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A NOTE ON π-REGULAR AND πS-UNITALITY OVER NOETHERIAN REGULAR δ-NEAR RINGS (π-R&πS-U-NR-δ-NR)

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Abstract: In this paper, we begin with to show that the characterization of π-regularity and πS-Unitality over Noetherian regular δ-near rings, also consider their application in near rings as well.

Next we introduce more general concepts of π-Regularity and πS-Unitality characterization over Noetherian Regular δ-near rings and then given some examples in near-rings also investigated their properties and characterization.

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1. Introduction

The concept of Von-Nuemann regularity of Noetherian Regular δ-Near Rings of Near Rings studied by many authors Biedleman,Choudari, Goel, Heathorly,Hongan, Ligh, Maron and Murthy. Their main results are suggested in the book G Pilz [12].

The Von-Neumann regularity of rings and its generalization were studied by Fischer, Snider, Hirano Tominga, Savage, Ligh, Schein and Ohori. In 1985, Ohori investigated the characterization of π-Regularity and strong π-regularity and π-Unitality of Rings.

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A Near Ring N is an algebraic system (N,+,.) with two binary operations '+' and '.' Such that (N,+) is a group where N is not necessarily abelian with neutral element say 0, (N,.) is a semi group and (a + b) c = ac + bc for all a, b, c $\in$ N.

If N has a unity 1, then N is called Unitary.

A Near Ring N is with the extra N0 = 0 = 0N and $\forall$a $\in$ N, a0 = 0 = 0a is said to be zero symmetric.

An element d $\in$ N is distributive if d (a + b) = da + db, $\forall$a, b $\in$ N.

We will use the notations as follows:

Given a Near Ring N, N_0 = {a $\in$ N / a0 = 0 }which is called the zero symmetric part of N and N_c = {a $\in$ N / a0 = a }which is called the constant part of N. The set of all distributive elements in N is denoted by N_d.

Obviously, we see that N_0 and N_c are sub Noetherian δ-delta near rings of N, but N_d is a semi group under multiplication of N.

Clearly, Noetherian Regular δ-near ring N is zero symmetric Near Ring in case N = N_0, in case of N=N_c, N is called a constant Near Ring and in case N = N_d then N is called distributive Noetherian Regular δ-near ring. For all basic results we shall refer to G Pilz [12] some preliminaries along with examples provided herewith in section 2.

2. Preliminaries

Definition 2.1. A Near – Ring is a set N together with two binary operations "+" and "." Such that

(i) (N, +) is a Group not necessarily abelian, (ii) (N,.) is a semi Group and (iii) for all n_1, n_2, n_3 $\in$N, (n_1+ n_2). n_3 = (n_1 . n_3 + n_2 . n_3) i.e. right distributive law.

Examples 2.2 Let M_{2x2} ={{(aij) / Z ; Z is treated as a near-ring}. M_{2x2} under the operation of matrix addition '+' and matrix multiplication '.'.

Example 2.3. Z be the set of positive and negative integers with 0. (Z, +) is a group. Define '.' on Z by a. b = a for all a, b $\in$ Z. Clearly (Z, +,.) is a near-ring.

Example 2.4. Let Z_{12} = {0, 1, 2, ..., 11}. (Z_{12}, +) is a group under '+' modulo 12.Define '.' on Z_{12} by a. b = a for all a $\in$ Z_{12}. Clearly (Z_{12}, +,.) is a near-ring.

Definition 2.5. A near-ring N is Regular Near-Ring if each element a N then there exists an element x in N such that a = axa.

Definition 2.6. A Commutative ring N with identity is a Noetherian Regular δ-Near Ring if it is Semi Prime in which every non-unit is a zero divisor and the Zero ideal is Product of a finite

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number of principle ideals generated by semi prime elements and N is left simple which has $N_0 = N$, $N_e = N$.

Definition 2.7. A near-ring N is called a δ-Near − Ring if it is left simple and N_0 is the smallest non-zero ideal of N and a δ-Near − Ring is a non-constant near ring.

Definition 2.8. A δ-Near-Ring N is isomorphic to δ-Near-Ring and is called a Regular δ-Near-Ring if every δ-Near-Ring N can be expressed as sub-direct product of near-rings $\{N_i\}$, N_i is a non-constant near-ring or a δ-Near-Ring N is sub-directly irreducible δ-Near-Rings N_i.

Definition 2.9. Let N be a Commutative Ring. Let N be a Noetherian Regular δ-Near-Ring if each $P \in A\ (N_N)$ is strongly prime i.e., P is a δ-Near − Ring of N.

Example 2.10. Let $N = \begin{bmatrix} F & F \\ 0 & F \end{bmatrix}$ where F is a field. Then $P(N) = \begin{bmatrix} 0 & F \\ 0 & 0 \end{bmatrix}$
Let, $\sigma : N \to N$ be defined by, $\sigma((a\ b = a\ 0\ 0\ c))\ 0\ c$

It can be seen that a σ endomorphism of N and N is a $\sigma(*)$-Ring or Noetherian Regular δ-Near– Ring.

Definition 2.11. Let $(N, +, \bullet)$ be a near-ring. A subset L of N is called a ideal of N provided that 1. $(N, +)$ is a normal subgroup of $(N, +)$, and 2. $m.(n + i)$ — $m.n \in L$ for all $i \in L$ and $m,n \in N$

3. General Concepts on π-Regularity Characterization Over Noetherian Regular δ-Near Rings of Near Rings

In this section we discussed some definitions, properties and general concepts on π-Regularity characterization over Noetherian Regular δ-Near Rings of Near Rings.

Definition 3.1. A Noetherian regular δ-Near– Ring N, an element $a \in N$ is called nilpotent if $\exists$ a positive integer $n \ni a^n = 0$.

Definition 3.2. A subset $S \subset N$ is called Nilpotent if $\exists$ a positive integer $n \ni S^n = 0$ and $S \subset N$ is called Nil if every element in S is Nilpotent which is introduced by G Pilz Near Rings, North Holans P.C., New Yark in 1983.

Note 3.3. Every Nilpotent subset of N is NIL.

Definition 3.4. A (two sided) N – subgroup of N is a subset K of N such that (i) (K, +) is a subgroup of (N, +), (ii) $NK \subset K$ and (iii) $KN \subset K$. If K satisfies the properties (i),(ii) then it is called a right N – subgroup of N.

Definition 3.5. A (two sided) N – subgroup of N is a subset M of N such that (i) (M, +) is a subgroup of (N, +), (ii) $NM \subset M$ and (iii) $MN \subset M$. If M satisfies the properties (i),(ii) then it is called a left N – subgroup of N, normal left N- Subgroup of N and right N- subgroup of N respectively.

Note 3.6. Hence Normal right N – subgroup of N are the same of right ideals of Noetherian regular δ-Near Ring N.

Definition 3.7. A subset L of a Noetherian regular δ-Near Ring N together with (i) NL $\subset$ L and (ii) LN $\subset$ L is called N – subset of N.

If this L satisfies (i) then it is called a left N – subset of Noetherian regular δ-Near Ring N (NR-δ-NR) and If this L satisfies (ii) then it is called a right N – subset of Noetherian regular δ-Near Ring N (NR-δ-NR).

Note 3.8. A Noetherian regular δ-Near Ring N is said to be 'Reduced' if N has no non-zero nilpotent elements i.e., $\forall$ a $\in$ N, $a^n = 0$ for some positive integer n $\Rightarrow$ a =0 Mc Coy already given proof for N is Reduced $\Leftrightarrow$ $\forall$ a $\in$ N, $a^2 = 0 \Rightarrow$ a =0.

Definition 3.9. A Noetherian δ-Near Ring is 'regular' (Von – Nuemann) if $\forall$ a $\in$ N, $\exists$ an element x $\in$ N such that a = axa, such an element 'a' is called regular.

Proposition 3.10. *Let N be a Noetherian Regular δ-Near Ring of a Near Ring. Then N is regular $\Leftrightarrow$ N has the condition "$\forall$ a $\in$ N, $\exists$ $e^2 = e \in$ N such that N a = N e^n and N is left S-Unital".*

Proof. Given N be a Noetherian Regular δ-Near Ring of a Near Ring.

$\Rightarrow$ To show that N is regular if and only if N has the condition "$\forall$ a $\in$ N, $\exists$ e^2 = e $\in$ N such that N a = N e^n and N is left S-Unital". For that, $\forall$ a $\in$ N, $\exists$ y $\in$ N such that a = aya. Since, ya and ay are idempotent elements in N, taking ya = e, Na = Naya = Nae $\subset$ Ne = Nya $\subset$ Na. Hence, Na = Ne. obviously, N is left S-Unital.

$\Leftarrow$ Let us assume that N has the given condition "$\forall$ a $\in$ N, $\exists$ e^2 = e $\in$ N such that N a = N e and N is left S- Unital".

Then S-Unitality $\Rightarrow$ that a $\in$ Na = Ne. so that $\exists$ y $\in$ N such that a = ye.

$\Rightarrow$ e = ee $\in$ Ne = Na, so that $\exists$ y $\in$ N such that e = ya.

Thus we obtain a = xe = xee = xeya = aya $\Rightarrow$ N is regular.

Hence the Proposition.

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Proposition 3.11. *Every Noetherian regular δ-Near Ring N has no non-zero nil left N- subset.*

Proof. Let N be a Noetherian regular δ-Near Ring and M be a nil left N- subset of N.

It is sufficient to show that M = {0 }.

Indeed, let a $\in$ M since N is Noetherian regular δ-Near Ring(NR-δ-NR),

N has the condition that there exists e^2 = e $\in$ N such that Na = Ne and N is left S-Unital.

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By known Proposition, since M is a left N – subset implies $a \in Na \in M$.

On the other hand, since M is Nil, there exists a positive integer n such that $a^n = 0$.

Given condition $e = ee \in Ne = Na \subset M$.

Also, $\exists$ a positive integer m such that $e = e^m = 0$.

By the two conditions, we have $a \in N0$, so that $a = r.0 = (r.0)^n = a^n = 0$.

Therefore, $M = \{0\}$. Hence the Proposition.

Corollary 3.12. *Every Noetherian regular δ-Near Ring N with identity has no non-zero nil left N-subgroup.*

Definition 3.13. A Noetherian regular δ-Near Ring N is said to be π-Regular if $\forall$ $a \in N$, $\exists$ a positive integer n such that a^n is a regular element i.e., $a^n = a^n x a^n$ for some $x \in N$. such an element 'a' is called π-Regular.

Note 3.14. Every Regular near ring and Noetherian regular δ-Near Ring is π-Regular but not every π-Regular is a Noetherian regular δ-Near Ring, Regular Near Ring.

Example 3.15. Let $N = \{0, a, b, c\}$be an additive klein 4 – group this is a Noetherian regular δ-Near Ring with the following multiplication table : -

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	c	b
c	0	a	b	c

Thus Noetherian regular δ-Near Ring N is zero symmetric Noetherian regular δ-Near Ring with identity say e. Moreover, N is a π-Regular but not regular.

Indeed, $0 = 0a0$, $a^2 = a^2\, b\, a^2$, $b^4 = b^4\, a\, b^4$ and $c^2 = c^2\, c\, c^2$ but a is not a regular element.

Example 3.16. Let $N = Z_4 = \{0, 1, 2, 3\}$be an additive group of integers mod. 4 and define multiplication as follows table : -

.	0	1	2	3
0	0	0	0	0
1	0	3	0	1
2	0	2	0	2
3	0	1	0	3

Thus Noetherian regular δ-Near Ring N is zero symmetric Noetherian regular δ-Near Ring with identity say e. Moreover, N is a π-Regular but not regular.

Indeed, $0 = 0a0$, $a^2 = a^2\, b\, a^2$, $b^4 = b^4\, a\, b^4$ and $c^2 = c^2\, c\, c^2$ but a is not a regular element.

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Proposition 3.17. *Let N be a Near Ring. Then N is regular if and only if N has the condition "$\forall$ a $\in$ N, $\exists$ $e^2 = e \in N$ such that Na = Na^n and N is left S-Unital.*

Proof. Refer Proposition 1. [21].

Proposition 3.18. *Every regular near ring N has no non-zero nil left N- Subset.*

Proof. Refer Proposition 3. [21].

4. General Concepts on πS-Unitality Characterization over Noetherian Regular δ-Near Rings of Near Rings

In this section we discussed some definitions, properties and general concepts on πS-Unitality characterization over Noetherian Regular δ-Near Rings of Near Rings.

Definition 4.1. A Noetherian regular δ-Near Ring N is said to be left S-Unital (respectively, right S-Unital) if $\forall$ a $\in$ N, a $\in$ Na (respectively, a $\in$ aN) such element 'a' is called left S-Unital(respectively, right S-Unital).

Definition 4.2. A Noetherian regular δ-Near Ring N is called S-Unital, if N is both left S-Unital and right S-Unital.

Note 4.3. Every Noetherian Regular δ-Near Ring N with left identity or identity is clearly left S-Unital.

Note 4.4. Every Noetherian Regular Near Ring N is S-Unital we studied and begin with to show that the characterization of regularity and S-Unitality in Noetherian Regular δ-Near Rings N of Near Ring.

Definition 4.5. A Near ring N is called left πS-Unital (respectively right πS-Unital) if for each element a $\in$ N, $\exists$ a positive integer n such that a^n is a S – Unital element, i.e., $a^n \in N\, a^n$ (respectively $a^n \in a^n\, N$) such an element 'a' is called left πS-Unital (respectively right πS-Unital).

N is called πS-Unital, if N is both left πS-Unital and right πS-Unital. Also, every left πS-Unital

(respectively right πS-Unital) Noetherian Regular δ-Near Ringis left πS-Unital (respectively right πS-Unital), but not conversely we can give counter example for this as below :

In above example 3.15 clearly, N is a left πS-Unital Noetherian Regular δ-Near Ring. But in example 3.15, N is left πS-Unital, indeed $0 = 1.0 = 2.0 = 3.0 \in N.0$, and $1 = 3.1 \in N.1$, $2^2 = 0 = 0.2^2 \in N\, 2^2$ and $3 = 3.3 \in N3$. But this Noetherian Regular δ-Near Ring N is not S- Unital, because 2 is not a left S- Unital element.

These are some Properties, concepts on π-Regularity and πS-Unitality characterization Over Noetherian Regular δ-Near Rings of Near Rings.

Theorem 4.6. *Let N be a Near Ring. Then N is a π – Regular if and only if N has the condition "$\forall$ a $\in$ N, $\exists$ $e^2 = e \in N$ and there exists n $\in$ $\mathbf{Z}^+$ such that $Na^n = Ne^n$" and N is left πS- unital.*

Proof. Refer Theorem 7. [21].

5. Main Results on π-Regularity and πS-Unitality Characterization over Noetherian Regular δ-Near Rings of Near Ring

We derived some important results on π-Regularity and πS-Unitality characterization over Noetherian Regular δ-Near Rings of Near Ring.

Theorem 5.1. *Let N be a Noetherian Regular δ-Near Ring. Then N is π-Regular if and only if N has the condition that "$\forall$ a $\in$ N, $\exists$ $e^2 = e \in N$ such that $\exists$ n such that n $\in Z^+$ such that $N a^n = N e^n$ " and N is left πS-unital.*

Proof. Suppose that N is π-Regular. Then for any a $\in$ N, there exists a positive integer n and x$\in$ N such that $a^n \in N a^n$. Hence N is left πS-Unital.

Next, since xa^n and a^nx are idempotent elements in N, putting $xa^n = e$, $N a^n = Ne^n xa^n \subset N a^n$.

Hence $N a^n = Ne$.

Conversely, assume that N has the given condition "$\forall$ a $\in$ N, $\exists$ $e^2 = e \in N$ such that $\exists$ n such that n $\in Z^+$ such that $N a^n = N e$", and N is left πS-Unital. Then the πS-Unitality $\Rightarrow a^n \in N a^n = N e$, so that there exists y $\in$ N such that $a^n = ye$ .... (Equation 1).

On the otherhand, we see that $e = ee \in Ne = Na^n$, so that there exists x $\in$ N such that $e = xa^n$............(Equation 2). From these two conditions of Equn 1 and Equn 2, we obtain that $a^n = a^n x a^n$.

Therefore, N is a πS-regular Noetherian Regular δ-Near Ring.

Hence the theorem.

Example 5.2. N = {0, a }is a non-zero N- subgroup which is Nil. Then for any Noetherian regular δ-Near Ring N, the center of N is denoted by the set Z(N) = {x $\in$ N / ax = xa, $\forall$ a$\in$ N}.

When N is distributive, i.e, $N = N_d$, Z(N) is a sub near ring of Noetherian Regular δ-Near Ring N. so that is a distributive π-Regular Near rings but which are not additive abelian.

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Theorem 5.3. *The center of a distributive π-Regular Noetherian regular δ-Near Ring N is also π-regular.*

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Proof. Let N be a distributive π-Regular Noetherian regular δ-Near Ring and let $a \in Z(N)$.

Then $\exists$ y $\in$ N and $\exists$ n $\in Z^+ \ni x^n a\, x^n \in Z(N)$. then our claim is done. Indeed, let q $\in$ N. since, a

$\in Z(N)$. Thus we can deduce that $q\,(a^n x) = (qa^n)x = a^n(qx) = a^n x a^n(qx) = a^n x(qx)a^n = a^n(xqx)a^n$

and $(a^n x)\ q = (xa^n)q = x(a^n q) = x(qa^n) = xq(a^n xa^n\ ) = (xq\ a^n\ x)\ xa^n = a^n(xqx)a^n$.

Hence, $a^n\ x \in Z\ (N)$. similarly we can obtain that $xa^n \in Z\ (N)$,

thus $q(xa^n x) = q(a^n xx) = (qa^n x)x = (a^n xq)x = x(a^n q)x$ &

$(xa^n x)\ q = x(a^n x)q = xq(a^n x) = x(qa^n)x = x(a^n q)x$

$\Rightarrow$ that $q(xa^n x) = (xa^n x)q$ i.e., $xa^n x \in Z\ (N)$.

Hence Z (N) is a π-Regular.

Hence the theorem.

References

[1] G. Pilz, *Near Rings*, North Holland Publishing Company, Amesterdam, New York (1983).

[2] J.C. Bieldleman, A note on regular near rings, *J. Indian Math. Soc.*, **33** (1969), 207-210.

[3] S.C. Choudari, A.K. Goyal, Generalized regular near rings, *Stud. Sci. Math. Hungar*, **14** (1982), 69-76.

[4] S. Leigh, On regular near rings, *J. Indian Math.*, **15** (1970), 7-13.

[5] M. Ohori, On strongly π-regular rings and periodic rings, *Math. J. Okyama. Univ.*, **27** (1998), 149-153.

[6] N.V. Nagendram, Ch. Padma, T.V. Pradeep Kumar, Y.V. Reddy, Ideal comparability over noetherian regular delta near rings (IC-NR-Delta-NR), International Journal of Advances in Algebra (IJAA), Accepted.

[7] N.V. Nagendram, S. Venu Madava Sarma, T.V. Pradeep Kumar, A note on sufficient condition of Hamiltonian path to complete graphs (SC-HPCG), *IJMA*, **2**, No. 11 (2011), 1-6.

[8] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, On Noetherian regular delta near rings and their extensions (NR-delta-NR), Bulgaria, **6**, No. 6 (2011), 255-262.

[9] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, On semi Noehterian regular matrix delta near rings and their extensions (SNRM-delta-NR), *Int. Journal of Advances in Algebra*, Jordan, Research India Publications, Rohini, New Delhi, **4**, No. 1 (2011), 51-55.

[10] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, On Boolean Noetherian regular delta near ring (BNR-delta-NR)s, *International Journal of Contemporary Mathematics*, **2**, No. 1-2 (2011), 23-27.

[11] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, On bounded matrix over a Noetherian regular delta near rings (BMNR-delta-NR), *Int. J. of Contemporary Mathematics*, **2**, No. 1-2 (2011), 11-16.

[12] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, On strongly semi prime over Noetherian regular delta near rings and their extensions (SSPNR-delta-NR), *Int. J. of Contemporary Mathematics*, **2**, No. 1 (2011), 69-74.

[13] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, On IFP ideals on Noetherian regular delta near rings (IFPINR-delta-NR), *Int. J. of Contemporary Mathematics*, **2**, No. 1-2 (2011), 43-46.

[14] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, On structure thoery and planar of Noetherian regular delta-near–rings (STPLNR-delta-NR), *International Journal of Contemporary Mathematics*, (2011), 79-83.

[15] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, On Matrix's maps over planar of Noetherian regular delta-near–rings (MMPLNR-delta-NR), *International Journal of Contemporary Mathematics*, **18** (2011), 203-211.

[16] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, Some fundamental results on P-regular delta-near–rings and their extensions (PNR-delta-NR), *International Journal of Contemporary Mathematics*, **2**, No.1-2 (2011), 81-85.

[17] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, Al Generalized ideal based-zero divisor graphs of Noetherian regular delta-near rings (GIBDNR-d-NR), *International Journal of Theoretical Mathematics and Applications*, **1**, No. 1 (2011), 59-71.

[18] N.V. Nagendram, T.V. Pradeep Kumar, Y.V. Reddy, Inverse localization of Noetherian regular delta-near rings (ICNR- Delta-NR), *International Journal of Pure And Applied Mathematics* (2011).

[19] N.V. Nagendram, S.V.M. Sarma, T.V. Pradeep Kumar, A note on relations between Barnette and sparse graphs, *International Journal of Mathematical Archive*, **2**, No. 12 (2011), 2538-2542.

[20] Y.V. Reddy, C.V.L.N. Murthy, On strongly regular near-rings, *Proc. Edinburgh Math.Soc.*, **27** (1984), 62-64.

[21] Yong Uk Cho, Some results on π-regularity and πS-unitality, *Korean J. Math.*, **17**, No. 4 (2009), 341-347.