appropriate boundary conditions as follows

$$\frac{d^2u}{dy^2} + 6\gamma\frac{d^2u}{dy^2}\left(\frac{du}{dy}\right)^2 - Ha^2u = -G, \quad u'\left(0\right) = 0 = u\left(1\right), \tag{5}$$

and

$$\frac{d^2\theta}{dy^2} + Br\delta\theta + Br\left(\frac{du}{dy}\right)^2\left(1 + 2\gamma\left(\frac{du}{dy}\right)^2\right) + BrHa^2u^2 = 0,\ \theta'\left(0\right) = 0 = \theta\left(1\right). \tag{6}$$

Here: $u'$is the fluid velocity, $T'$the fluid temperature, $p'$the pressure, $\beta_3$is the material coefficient, k the thermal conductivity, Q_0 is the internal heat generation term,$\mu$is the dynamic viscosity, γ is the dimensionless third grade material parameter, δrepresents internal heat generation parameter, Brviscous heating parameter, θ is the dimensionless temperature, $u$is the dimensionless velocity, G is the dimensionless pressure gradient, Ha^2 is the Hartmann number, B_0 the electromagnetic induction, $\sigma_e$the conductivity of the fluid.

3. Method of Solution

In order to obtain the solution by perturbation method, we assume that $0 < \gamma << 1, \quad 0 < Br << 1$, and we can form solutions in the forms

$$u\left(y\right) = \sum_{n=0}^{k} \gamma^n u_n\left(y\right), \tag{7}$$

$$\theta\left(y\right) = \sum_{n=0}^{k} \left(Br\right)^n \theta_n\left(y\right). \tag{8}$$

From equations (5) - (6), we obtain

$$\gamma^0 : \qquad \frac{d^2u_0}{dy^2} - Ha^2u_0 = -G\ ; \ \ u_0'\left(0\right) = 0 = u_0\left(1\right), \tag{9}$$

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$$\gamma^1 : \quad \frac{d^2u_1}{dy^2} + 6\frac{d^2u_0}{dy^2}\left(\frac{du_0}{dy}\right)^2 - Ha^2u_1 = 0 \; ; \;\; u_1'(0) = 0 = u_1(1) \,, \tag{10}$$

$$\gamma^2 : \quad \frac{d^2u_2}{dy^2} + 6\frac{d^2u_1}{dy^2}\left(\frac{du_0}{dy}\right)^2 + 12\frac{du_0}{dy}\frac{du_1}{dy}\frac{d^2u_0}{dy^2} - Ha^2u = 0; \quad u_2'(0) = 0 = u_2(1) \,, \tag{11}$$

$$Br^0 : \quad \frac{d^2\theta_0}{dy^2} = 0 \; ; \qquad \theta_0'(0) = 0 = \theta_0(1) \,, \tag{12}$$

$$Br^1 : \quad \frac{d^2\theta_1}{dy^2} + \delta\theta_1 + \left(\frac{du}{dy}\right)^2\left(1 + 2\gamma\left(\frac{du}{dy}\right)^2\right) + Ha^2u^2 = 0; \quad \theta_1'(0) = 0 = \theta_1(1) \,, \tag{13}$$

$$Br^2 : \quad \frac{d^2\theta_2}{dy^2} + \delta\theta_2 = 0 \; ; \quad \theta_2'(0) = 0 = \theta_2(1) \,. \tag{14}$$

By solving equations (9)-(11) subject to boundary conditions, we have

$$\begin{aligned} u(\mathrm{y}) = & -\frac{e^{-Hay}(e^{Ha} - e^{Hay} - e^{2Ha+Hay} + e^{Ha+2Hay})G}{(1+e^{2Ha})Ha^2} \\ & + \frac{\gamma}{4\left(1+e^{2Ha}\right)^4 Ha^4} 3e^{-Hay}\Big(-e^{Ha}G^3 - 2e^{3Ha}G^3 - 2e^{5Ha}G^3 - e^{7Ha}G^3 \\ & + 2e^{2Ha+Ha(3-2y)}G^3 + 2e^{Ha(3-2y)}G^3 - e^{Ha+2Hay}G^3 - 2e^{3Ha+2Hay}G^3 - 2e^{5Ha+2Hay}G^3 \\ & - e^{7Ha+2Hay}G^3 - e^{Ha(3-4y)+2Hay}G^3 - e^{2Ha+Ha(3-4y)+2Hay}G^3 \\ & + 2e^{Ha(3-4y)+4Hay}G^3 + 2e^{2Ha+Ha(3-4y)+4Hay}G^3 + 2e^{Ha(3-2y)+4Hay}G^3 \\ & + 2e^{2Ha+Ha(3-2y)+4Hay}G^3 - e^{Ha(3-2y)+6Hay}G^3 - e^{2Ha+Ha(3-2y)+6Hay}G^3 \\ & + 2e^{Ha(3-4y)+8Hay}G^3 + 2e^{2Ha+Ha(3-4y)+8Hay}G^3 - 4e^{3Ha}G^3Ha + 4e^{5Ha}G^3Ha \\ & - 4e^{3Ha+2Hay}G^3Ha + 4e^{5Ha+2Hay}G^3Ha + 4e^{Ha(3-2y)+2Hay}G^3Hay \\ & + 4e^{2Ha+Ha(3-2y)+2Hay}G^3Hay - 4e^{Ha(3-4y)+6Hay}G^3Hay \\ & - 4e^{2Ha+Ha(3-4y)+6Hay}G^3Hay\Big) + O\left(\gamma^2\right). \end{aligned} \tag{15}$$

Now using Mathematica, (15) is used to obtain the solution of (12) – (14) and substituted in (8). The graphical solutions are given in the Figures (3)-(6).

The flow rate Q per unit width of the plates is given by

$$Q = \int_{-1}^{1} u(y)\, dy. \tag{16}$$

And its numerical result is shown as Table 01.

The skin friction Sf at both walls is given by

$$Sf = -\frac{du}{dy} - 2\gamma\left(\frac{du}{dy}\right)^3\Bigg|_{y=-1,1}. \tag{17}$$

And the rate of heat transfer Nu is given by

$$Nu = \frac{d\theta}{dy}\Bigg|_{y=-1,1}. \tag{18}$$

4. Discussion of Results

For us to understand the dynamics of the flow, analytical results are shown graphically for both temperature and velocity fields. Table 1 shows the effect of Hartmann's number on the flow rate. The present result agreed perfectly with that obtained in [14] when Ha=0. Further increase in Ha is observed to decrease the flow rate within the channel. Table 2 shows the variation of flow parameters on the skin friction at both walls. As observed from the Table, an increase in Hartmann's number increases the skin friction at the lower wall and decreases it at the upper wall. From the same Table an increase in the non-Newtonian material parameter is seen to decrease the skin friction at the lower wall and enhancing skin friction at the upper wall. Just like the non-Newtonian material parameter, an increase in the Hartmann's number decreases the rate of heat transfer at the lower wall but increases the heat transfer rate at the upper wall as seen in Tables 3 and 4 respectively. However an increase in internal heat generation parameter enhances the rate of heat transfer at the lower wall while decreasing it at the upper wall.

Table 1: Variation of Hartmann number with flow rate for G=1

γ	Ha=0	Ha=0.1	Ha=1
0	0.66667	0.664011	0.476812
0.4	0.89524	0.88372	0.455781
0.7	1.78667	1.75053	0.531338
1	3.29524	3.21909	0.685178

Figure 1 shows the plot of velocity against space for variations in the non-Newtonian parameter. As observed from the graph that increase in non-Newtonian parameter reduces the flow velocity. This contributed to fluid viscosity due to increase in frictional force. The effect of magnetic field placed across the channel is

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Table 2: Effect of Hartmann number skin friction for G=1

γ	Ha	$Sf\,(y=-1)$	$Sf\,(y=1)$
0.1	1	-0.780544	0.780544
0.1	2	-0.487139	0.487139
0.1	3	-0.333464	0.333464
0.1	1	-0.761594	0.761594
0.2	1	-0.846873	0.846873
0.3	1	-1.00756	1.00756

Table 3: Effect of Hartmann's number on Nu ($\gamma = 0.1$, $Br = 0.71$, $\delta = 1 = G$)

Ha	$Nu\,(y=-1)$	$Nu\,(y=1)$
0.25	0.22757	-0.22757
0.5	0.220722	-0.220722
1	0.189288	-0.189288

shown in Figure 2. From the graph maximum flow occurs at minimum value of the Hartmann number showing that increase in this parameter has reducing effect on the flow velocity. This is so due to the retarding effects of the Lorentz forces which are present in the magnetic field applied across the channel. It is observed in Figure 3 that an increase in magnetic field intensity leads to an increase in fluid temperature. This is true since heat is generated when fluid passes through a magnetic field therefore contributing to an increase in the fluid temperature. Figure 4 describes the influence of internal heat generation on the temperature distribution within the channel. It is noticed that as internal heat generation increases there is corresponding increase in the fluid temperature distribution. Accumulation of

Table 4: Effect of non-Newtonian material on Nu ($Ha = 1$, $Br = 0.71$, $\delta = 1 = G$)

γ	$Nu\,(y=-1)$	$Nu\,(y=1)$
0.1	0.189288	-0.189288
0.3	0.162872	-0.162872
0.5	0.135722	-0.135722

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Table 5: Effect of non-Newtonian material on Nu ($Ha = 1$, $Br = 0.71$, $Ha = 1 = G$)

δ	$Nu\,(y=-1)$	$Nu\,(y=1)$
1	0.189288	-0.189288
2	0.218674	-0.218674
3	0.24806	-0.24806

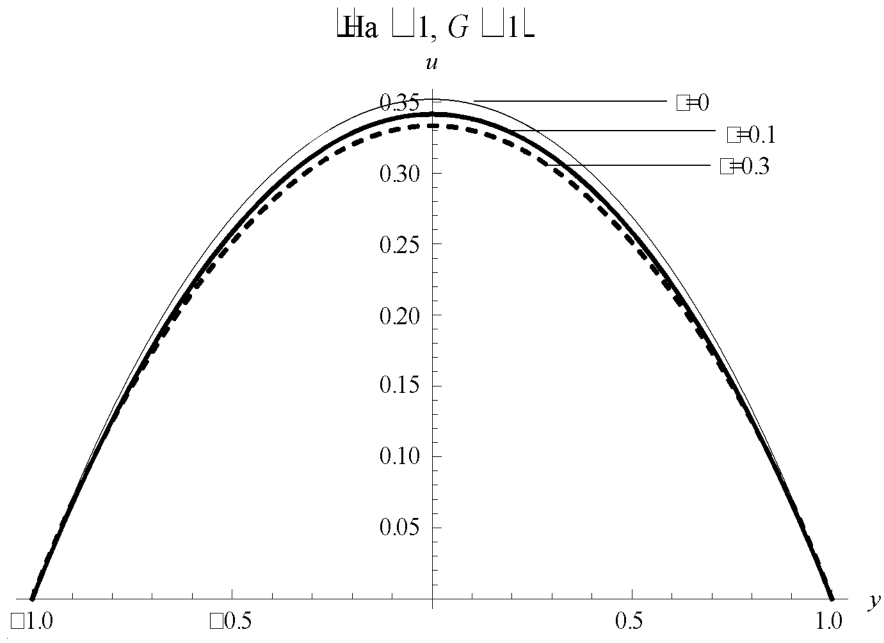

Figure 1: Variation of velocity with non-Newtonian parameter

excessive heat could lead to self-heating which has undesirable effect on the production set-up, safety of lives and the environment. The effect of Brinkman number on the temperature distribution as presented in Figure 5. As seen from the plot, minimum temperature occurred at minimum Brinkman number therefore increase in Brinkman number enhances the flow due to rise in the fluid kinetic energy. Finally, Figure 6 displays the effect of increase in fluid non-Newtonian material parameter on the temperature profile. From the graph, the maximum temperature is observed at the purely Newtonian case and the increase in non-Newtonian behaviour decreases the temperature. This is due to the rise in the fluid viscosity.

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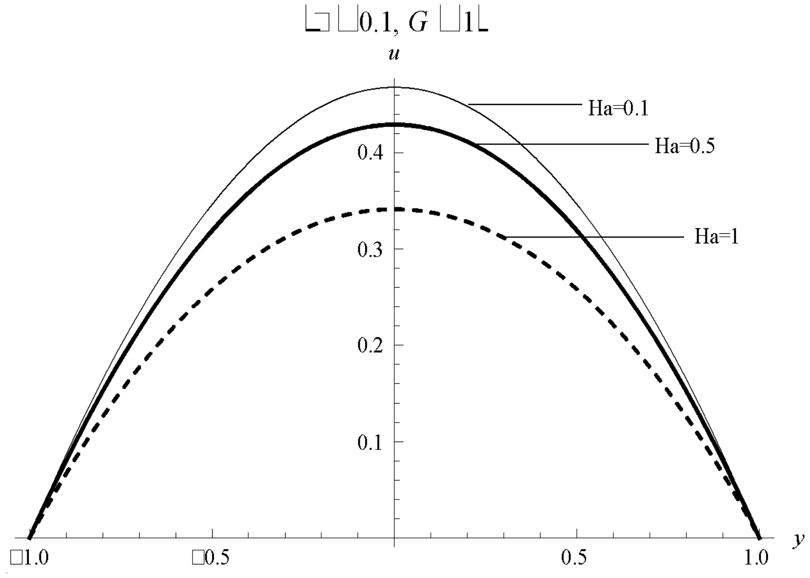

Figure 2: Variation of velocity with Hartmann number

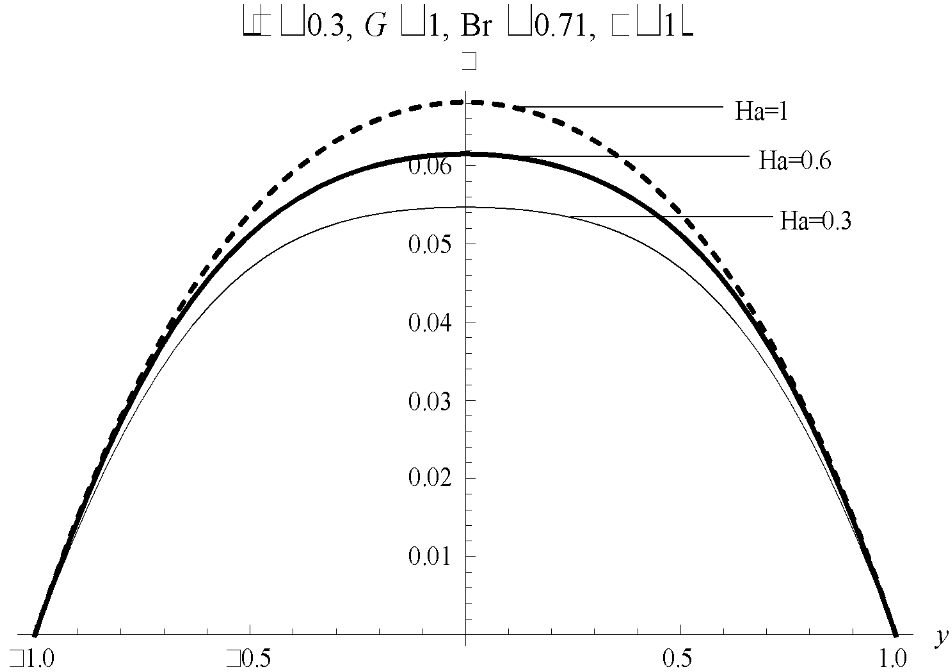

Figure 3: Temperature with Hartmann number

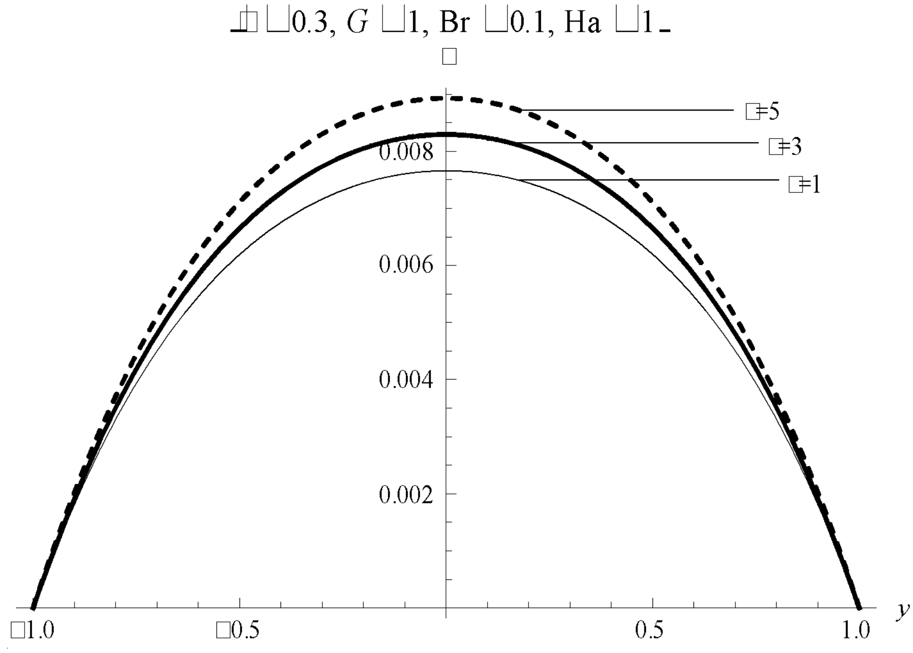

Figure 4: Temperature with internal heat generation parameter

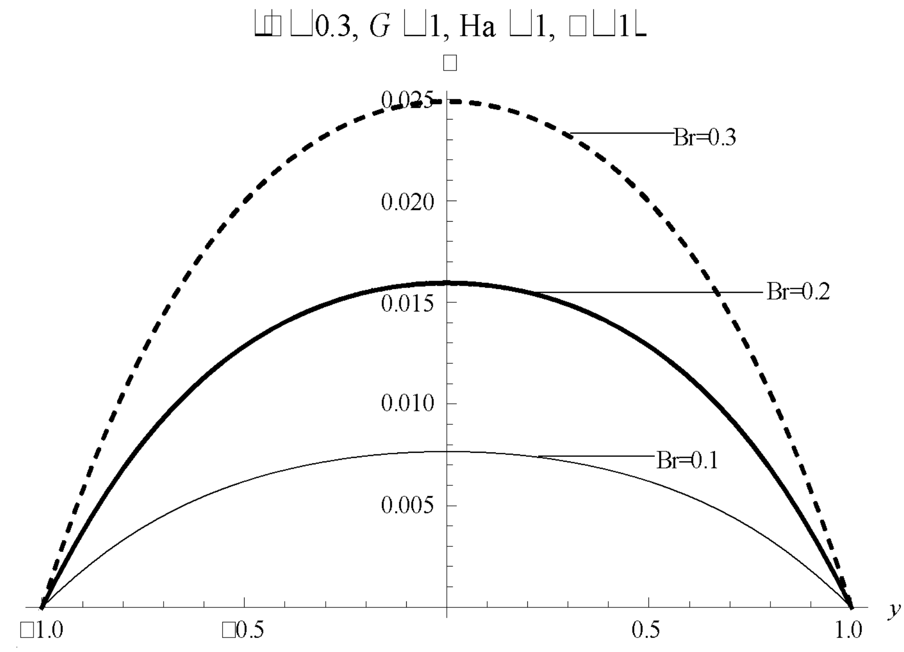

Figure 5: Temperature with Brinkman number

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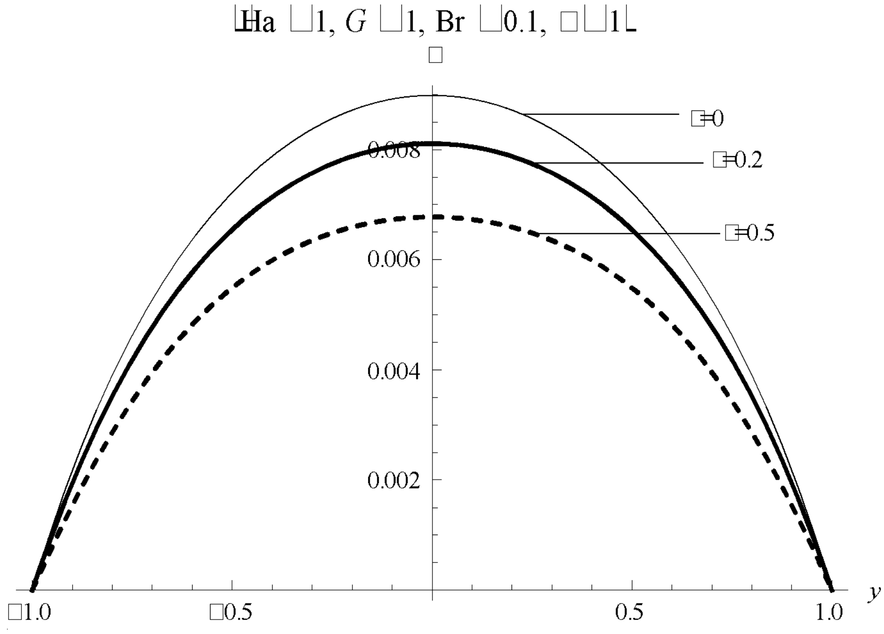

Figure 6: Temperature with non-Newtonian parameter

5. Conclusion

This paper presents the combined effects of temperature dependent internal heat generation and Ohmic heating on the third-grade fluid flow through parallel isothermal plates. We present analytical solutions in the form of regular perturbation method via Wolfram Mathematica for both the velocity and temperature fields. Generally, the result shows that an increase in the magnetic field intensity weakens the skin friction at the upper plate and increases the skin friction at the lower plate. On the other hand, an increase in the non-Newtonian material parameter weakens the skin friction at the lower plate while enhancing skin friction at the upper plate. However, an increase in both Hartmann's number and internal heat generation parameters decreases the rate of heat transfer from the fluid to the lower plate and at the upper plate, the rate of heat transfer from the fluid to the wall increases. Finally, the reverse is the case with internal heat generation. This is because an increase in this parameter brings about rise in the rate of heat transfer from the fluid to the lower and at the upper wall the rate decreases with an increase in the internal heat generation parameter.

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