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ON WEAKLY γ-CONTINUOUS FUNCTIONS

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Abstract: We introduce a new class of functions called weakly γ-continuous functions which is contained in the class of weakly continuous functions and contains the class of γ-continuous functions.

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1. Introduction

Kasahara [5] defined an operation α on a topological space to introduce α-closed graphs. Following the same technique, Ogata [10] defined an operation γ on a topological space and introduced γ-open sets. Basu et al [3] introduced a type of continuity called γ-continuous function.

In this paper, we introduce a new class of functions called weakly γ-continuous functions which is contained in the class of weakly continuous functions and contains the class of γ-continuous functions. We obtain basic properties of weakly γ-continuous functions.

2. Preliminaries

Throughout the present paper, (X, τ) and (Y, σ) (or simply X and Y) denotes a topological spaces on which no separation axioms is assumed unless explicitly stated. Let A be a subset of a topological space X. The closure and interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively.

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Definition 2.1. [5] Let (X, τ) be a topological space. An operation γ on the topology τ is a mapping from τ in to power set $P(X)$ of X such that $V \subseteq \gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of γ at V.

Definition 2.2. [10] A subset A of a topological spac (X, τ) is called γ-open set if for each $x \in A$ there exists an open set U such that $x \in U$ and $\gamma(U) \subseteq A$. Then, τ_γ denotes the set of all γ-open set in X. Clearly $\tau_\gamma \subseteq \tau$. Complements of γ-open sets are called γ-closed.

Definition 2.3. [10] Let (X, τ) be a topological space and A be subset of X, then τ_γ-$Cl(A) = \bigcap\{F : A \subseteq F, X \setminus F \in \tau_\gamma\}$.

Definition 2.4. [11] Let (X, τ) be a topological space and A be subset of X, then τ_γ-$Int(A)$= $\bigcup\{U : U$ is a γ-open set and $U \subseteq A\}$.

Definition 2.5. A subset A of a space X is said to be

1. semi-open [8] if $A \subseteq Cl(Int(A))$.

2. pre-open [9] if $A \subseteq Int(Cl(A))$.

3. β-open [1] if $A \subseteq Cl(Int(Cl(A)))$.

Definition 2.6. [12] The θ-interior of a subset A of X is the union of all open sets of X whose closures are contained in A. The subset A is called θ-open if $A = Int_\theta(A)$. The complement of a θ-open set is called θ-closed. Alternatively, a set $A \subseteq X$ is called θ-closed if $A = Cl_\theta(A)$, where $Cl_\theta(A) = \{x \in X : A \cap Cl(U) \neq \phi$, $U \in \tau$ and $x \in U\}$. The family of all θ-open sets forms a topology on X. For each subset A of X, $Cl_\theta(A)$ is closed in X.

Theorem 2.7. *Let A be a subset of a topological space (X, τ). Then:*

1. *If $A \in SO(X)$, then $pCl(A) = Cl(A)$ [4].*

2. *If $A \in \tau$, then $Cl_\theta(A) = Cl(A)$ [12].*

Proposition 2.8. *[2] A subset A of a space X is β-open if and only if $Cl(A)$ is regular closed.*

Theorem 2.9. *[1] Let A be any subset of a space X. Then $A \in \beta O(X)$ if and only if $Cl(A) = Cl(Int(Cl(A)))$.*

Proposition 2.10. *[6] Let A be any subset of a topological space (X, τ) and γ be an operation on τ. Then the following statements are true:*

1. *$X \setminus \tau_\gamma$-$Int(A) = \tau_\gamma$-$Cl(X \setminus A)$.*

2. *$X \setminus \tau_\gamma$-$Cl(A) = \tau_\gamma$-$Int(X \setminus A)$.*

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Definition 2.11. [3] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be γ-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-open set U containing x such that $f(U) \subseteq V$.

3. Weakly γ-Continuous

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly γ-continuous (resp. weakly continuous [7]) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-open (resp. open) set U of X containing x such that $f(U) \subseteq Cl(V)$. "weakly γ-continuous" will be denoted by "w.γ.c.".

Every w.γ.c. function is weakly continuous. However, but the converse is not true by following example.

Example 3.2. Consider $X = \{a, b, c\}$ with the topology $\tau = \sigma = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Define an operation γ on τ by

$$\gamma(A) = \begin{cases} A & \text{if } A = \{a, b\} \\ X & \text{if } A \neq \{a, b\} \end{cases}$$

Define a function $f : (X, \tau) \to (X, \sigma)$ as follows:

$$f(x) = \begin{cases} a & \text{if } x = a \\ b & \text{if } x = b \\ c & \text{if } x = c \end{cases}$$

Then f is weakly continuous, but not w.γ.c., because $\{a\}$ is an open set in (X, σ) containing $f(a) = a$, but there exist no γ-open set U in (X, τ) containing a such that $f(U) \subseteq Cl(\{a\}) = \{a\}$.

Proposition 3.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is w.γ.c. if for each $x \in X$ and each regular closed set F of Y containing $f(x)$, there exists a γ-open set U of X containing x such that $f(U) \subseteq F$.*

Proof. Suppose that each regular closed set F of Y containing $f(x)$, there exists a γ-open set U of X containing x such that $f(U) \subseteq F$. Let V be an open set in Y, so $Cl(V) = F$ (say) is a regular closed set of Y containing $f(x)$, then by hypothesis there exists a γ-open set U of X containing x such that $f(U) \subseteq F = Cl(V)$. Hence f is w.γ.c.. □

Proposition 3.4. *A function $f : (X, \tau) \to (Y, \sigma)$ is w.γ.c. if and only if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-open set U in X containing x such that $f(U) \subseteq pCl(V)$.*

Proof. Follows from Theorem 2.7. □

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Every γ-continuous is w.γ.c., but the converse is not true in general as shown in the following example:

Example 3.5. Consider $X = \{a, b, c\}$ with the topology $\tau = \sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Define an operation γ on τ by $\gamma(A) = A$ for all $A \in \tau$. Define a function $f : (X, \tau) \to (X, \sigma)$ as follows:

$$f(x) = \begin{cases} c & \text{if } x = a \\ b & \text{if } x = b \\ a & \text{if } x = c \end{cases}$$

Then f is w.γ.c. but not γ-continuous, because $\{a\}$ is an open set in (X, σ) containing $f(c) = a$, but there exist no γ-open set U in (X, τ) containing c such that $f(U) \subseteq \{a\}$.

Proposition 3.6. *A function $f : (X, \tau) \to (Y, \sigma)$ is w.γ.c. if and only if $f^{-1}(Cl(V))$ is γ-open set in X, for each open set V in Y.*

Proof. Let V be any open set in Y. We have to show that $f^{-1}(Cl(V))$ is γ-open set in X. Let $x \in f^{-1}(Cl(V))$, implies that $f(x) \in Cl(V)$. Since f is w.γ.c., then there exists a γ-open set U of X containing x such that $f(U) \subseteq Cl(V)$. Which implies that $x \in U \subseteq f^{-1}(Cl(V))$. Therefore, $f^{-1}(Cl(V))$ is a γ-open set in X.

Conversely, let $x \in X$ and let V be any open set of Y containing $f(x)$. Then $x \in f^{-1}(Cl(V))$, by hypothesis $f^{-1}(Cl(V))$ is a γ-open set in X containing x, so clearly $f(f^{-1}(Cl(V)) \subseteq Cl(V)$. Therefore f is w.γ.c.. □

Proposition 3.7. *A function $f : (X, \tau) \to (Y, \sigma)$ is w.γ.c. if and only if $f^{-1}(Int(F))$ is a γ-closed set in X, for each closed set F of Y.*

Proof. Let F be any closed set of Y. Then $Y \setminus F$ is an open set of Y, since f is w.γ.c.. Then by Proposition 3.6, $f^{-1}(Cl(Y \setminus F))$ is a γ-open set in X and $f^{-1}(Cl(Y \setminus F)) = f^{-1}(Y \setminus Int(F)) = X \setminus f^{-1}(Int(F))$ is a γ-open set in X and hence $f^{-1}(Int(F))$ is a γ-closed set in X.

Conversely, let V be any open set of Y. Then $Y \setminus V$ is closed, and by hypothesis $f^{-1}(Int(Y \setminus V)) = f^{-1}(Y \setminus Cl(V)) = X \setminus f^{-1}(Cl(V))$ is a γ-closed set in X, so $f^{-1}(Cl(V))$ is a γ-open set in X. Therefore, by Proposition 3.6, f is w.γ.c.. □

Theorem 3.8. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is w.γ.c..*
2. *$f^{-1}(V) \subseteq \tau_\gamma$-$Int(f^{-1}(Cl(V)))$, for every open set V of Y.*
3. *τ_γ-$Cl(f^{-1}(Int(F))) \subseteq f^{-1}(F)$, for every closed set F of Y.*
4. *τ_γ-$Cl(f^{-1}(Int(Cl(B)))) \subseteq f^{-1}(Cl(B))$, for every subset B of Y.*

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5. $f^{-1}(Int(B)) \subseteq \tau_\gamma\text{-}Int(f^{-1}(Cl(Int(B))))$, *for every subset* B *of* Y.

6. $\tau_\gamma\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$, *for every open set* V *of* Y.

Proof. (1) $\Rightarrow$ (2). Let V be an open set of Y such that $x \in f^{-1}(V)$. Then $f(x) \in V$. There exists $U \in \tau_\gamma$ containing x such that $f(U) \subseteq Cl(V)$. Thus, we obtain $x \in U \subseteq f^{-1}(Cl(V))$. This implies that $x \in \tau_\gamma\text{-}Int(f^{-1}(Cl(V)))$.

(2) $\Rightarrow$ (3). Let F be any closed set of Y. Suppose that $x \notin f^{-1}(F)$. Then $Y \setminus F$ is open in Y and $x \in X \setminus f^{-1}(F) = f^{-1}(Y \setminus F)$. By (2) and Proposition 2.10, we have
$x \in \tau_\gamma\text{-}Int(f^{-1}(Cl(Y \setminus F))) = \tau_\gamma\text{-}Int(f^{-1}(Y \setminus Int(F))) = \tau_\gamma\text{-}Int(X \setminus f^{-1}(Int(F)))$
$= X \setminus (\tau_\gamma\text{-}Cl(f^{-1}(Int(F))))$.

Therefore, we obtain $x \notin \tau_\gamma\text{-}Cl(f^{-1}(Int(F)))$.

(3) $\Rightarrow$ (4). Let B be any subset of Y. Then $Cl(B)$ is closed in Y and by (3), we have that if $x \in \tau_\gamma\text{-}Cl(f^{-1}(Int(Cl(B))))$ then $x \in f^{-1}(Cl(B))$.

(4) $\Rightarrow$ (5). Let B be any subset of Y and $x \in f^{-1}(Int(B))$. Then we have $x \in f^{-1}(Int(B)) = X \setminus f^{-1}(Cl(Y \setminus B))$. Then $x \notin f^{-1}(Cl(Y \setminus B))$ and by (4), $x \in X \setminus \tau_\gamma\text{-}Cl(f^{-1}(Int(Cl(Y \setminus B)))) = \tau_\gamma\text{-}Int(f^{-1}(Cl(Int(B))))$.

(5) $\Rightarrow$ (6). Let V be any open set of Y. Suppose that $x \notin f^{-1}(Cl(V))$. Then $f(x) \notin Cl(V)$ and there exists an open set W containing $f(x)$ such that $W \cap V = \phi$; hence $Cl(W) \cap V = \phi$. By (5), we have $x \in \tau_\gamma\text{-}Int(f^{-1}(Cl(W)))$ and hence there exists $U \in \tau_\gamma$ such that $x \in U \subseteq f^{-1}(Cl(W))$. Since $Cl(W) \cap V = \phi$; $U \cap f^{-1}(V) = \phi$ implies that $x \notin \tau_\gamma\text{-}Cl(f^{-1}(V))$. Therefore, if $x \in \tau_\gamma\text{-}Cl(f^{-1}(V))$, then $x \in f^{-1}(Cl(V))$.

(6) $\Rightarrow$ (1). Let $x \in X$ and V any open set of Y containing $f(x)$. Then, we have $x \in f^{-1}(V) \subseteq f^{-1}(Int(Cl(V))) = X \setminus f^{-1}(Cl(Y \setminus Cl(V)))$. By (6), $x \notin \tau_\gamma\text{-}Cl(f^{-1}(Y \setminus Cl(V)))$ and hence $x \in \tau_\gamma\text{-}Int(f^{-1}(Cl(V)))$. Therefore, there exists $U \in \tau_\gamma$ such that $x \in U \subseteq f^{-1}(Cl(V))$; hence $f(U) \subseteq Cl(V)$. This shows that f is w.γ.c.. □

Theorem 3.9. *For a function* $f : (X, \tau) \to (Y, \sigma)$, *the following statements are equivalent:*

1. f *is w.γ.c..*

2. $\tau_\gamma\text{-}Cl(f^{-1}(Int(Cl_\theta(B)))) \subseteq f^{-1}(Cl_\theta(B))$, *for every subset* B *of* Y.

3. $\tau_\gamma\text{-}Cl(f^{-1}(Int(Cl(B)))) \subseteq f^{-1}(Cl_\theta(B))$, *for every subset* B *of* Y.

4. $\tau_\gamma\text{-}Cl(f^{-1}(Int(Cl(O)))) \subseteq f^{-1}(Cl(O))$, *for every open set* O *of* Y.

5. $\tau_\gamma\text{-}Cl(f^{-1}(Int(Cl(V)))) \subseteq f^{-1}(Cl(V))$, *for every preopen set* V *of* Y.

6. $\tau_\gamma\text{-}Cl(f^{-1}(Int(F))) \subseteq f^{-1}(F)$, *for every regular closed set* F *of* Y.

7. $\tau_\gamma\text{-}Cl(f^{-1}(Int(Cl(O)))) \subseteq f^{-1}(Cl(O))$, *for every* β*-open set* O *of* Y.

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8. τ_γ-$Cl(f^{-1}(Int(Cl(O)))) \subseteq f^{-1}(Cl(O))$, *for every semi-open set O of Y.*

Proof. (1) ⇒ (2). Let B be any subset of Y. Then $Cl_\theta(B)$ is closed in Y. Then by Theorem 3.8 (3), if $x \in \tau_\gamma$-$Cl(f^{-1}(Int(Cl_\theta(B))))$, then $x \in f^{-1}(Cl_\theta(B))$.

(2) ⇒ (3). This is obvious since $Cl(B) \subseteq Cl_\theta(B)$ for every subset B.

(3) ⇒ (4). This is obvious since $Cl(O) = Cl_\theta(O)$ for every open set O.

(4) ⇒ (5). Let $V \in PO(Y)$ and $x \in \tau_\gamma$-$Cl(f^{-1}(Int(Cl(V))))$. Then we have $V \subseteq Int(Cl(V))$ and $Cl(V) = Cl(Int(Cl(V)))$. Now, set $O = Int(Cl(V))$, then O is open in Y and $Cl(O) = Cl(V)$. By (4), we have $x \in f^{-1}(Cl(O))$ and hence $x \in f^{-1}(Cl(V))$.

(5) ⇒ (6). Let F be any regular closed set of Y and $x \in \tau_\gamma$-$Cl(f^{-1}(Int(F)))$. Then we have $Int(F)$ is preopen in Y and $Int(F) = Int(Cl(Int(F)))$. Hence by (5), $x \in f^{-1}(Cl(Int(F))) = f^{-1}(F)$.

(6) ⇒ (7). Let $O \in \beta O(Y)$ and $x \in \tau_\gamma$-$Cl(f^{-1}(Int(Cl(O))))$. Then $O \subseteq Cl(Int(Cl(O)))$. Since $Cl(O)$ is regular closed, by (6), $x \in f^{-1}(Cl(O))$.

(7) ⇒ (8). This is obvious since $SO(Y) \subseteq \beta O(Y)$.

(8) ⇒ (1). Let V be any open set of Y and $x \in \tau_\gamma$-$Cl(f^{-1}(V))$. Then, V is semi-open and $x \in \tau_\gamma$-$Cl(f^{-1}(Int(Cl(V))))$. By (8), $x \in f^{-1}(Cl(V))$. It follows from Theorem 3.8, that f is w.γ.c.. □

Theorem 3.10. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is w.γ.c..*
2. *$f(\tau_\gamma$-$Cl(A)) \subseteq Cl_\theta(f(A))$, for every subset A of X.*
3. *τ_γ-$Cl(f^{-1}(B)) \subseteq f^{-1}(Cl_\theta(B))$, for every subset B of Y.*

Proof. (1) ⇒ (2). Let A be any subset of X. Let $x \in \tau_\gamma$-$Cl(A)$ and V be any open set of Y containing $f(x)$. Since f is w.γ.c. there exists $U \in \tau_\gamma$ containing x such that $f(U) \subseteq Cl(V)$. Since $x \in \tau_\gamma$-$Cl(A)$, implies that $U \cap A \neq \phi$ and hence $\phi \neq f(U) \cap f(A) \subseteq Cl(V) \cap f(A)$. Therefore, we have $f(x) \in Cl_\theta(f(A))$ and hence $f(\tau_\gamma$-$Cl(A)) \subseteq Cl_\theta(f(A))$.

(2) ⇒ (3). Let B be any subset of Y and $x \in \tau_\gamma$-$Cl(f^{-1}(B))$. By (2), we have $f(x) \in Cl_\theta(f(f^{-1}(B))) \subseteq Cl_\theta(B)$ and hence $x \in f^{-1}(Cl_\theta(B))$.

(3) ⇒ (1). Let V be any open set of Y containing $f(x)$. Since $Cl(V) \cap (Y \setminus Cl(V)) = \phi$, we have $f(x) \notin Cl_\theta(Y \setminus Cl(V))$ and hence $x \notin f^{-1}(Cl_\theta(Y \setminus Cl(V)))$. By (3), $x \notin \tau_\gamma$-$Cl(f^{-1}(Y \setminus Cl(V)))$, then there exists $U \in \tau_\gamma$ containing x such that $U \cap f^{-1}(Y \setminus Cl(V)) = \phi$, hence $f(U) \cap (Y \setminus Cl(V)) = \phi$. This shows that $f(U) \subseteq Cl(V)$. Therefore, f is w.γ.c.. □

Proposition 3.11. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

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1. *f is w.γ.c. at $x \in X$.*

2. *$x \in \tau_\gamma$-$Int(f^{-1}(Cl(V)))$, for each open set V containing $f(x)$.*

Proof. (1) $\Rightarrow$ (2). Let V be any open set containing $f(x)$. There exists a γ-open set U containing x such that $f(U) \subseteq Cl(V)$. We have $U \subseteq f^{-1}(Cl(V))$. Since U is γ-open, then $x \in U = \tau_\gamma$-$Int(U) \subseteq \tau_\gamma$-$Int(f^{-1}(Cl(V)))$.

(2) $\Rightarrow$ (1). Let $x \in \tau_\gamma$-$Int(f^{-1}(Cl(V)))$ for each open set V containing $f(x)$. Take $U = \tau_\gamma$-$Int(f^{-1}(Cl(V)))$. Then $f(U) \subseteq Cl(V)$. Moreover, U is γ-open. Thus, f is w.γ.c. at $x \in X$. □

Proposition 3.12. *The following hold for a function $f : (X, \tau) \to (Y, \sigma)$:*

1. *If f is w.γ.c., then $f^{-1}(F)$ is γ-closed in X, for every θ-closed set F of Y.*

2. *If f is w.γ.c., then $f^{-1}(V)$ is γ-open in X, for every θ-open set V of Y.*

3. *If $f^{-1}(Cl_\theta(B))$ is γ-closed in X, for every subset B of Y, then f is w.γ.c..*

Proof. (1) and (2) follows from Theorem 3.10.

(3) Let $B \subseteq Y$. Since $f^{-1}(Cl_\theta(B))$ is γ-closed in X, then τ_γ-$Cl(f^{-1}(B)) \subseteq \tau_\gamma$-$Cl(f^{-1}(Cl_\theta(B))) = f^{-1}(Cl_\theta(B))$. By Theorem 3.10, f is w.γ.c.. □

Theorem 3.13. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is w.γ.c..*

2. *τ_γ-$Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$, for every $V \in PO(Y)$.*

3. *$f^{-1}(V) \subseteq \tau_\gamma$-$Int(f^{-1}(Cl(V)))$, for every $V \in PO(Y)$.*

Proof. (1) $\Rightarrow$ (2). Let V be any preopen set of Y such that $x \in \tau_\gamma$-$Cl(f^{-1}(V))$. Suppose that $x \notin f^{-1}(Cl(V))$. Then there exists an open set W containing $f(x)$ such that $W \cap V = \phi$. Hence we have $W \cap Cl(V) = \phi$ and hence $Cl(W) \cap Int(Cl(V)) = \phi$. Since V is preopen, $V \subseteq Int(Cl(V))$ and we have $V \cap Cl(W) = \phi$. Since f is w.γ.c. at $x \in X$ and W is an open set containing $f(x)$, there exists $U \in \tau_\gamma$ containing x such that $f(U) \subseteq Cl(W)$. Then $f(U) \cap V = \phi$ and hence $U \cap f^{-1}(V) = \phi$. This shows that $x \notin \tau_\gamma$-$Cl(f^{-1}(V))$. This is a contradiction. Therefore, we have $x \in f^{-1}(Cl(V))$.

(2) $\Rightarrow$ (3). Let $V \in PO(Y)$ and $x \in f^{-1}(V)$. Then, we have

$$f^{-1}(V) \subseteq f^{-1}(Int(Cl(V))) = X \setminus f^{-1}(Cl(Y \setminus Cl(V))).$$

Therefore, $x \notin f^{-1}(Cl(Y \setminus Cl(V)))$. Then by (2), $x \notin \tau_\gamma$-$Cl(f^{-1}(Y \setminus Cl(V)))$. Hence $x \in X \setminus \tau_\gamma$-$Cl(f^{-1}(Y \setminus Cl(V))) = \tau_\gamma$-$Int(f^{-1}(Cl(V)))$.

(3) $\Rightarrow$ (1). This follows from Theorem 3.8, since every open set is preopen. □

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Proposition 3.14. *If $f : X \to Y$ is w.γ.c., then for each $x \in X$ and each subset B of Y with $f(x) \subseteq Int_\theta(B)$, there exists a γ-open set U containing x such that $U \subseteq f^{-1}(B)$.*

Proof. Since $f(x) \subseteq Int_\theta(B)$, there exists a non-empty open set V of Y such that $V \subseteq Cl(V) \subseteq B$ and $f(x) \subseteq V$. Since f is w.γ.c., there exists a γ-open set U containing x such that $f(U) \subseteq Cl(V)$ and hence $U \subseteq f^{-1}(B)$. □

Proposition 3.15. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties hold:*

1. $D_{w\gamma c}(f) = \bigcup_{O\in\sigma}\{f^{-1}(O) \setminus \tau_\gamma\text{-}Int(f^{-1}(Cl(O)))\}$.
2. $D_{w\gamma c}(f) = \bigcup_{H\in F}\{\tau_\gamma\text{-}Cl(f^{-1}(Int(H))) \setminus f^{-1}(H)\}$.
3. $D_{w\gamma c}(f) = \bigcup_{B\in P(Y)}\{\tau_\gamma\text{-}Cl(f^{-1}(Int(Cl(B)))) \setminus f^{-1}(Cl(B))\}$.
4. $D_{w\gamma c}(f) = \bigcup_{B\in P(Y)}\{f^{-1}(Int(B)) \setminus \tau_\gamma\text{-}Int(f^{-1}(Cl(Int(B))))\}$.
5. $D_{w\gamma c}(f) = \bigcup_{O\in\sigma}\{\tau_\gamma\text{-}Cl(f^{-1}(O)) \setminus f^{-1}(Cl(O))\}$.

   Where $D_{w\gamma c}(f) = \{x \in X$: f is not w.γ.c. at $x\}$ and F is the family of closed sets of (Y, σ).

Proof. We shall show only the first equality since the proofs of the other are similar to the first. Let $x \in D_{w\gamma c}(f)$. By Theorem 3.8, there exists an open set V of Y such that that $x \in f^{-1}(V)$ and $x \notin \tau_\gamma\text{-}Int(f^{-1}(Cl(V)))$. Therefore, we have $x \in f^{-1}(V) \setminus \tau_\gamma\text{-}Int(f^{-1}(Cl(V))) \subseteq \bigcup_{O\in\sigma}\{f^{-1}(O) \setminus \tau_\gamma\text{-}Int(f^{-1}(Cl(O)))\}$.

Conversely, let $x \in \bigcup_{O\in\sigma}\{f^{-1}(O) \setminus \tau_\gamma\text{-}Int(f^{-1}(Cl(O)))\}$. Then, there exists $V \in \sigma$ such that $x \in f^{-1}(V) \setminus \tau_\gamma\text{-}Int(f^{-1}(Cl(V)))$. By Theorem 3.8, we obtain $x \in D_{w\gamma c}(f)$. □

Proposition 3.16. *If $f : X \to Y$ is w.γ.c. and $g : Y \to Z$ is continuous, then the composition $gof : X \to Z$ is w.γ.c..*

Proof. Let $x \in X$ and A be an open set of Z containing $g(f(x))$. Since g is continuous, then $g^{-1}(A)$ is an open set of Y containing $f(x)$. But f is w.γ.c., then there exists a γ-open set B of X containing x such that $f(B) \subseteq Cl(g^{-1}(A))$. Also, since g is continuous, then we obtain $(gof)(B) \subseteq g(Cl(g^{-1}(A))) \subseteq Cl(A)$. Therefore, gof is w.γ.c.. □

Corollary 3.17. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then the composition function $gof : X \to Z$ is w.γ.c. if f and g satisfy one of the following conditions.*

1. *f is γ-continuous and g is continuous.*

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2. *f is γ-continuous and g is γ'-continuous.*

3. *f is w.γ.c. and g is γ'-continuous.*

Where γ and γ' are operations on τ and σ, respectively.

Proof. Obvious. □

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