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SIMULATION MODEL OF SPHERICAL SHOCK WAVE IN A MEDIUM WITH VARIABLE DENSITY

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Abstract: Self similar solutions of spherical shock wave for one dimensional flow. Spherical shock wave propagate into an uniform atmosphere with variable density and pressure, spherical shock wave obtained where the uniform atmosphere assumed to be at rest.

Key Words: simulation, spherical shock, self similar, density,

1. Introduction

Witham [1], studied the problem of spherical shock wave. Sakurai [2], Verma and Singh [3], Singh and Srivastava [4] have considered the problems of spherical shock waves in an exponentially increasing medium under the uniform pressure. Ojha and Onkar [5] studied the propagation of shock waves in an inhomogeneous self gravitating gaseous mass in which the disturbances are headed by a shock of variable strength Singh and Srivastava [6] have discussed the problems of magnetoradiative shock in a conducting plasma. Similarity solutions for shock waves phenomena in magnetogasdynamics have been obtained by number of authors eg Vishwakarma [7], Michaut Haut and et al [8]. Shilpa Shinde [9] Propagation of Cylindrical shock wave in a non uniform rotating stellar atmosphere under the action of monochromatic radiation and gravitation, [10] Kishore Kumar Srivastava, ReshmaLitoria and Jitendra Kumar Soni, Propagation of Exponential Magneto Radiative Shock Waves.

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In this paper a self similar model of the flow behind a spherical shock wave has been considered in which we have assumed that the disturbance is headed by a shock surface of variable strength and is propagating into a medium with variable density and pressure. The shock wave propagate in an uniform atmosphere which is assumed to be at rest.

The shock position in this problem is given by

$$R = At^{\mu}, \tag{1.1}$$

where A and μ are constants, $\mu < 1$.

We assume that the density distribution is given by

$$\rho_0 = br^{\beta},$$

where b and β are constants.

2. Equation of Motion

The basic equations governing the motion of the fluid in spherical symmetry are

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} = 0, \tag{2.2}$$

$$\frac{\partial P}{\partial t} + u\frac{\partial P}{\partial r} - \frac{(\gamma - 1)\,P}{\rho}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0. \tag{2.3}$$

Here $\rho, u, P, tand\gamma$ are the density, velocity, pressure, time and ratio of specific heat of the gas.

3. Similarity Solutions

We introduce the similarity variable

$$\eta = r/R\,(t)\,, \tag{3.1}$$

and re-write the solution of the basic equations in the form

$$u = \dot{R}V\,(\eta)\,, \tag{3.2}$$

$$\rho = \rho_0 D\,(\eta)\,, \tag{3.3}$$

$$p = P_0\dot{R}^2 P\,(\eta)\,, \tag{3.4}$$

where V, D, P are functions of η, only; $\dot{R} = dR/dt$ is the shock velocity.

4. Solution of Equations of Motion

In view of similarity transformation (3.1)-(3.4), the basic equations (2.1)-(2.3) take the form

$$\eta(1-\eta)D' + \left(\beta + \eta V'\right)D = 0, \tag{4.1}$$

$$\left(\frac{\mu-1}{\mu}\right)V + (V-\eta)V' + P' + \frac{\beta}{\eta}P = 0, \tag{4.2}$$

$$\left[2\left(\frac{\mu-1}{\mu}\right) + \frac{\beta V}{\eta} - \frac{(\gamma-1)\beta}{\eta}\right]P + (V-\eta)P' - (1-\eta)(\gamma-1)\frac{PD'}{D} = 0. \tag{4.3}$$

5. He Jump Conditions

The jump conditions for a strong shock wave are

$$\mu_1 = \frac{2\dot{R}}{\gamma+1}, \tag{5.1}$$

$$P_1 = \frac{2\rho_0\dot{R}^2}{\gamma+1}, \tag{5.2}$$

$$\rho_1 = \frac{(\gamma-1)\rho_0}{(\gamma-1)}, \tag{5.3}$$

where suffix 1 denotes the values of flow variables immediately behind the shock front.

The system of equations (4.1)-(4.2) can be reduced to

$$D' = \frac{D\left[(V-\eta)\left(\frac{\mu-1}{\mu}\right)V - \frac{\beta}{\eta}(V-\eta)^2 + \frac{\beta P}{\eta}(V-\eta) - \left\{2\left(\frac{\mu-1}{\mu}\right) + \frac{\beta V}{\eta} - \frac{(\gamma-1)\beta}{\eta}\right\}P\right]}{(1-\eta)\left[(V-\eta)^2 - (\gamma-1)P\right]}, \tag{5.4}$$

$$V' = -\frac{\alpha}{\beta} - \left\{\left[(V-\eta)\left(\frac{\mu-1}{\mu}\right)V - \frac{\beta}{\eta}(V-\eta) - 2\left(\frac{\mu-1}{\mu}\right) + \frac{\beta V}{\eta} - \frac{(\gamma-1)}{\eta}\beta\right]P\right\} / \left[(V-\eta)^2 - (\gamma-1)P\right], \tag{5.5}$$

$$P' = -\left(\frac{\mu-1}{\mu}\right)V - \frac{\beta}{\eta}P + (V-\eta)\frac{\beta}{\eta} + (V-\eta)$$
$$\left\{\left[(V-\eta)\left(\frac{\mu-1}{\mu}\right)V - \frac{\beta}{\eta}(V-\eta)^2 + \frac{\beta P}{\eta}(V-\eta)\right.\right.$$
$$\left.\left. - \left\{2\left(\frac{\mu-1}{\mu}\right) + \frac{\beta V}{\eta} - \frac{(\gamma-1)\beta}{\eta}\ \right\}\right] P\right\} / \left[(V-\eta)^2 - (\gamma-1)P\right]. \quad (5.6)$$

6. Results and Discussion

In this paper we have studied the propagation of spherical shock. The case $\mu \leq 1$ corresponds to a blast wave problem while $\mu = 0$ gives the problems of uniformly expanding shock wave in a medium with zero temperature gradient. For other value of $\mu = 1 to 0$, neither the total energy of the wave is constant nor shock wave expands uniformly. The kinematics condition at the inner expanding surface is $V(\overline{\eta}) = \mu$ where $\bar{\eta}$ is the value of η at the inner expanding surface. The kinematics condition demands that the velocity of the fluid particle at the expanding surface is equal to the velocity of the surface it self.

For exhibiting the numerical solution it is convenient to write the field variables in non dimensional form as.

$$\gamma = \frac{2}{\gamma+1}, \quad P(1) = 1, \quad D(1) = 1.$$

The numerical integration of equations (3.4.18) – (3.4.20) is carried out by using Runge-Kutta method for $\gamma^2 = \frac{4}{3}$. $\mu = 0.6, 0.8$ and$\beta = 0.4$. Here we take $M_A^2 = 0$ is pure non magnetic case. Thus there is no magnetically dominated layer in the flow field behind the shock ie there is no influence of magnetic forces. The nature of the field variables are illustrated by Figures 3.1-3.3. From Figure 1 and Figure 3.2, we see that velocity and density distribution is minimum at shock front but it increases sharply as we move inwards from shock front. But from Figure 3.3 it is clear that discontinuity in pressure distribution is maximum at shock front and decreases rapidly as we move away from shock front.

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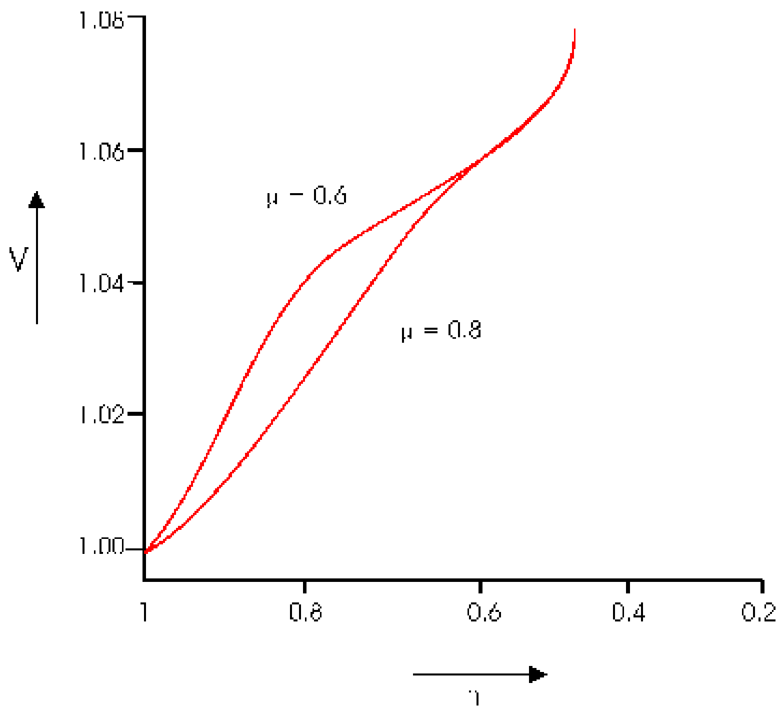

Fig. 3.1
Velocity distribution

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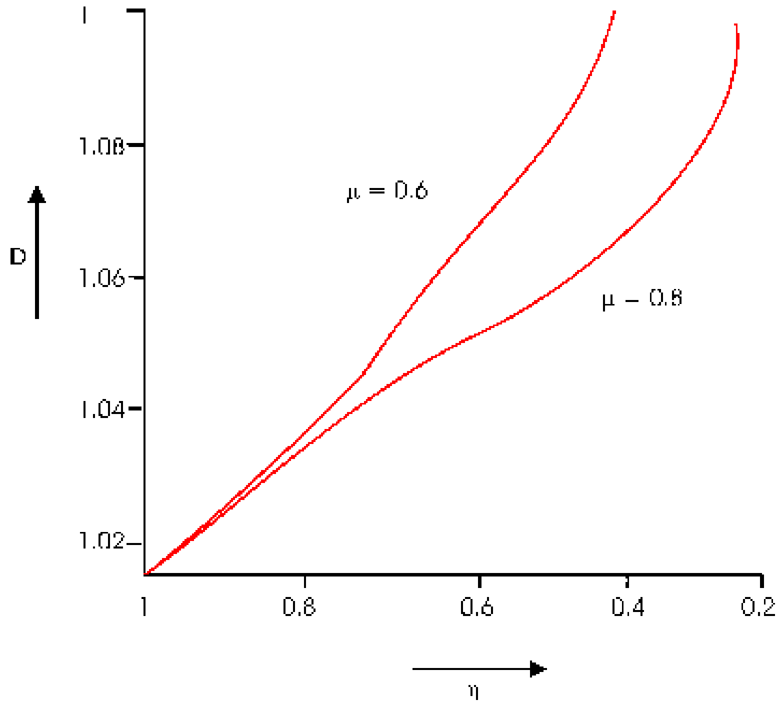

Fig. 3.2
Density distrib..tion

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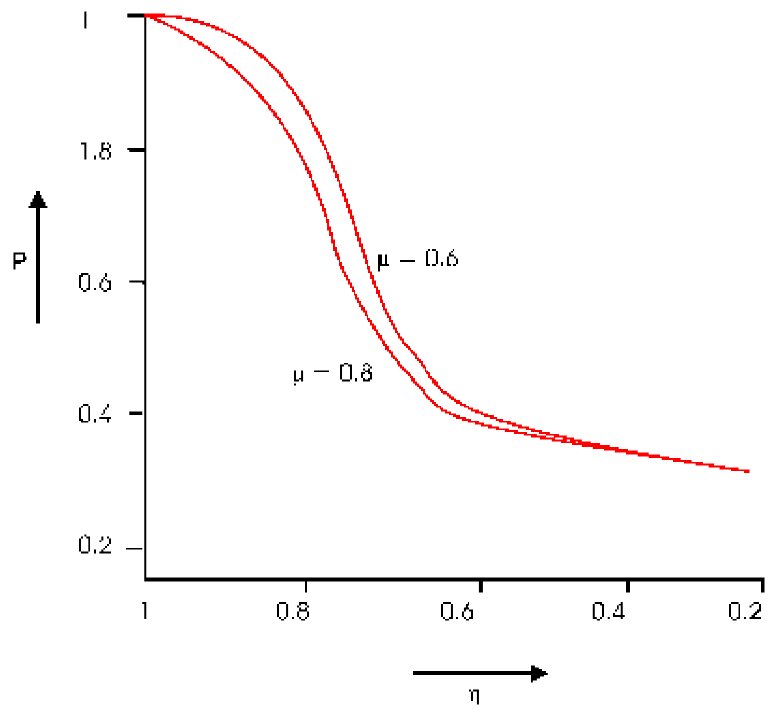

Fig. 3.3
Pressure distr bution

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