International Electronic Journal of Pure and Applied Mathematics

Volume 6 No. 4 2013, 277-281
ISSN: 1314-0744
url: http://www.e.ijpam.eu
doi: http://dx.doi.org/10.12732/iejpam.v6i4.6

ON EDGE MAGIC LABELING OF SOME GRAPHS

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Abstract: In this paper, we present an algorithm to get a multi magic edge labeling of Wheel $k_{1,n+e}$, Book with Triangular pages.

AMS Subject Classification: 05C78
Key Words: graph labeling, edge magic, Wheel graph, triangular book

1. Introduction

Graph labelings have lately aroused considerable attention. They gave birth to families of graphs with attractive names such as magic, graceful, harmonious, felicitous, sequential, and elegant. Labeled graphs serve as useful models for a broad range of applications such as; coding theory problems, including the design of good radar type codes, missile guidance codes and convolution codes with optimal auto correlation properties. They facilitate the optimal non standard encoding of integers. Labeled graphs have also been applied in determining ambiguities in x-ray crystallographic analysis, to design a communication network addressing system, in determining optimal circuit layouts and radio astronomy problems etc. [1, 2]. A labeling of a graph G is an assignment f of labels to either the vertices or the edges of G that induces for each edge uv in the former a label depending on the vertex labels $f(u)$ and $f(v)$ and in the latter for each vertex u a label depending on the labels of the edges incident with it. Kotzig and Rosa [4] defined a magic labeling to be a total labeling on the vertices and edges in which the labels are the integers from 1 to n. The sum of labels on an edge and its two end points is constant. In 1996 Ringel and Llado [5] redefined this type of labeling as edge magic. Also Enomoto et al [3] have introduced the name super edge-matrix for magic labeling in the sense of Kotzig and Rosa, with the added property that the v, vertices receive the smaller labels $\{1, 2, \ldots, v\}$. A

Received: November 29, 2012

one-to-one map f from $V \cup E$ on to the integers $\{1, 2, \ldots, v + e\}$ is an edge-magic labeling if there is a constant k so that for any edge xy, $f(x) + f(x, y) + f(y) = k$. The constant k is called the edge magic number for f. An edge-magic labeling f is called super edge-magic if $f(\nu) = \{1, 2, \ldots, v\}$ and $f(e) = \{v + 1, v + 2, \ldots, v + e\}$. A graph G is called edge-magic [respectively super edge magic] if there exists an edge-magic [respectively super edge magic] labeling of G.

2. Preliminaries

In this section, we give the basic notation relevant to this paper. Let $G = G(u, v)$ be a finite simple and undirected graph with u vertices and v edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers called labels.

Definition 1 (Antimagic Labeling)**.** A graph with v edges is called anti magic, if its edges can be labeling with $1, 2, \ldots, v$, so that the sum of the labels of the edges incident to each vertex are distinct.

Definition 2 (Edge-magic Labeling)**.** A (u, v) graph G is said to have consecutive and constant edge-magic labeling if, for some edges in the graph the labels of $f(u)+f(v)+f(uv)$ are consecutive integers and for the rest of the edges the labeling is constant integer. We define such labeling as edge magic labeling.

Definition 3 (Wheel Graph)**.** A Wheel graph W_n is a graph with n vertices formed by connecting a single vertex to all vertices of an $(n - 1)$ cycle.

Definition 4 (Triangular Book)**.** Triangular Book is the complete tri partite graph $k_{1,1,p}$. It is a graph consisting of p triangles sharing a common edge.

3. Main Results

In this section, we present an algorithm to get a multi magic labeling of wheel, book with triangular pages and $K_{1,n+e}$.

Theorem 5. *For every $n \geq 4$, there exists a $W_n = C_n + k_1$, having edge multi magic constants is of the form $2n + x$, where $x = 3, 4, 5, 6, \ldots$.*

Proof. Consider $W_n = C_n + K_1$, having $n \geq 4$ vertices and $n + k$ edges where $k = 2, 3, 4, 5, \ldots$ respectively. The vertex labeling of this graph defined by

i) $f(v_1) = 1$

ii) $f(v_i) = i + 1$ for $2 \leq i \leq n - 1$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 6, No. 4 (2013)

Define the map f on E as follows. Let $f : E \to \{n+1, n+2, \ldots, n+k\}$ where $k = 2, 3, 4, \ldots$ respectively.

i) When $n = 4, 5, 6, 7, 8, 9, \ldots$ $f(v_1, v_2) = 2n + t$ where $t = 2, 3, 4, 5, 6, 7$ respectively.

ii) $f(v_2, v_n) = n + 1$

iii) When $i = 2, 3, 4, 5, \ldots$ then $f(v_i, v_{i+1}) = 2n - k$, where $k = 1, 2, 3, 4, \ldots$ respectively.

iv) When $i = 0, 1, 2, 3, \ldots$ then $f(v_1, V_{n-j}) = 2n + s$ where $s = 0, 1, 2, 3, \ldots$ respectively, and also the spokes of wheel have same edge labeling.

□

Example 6. $n = 6$

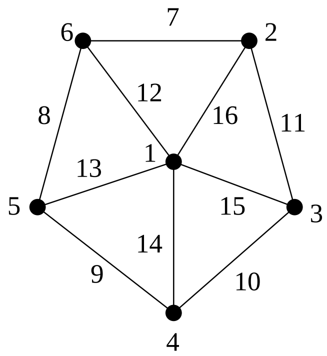

Figure 1

Now the multi magic constants of w_6 are 15, 16, 17, 18, 19.

Theorem 7. *Let G be the graph with $V(G) = \{u, u_i, w, z : 1 \leq i \leq n\}$ and $E(G) = \{uv, vw, wu, uu_i, 1 \leq i \leq n\}$.*
The vertex labeling of this graph defined by

i) $f(u) = 1$

ii) $f(v) = 2$

iii) $f(w) = 3$

iv) $f(u_i) = 3 + i$, $1 \leq i \leq n$.

The edge labeling of this graph defined by

i) $f(u, v) = 2n$

ii) $f(v, w) = 2n - 1$

iii) $f(w, u) = 2n - 2$

iv) *When* $j = 0, 1, 2, 3, \ldots$ *then* $f(u, u_{n-j}) = n + i$, *where* $i = 1, 2, 3, 4, \ldots$ *respectively.*

The edge multi magic constants is of the form

i) $f(v, w) = 2n + 4$

ii) $f(v, u) = 2n + 3$

iii) $f(u, w) = 2n + 2$

iv) $f(u, u_i) = 2n + 2$ *for all* $i = 1, 2, 3, \ldots, n$.

Example 8. When $n = 7$,

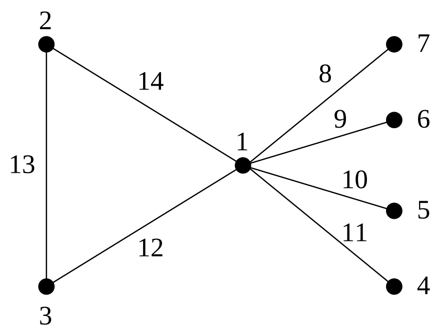

Figure 2

Now here the multi magic constants are 16,17,18.

Theorem 9. *For every* $n \geq 3$, *there exists a* $(n, n+k)$ *(where* $k = 0, 1, 2, 3, \ldots$ *respectively), Book with triangular pages graph having edge multi magic constants as follows. The vertex set has initially '3' vertices that are* u, v, w, *make a triangular book. From* u *and* v *increasing the triangular pages that are namely,* $x_1, x_2, x_3, x_4, \ldots$.

Define a vertex label,

i) $f(u) = 1$,

ii) $f(v) = 2$,

iii) $f(x_i) = 3, 4, 5, \ldots$ *where* $i = 1, 2, 3, \ldots$ *respectively.*

Define the edge labeling as follows;

i) For $n = 3, 4, 5, 6, \ldots$
$f(u, v) = 2n + i$, *where* $i = 0, 1, 2, 3, \ldots$ *respectively*

ii) For $i = 1, 2, 3, \ldots$
$f(v, x_i) = 3n - 2t$ *where* $t = 2, 3, 4, \ldots$ *respectively*

iii) For $i = 1, 2, 3, \ldots$
$f(u, x_i) = 3n - k$ *where* $k = 5, 7, 9, 11, 13, \ldots$ *respectively*

When $n \geq 3$*, the edge multi magic constant is of the form* $2n + k$ *where* $k = 3, (3, 4), (3, 4, 5), (3, 4, 5, 6), \ldots$ *respectively.*

Example 10. $n = 4$

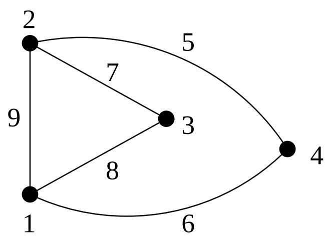

Figure 3

Here $n = 4$ and edges are 5. Now the edge multi magic constants are 11 and 12.

4. Conclusion

Graph labeling have been applied to many areas like communication network, circuit analysis, radio astronomy etc. There are different kinds of labeling of graphs. It is of interest to study edge multi magic labeling as the uniqueness of this labeling helps in the optimal paths of circuit theory, thereby minimizing the cost of the network. We study in multi magic labeling of some simple graphs like wheel, book with triangular pages and $k_{1,n+e}$. We explore graphs like product graphs, harmonious graphs and arithmetic graphs.

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