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COMPUTATIONAL SOLUTIONS TO THE RIPPLE PROBLEM IN LARGE Magic DECKS

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Abstract: We consider a large shuffled deck of **Magic:The Gathering** cards, where some fraction of the deck is Simian Spirit Guides and the remainder is Surging Flames. We compute in exact form the probability of an immediate win as a function of the deck's fraction of Surging Flames. We determine that the ideal fraction of the deck to be Surging Flames is an irrational root of a 32^{nd} order polynomial and proceed numerically, finding an 88% immediate win probability at a Surging Flame fraction of 0.4873655 ± 0.0000005.

1. Introduction and Problem Statement

In a game of **Magic: the Gathering** two or more players each begin with a life total of twenty and a deck, from which they draw a seven card hand. Mana is used to play cards which, whether directly or indirectly, typically damage the opponent. The last player with a positive life total is the winner.

A continuous generation of expansion sets has brought the number of distinct **Magic** cards into the tens of thousands, many of which interact nontrivially with others. As such, the space of puzzles generated by the card game is large. We work out an elegant puzzle involving only two distinct cards and a single noninteracting opponent. Even so, an exhaustive evaluation of possible cases would be too expensive to compute manually.

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We consider a deck composed of some fraction Surging Flames and the rest Simian Spirit Guides. Any number of Simian Spirit Guides may be discarded from the hand for one mana each. Each copy of Surging Flame costs two mana to play and deals two damage to the opponent.

Additionally, Surging Flame has the ability *Ripple 4*. As part of the effect of Surging Flame, the top four cards of the deck are revealed to all players. Any revealed copies of Surging Flame are played without paying their mana cost. Other revealed cards are put on the bottom of the deck. *Ripple* takes effect regardless of whether Surging Flame was played from the hand or from the top of the deck, making it possible for a single ripple to start a chain reaction.

Between those played from the hand and those revealed from the deck, we must play ten copies of Surging Flame to win. Further copies are irrelevant. We compute the deck's fraction of Surging Flames necessary to maximize the probability of winning immediately.

2. Technique

For each set of one Surging Flame and two Simian Spirit Guides, we can play one Surging Flame. That is, if exactly one, four, or five Surging Flames are drawn in the seven card opening hand then one copy may be played. With exactly two or three Surging Flames two can be played. Otherwise none can be played and the hand fails to win immediately.

Let x be the deck's fraction of Surging Flames. Then of course the probability of having exactly k of them in the opening hand and in a four card pile are respectively given by

$$P(k) = \binom{7}{k} x^k (1-x)^{7-k} \tag{1}$$

$$Q(k) = \binom{4}{k} x^k (1-x)^{4-k} \tag{2}$$

We will compute the probabilities of each winning case. Let $\texttt{odds[}i\texttt{]}$ be the probability of case i and $\texttt{plays[}i\texttt{]}$ be the number of times that the i^{th} case has played Surging Flame. We seed these arrays with

$$\begin{aligned} &\texttt{odds[0]} \leftarrow P(0) + P(6) + P(7), && \texttt{plays[0]} \leftarrow 0; \\ &\texttt{odds[1]} \leftarrow P(1) + P(4) + P(5), && \texttt{plays[1]} \leftarrow 1; \\ &\texttt{odds[2]} \leftarrow P(2) + P(3), && \texttt{plays[2]} \leftarrow 2 \end{aligned} \tag{3}$$

Then we proceed by incrementally investigating all possible ways in which a game can evolve. Our algorithm is simple enough:

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```
for n ← 0 to 9
  ℓ ← plays.length()
  for i ← 0 to ℓ − 1
    if plays[i] > n then
      for k ← 0 to 4
        plays.append( plays[i]+k )
        odds.append( odds[i]·Q(k) )                    (4)
      end for
    end if
    plays[i] ← 0
    odds[i] ← 0
  end for
end for
```

Each iteration in n corresponds to revealing a pile of four cards from the top of the deck. A case that is allowed to reveal that pile is bifurcated into five new cases, one for each number of Surging Flames that could appear in that pile. The parent case is then made null, as is any case which has played fewer than ten copies of Surging Flame and will reveal no more piles. After all iterations have completed, the sum of all elements of `odds` gives the probability of immediate victory.

Since our number of iterations is small we are unconcerned by our exponential asymptotic complexity. In fact we prefer to keep the code as simple as possible by omitting even obvious optimizations. We elect to carry out the computation in `C++` after writing a short class to enable the addition and multiplication of polynomials with coefficients of type `long long int`; that is, the coefficients of the polynomial are integers in some cases as large as 10^{18}. Even so, the routine can be completed by a desktop computer in a matter of moments.

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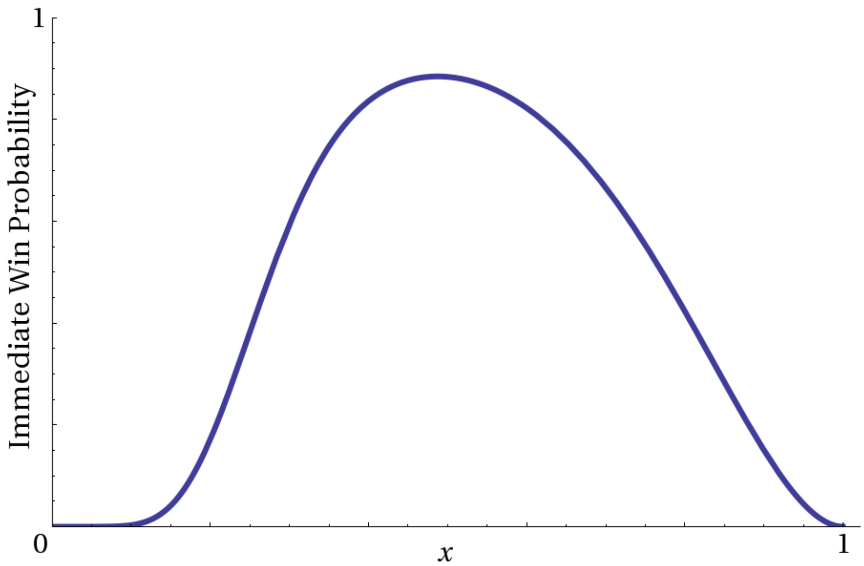
3. Exact Results

Our result is the following polynomial in x, the deck's fraction of Surging Flames.

$$
\begin{aligned}
\textstyle\sum_i \texttt{odds}[i] = \; & 514406893x^{10} - 14208915084x^{11} + 192367138782x^{12} \\
& - 1696557565122x^{13} + 10933445396335x^{14} - 54767633628036x^{15} \\
& + 221527650090978x^{16} - 742218462064618x^{17} + 2096945656690556x^{18} \\
& - 5060203362464656x^{19} + 10527016434129608x^{20} - 19004539331441699x^{21} \\
& + 29903265189622188x^{22} - 41106973719324905x^{23} + 49385458987881149x^{24} \\
& - 51750097201757325x^{25} + 47058841677902019x^{26} - 36766332553082013x^{27} \\
& + 24210423037041920x^{28} - 12901223465477340x^{29} + 4980295115307946x^{30} \\
& - 742462788195296x^{31} - 788510007017460x^{32} + 911191645532532x^{33} \\
& - 587929736780232x^{34} + 281793447803700x^{35} - 106870948771160x^{36} \\
& + 32582727821952x^{37} - 7953506734656x^{38} + 1526444387244x^{39} \\
& - 222589611860x^{40} + 23229854340x^{41} - 1547683060x^{42} \\
& + 49506380x^{43}
\end{aligned}
\tag{5}
$$

We note, as expected, that this function has exactly one extremum on the unit interval.



(6)

To find the extrema we differentiate our probability function. We find a nine-time degenerate root at $x = 0$ and a double root at $x = 1$ which leaves the extrema

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to be determined by solving

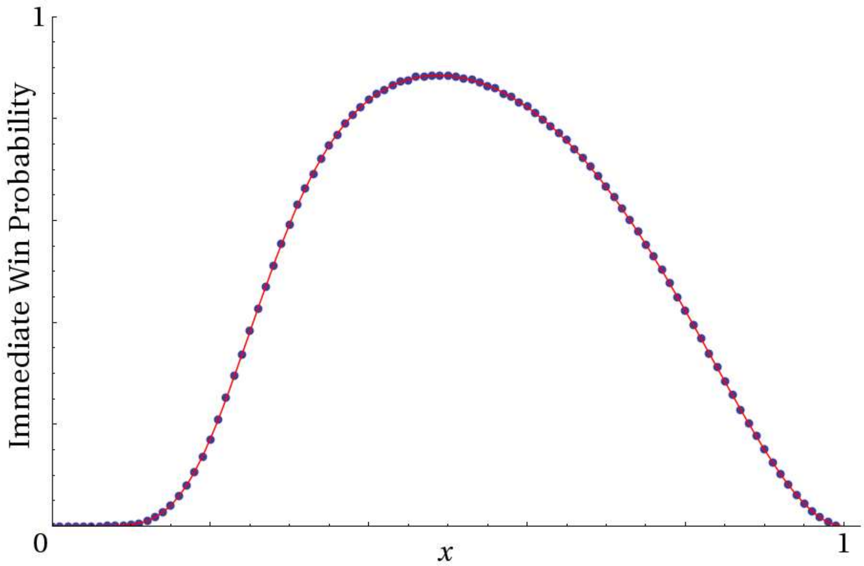
$$\begin{aligned} 0 = \;& -734866990 && +21593428142x && -308178809770x^2 \\ & +2842570954028x^3 && -19024319838642x^4 && +98334895078578x^5 \\ & -408014019415086x^6 && +1394516531313272x^7 && -3997629443033872x^8 \\ & +9737208255084480x^9 && -20339981556714400x^{10} && +36673636437610697x^{11} \\ & -57308054158344751x^{12} && +77757716633722794x^{13} && -91563857039012574x^{14} \\ & +93257918681549301x^{15} && -81532064693515341x^{16} && +60280932296943852x^{17} \\ & -36560759851223828x^{18} && +16887165934325152x^{19} && -4456955988423188x^{20} \\ & -1168906497844020x^{21} && +2435710677092940x^{22} && -1859907080417568x^{23} \\ & +995751641086416x^{24} && -413215597932084x^{25} && +136406424319596x^{26} \\ & -35816565596436x^{27} && +7359613820268x^{28} && -1144862051520x^{29} \\ & +127078587680x^{30} && -8981987740x^{31} && +304110620x^{32} \end{aligned} \tag{7}$$

In accordance with the rational root theorem, any rational root must be of the form $\frac{p}{q}$ where p divides the constant term and q divides the leading coefficient. We find by exhaustive search of such values that there are no rational roots.

4. Numerical Computations

From the plot it's evident that the solution is near $x = \frac{1}{2}$. As such, we expand the polynomial in a Taylor Series around $\frac{1}{2}$. Double precision values become dominated by roundoff error when raised to large powers so we prefer the low order terms to dominate. We conclude the extrema to be at 0.4873655 ± 0.0000005.

We compare our results with the outcome of a straightforward Monte Carlo routine.



(8)

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In red we have superimposed our polynomial over the Monte Carlo results and note agreement. Our Monte Carlo uncertainties are comparable to the radius of the blue circles. With 10^5 trials at each of 100 evenly spaced bins we conclude that the maximum immediate win probability is just over 88 percent, at $x = 0.48 \pm 0.02$. The peak is sufficiently flat that a notable increase in Monte Carlo result precision would require a dramatic increase in computational expense.

5. Acknowledgements

This problem was suggested by Matthias Hunt. Surging Flame and Simian Spirit Guide are elements of the game **Magic: the Gathering**, which is owned by Wizards of the Coast, a subsidiary of Hasbro.