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APPLICATIONS OF LINEAR PROGRAMMING ON OPTIMIZATION OF COOL FREEZERS (ALP-on-OCF)

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Abstract: In this paper, we studied the applications of Linear Programming Problems on Optimization of a Cool Freezers as well as by using existence basic concepts and obtained some fundamental results after applying the LPP on these concepts of Optimization of Steam Boilers by linear programming methods.

As energy and equipment costs increase an efficient cool system becomes more important in the over all economics of processing cool tower plant. Linear Programming Model for the Optimization of a cool freezer is presented in this paper by me as author N V Nagendram and co-author T Radha Rani, as guide Dr T V Pradeep Kumar and as advisor Dr Y V Reddy. We have developed mathematical model for analyzing the performance of total cost of a cool freezer. The Optimal solution in the model is the size of cool exchangers of a cool freezer which is characterized by the lowest total cost, which consists of investment cost and cost for ten years.

Investigation shows that total cost might minimally / maximally same upto 30%.

Key Words: cool freezers, optimization, linear programming problem, basic solution, feasible solution, optimal basic feasible solution, cost benefit analysis

1. Introduction

Technology process in industry need cool. Cool is needed for space cooling and different technological processes. When cool freezer produces cool there is a need

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for its optimization. In this paper, a linear programming LP model is developed for that purpose.

There are many methods for the optimization of energy system performance linear programming is one of the several mathematical techniques that attempt to solve problems by minimizing / maximizing a function of several independent variables. This method is widely used and is among the best ones for analyzing complex systems. Typical applications of linear programming is for the optimization of cool and exchanger networks [1], industrial buildings [2], energy systems [3,4], large electric utility etc.,

We have studied a cool freezer which converts the feed water to super cooled at pressure 36 bar and temperature 0 ^{0}C i.e. -4^{0}C A ccol freezer comprises the following major elements Furnace(F), First (FSC) and second (SSC) super cooler, connective surface(CS), drum(D), economizer(ECO) and air cooler(AC), steam boiler(SB) water enters at ECO where it is cooled at temperature below or above the cooling point about – 4^{0}C from 0^0 C cooled water is sent to F and CS where water turn into ice. The cool water mixture rises into a cool fridge drum where water separates from ice and generation of dry saturated cool at at 39 bar is ensured. Super cooler is made up of two parts.

In FSC saturated cool is super cooled at -4^{0}C. Then cool flow through cooler (C) when it mixes with feed water which reduces its temperature to -4^{0}C. The cool is sent to SSC and from that point super cooled i.e., at 30 bar and – 4 ^{0}Cleaves the cool freezer.

For a particular cool exchanger we assumed that the product of its cooling size and co-efficient of thermal transmittance Z = FU is a constant and characteristic value of this cool exchanger depending on the type and cost of fuel mass flow rate at a cool fridge exit and costs we obtained characteristic value for minimum total cost of a cool freezer.

Im our case total cost consist of investment cost of cool exchanger and costs of fuel for period of ten years.

The cool exchanger network used to model the energy system of the cool freezer. The mathematical representation of the mass and energy below consists of a set of linear and non-linear equations. The linear equations of mathematical model of the system are as below :-

$$M_{\mathrm{w}} = M_{\mathrm{w1}} + M_{\mathrm{w2}}, \tag{1}$$

$$M_{\mathrm{A}} = L.B. \tag{2}$$

$$M_{\mathrm{cp}} = V.B., \tag{3}$$

$$M_{\mathrm{out}} = M_{\mathrm{w}} + M_1, \tag{4}$$

$$M_{\mathrm{w}}.h_4 + M_1.h_1 = M_{\mathrm{out}\ .}\,h_5. \tag{5}$$

Here $\mathrm{M_w}$ = the mass flow of water or cool, M1 is the mass flow of water through F, Mw2 is the mass flow of water through Cs, MA id the mass flow of air, L is the mass of air required for combustion, B is the consumption of fuel, Mcp is the mass of flow of combustion products, V is the mass of combustion products for 1 kg of fuel, Mout = (6+10(is the mass of flow of cool at a cool freezer exit, M1 is the mass flow of feed water, h4 is 3211 is the specific enthalpy of cool from FSH, H1 = 591.9 KJ/Kg is the specific enthalpy of feed water and H5 = 3113 Kg/s is the specific enthalpy of cool at exit of C.

Non-linear equations of mathematical model of the system are the following energy balances at:

Furnace F:

$$Mc_{\mathrm{p}}.c_{\mathrm{pcp}}\,(T_{\mathrm{c}}\ - T_{\mathrm{cpt}}) = Mw1.(h3 - h2) \tag{6}$$

$$\underline{Z}_{\mathrm{F}}\,(T_{\mathrm{c}} + T_{\mathrm{cpt}}\ - T_2 - T_3) = M_{\mathrm{w}}1.(h6 - h5).$$

The second super cooler SSC:

$$Mc_{\mathrm{p}}.c_{\mathrm{pcp}}\,(T_{\mathrm{cp1}}\ - T_{\mathrm{cp2}}) = M_{\mathrm{out}}.(h6 - h5) \tag{8}$$

$$\underline{Z}_{\mathrm{SSC}}\,(T_{\mathrm{cp1}} + T_{\mathrm{cp2}}\ - T5 - T6) = M_{\mathrm{out}}.(h6 - h5). \tag{9}$$

Considering these mass and balances we assumed the following: The cool exchangers of the cool freezers are counter flow type. The co-efficient of thermal transmittance is constant through out the exchanger, instead of mean logarithmic temperature difference we used the mean arithmetic temperature difference. The excess air co-efficient is constant value of $\lambda = 1.2$ and there are no cool losses through connecting piping and passages. To solve this problem we have used software "SYSTEM".

2. Preliminaries: General Concepts of Linear Programming Problem

Definition 2.1. (Linear Programming Problem (LPP)) The general L.P.P. can be described as follows : Given the set of conditions in the form of 'm' Linear inequalities or equations in n variables. We wish to find non negative values of these variables and optimize (Maximize or Minimize same linear function of the variables.

Mathematically a general L.P.P. as follows optimize the function

$$Z = C_1X_1\ + C_2X_2 + \cdots + C_{\mathrm{n}}X_{\mathrm{n}} \tag{1}$$

$$A_{\mathrm{i}1}x_1\ + a_{\mathrm{i}2}x_2\ + \cdots + a_{\mathrm{in}}x_{\mathrm{n}}(\geq, =, \leq)b_{\mathrm{i}} \tag{2}$$

$$I = 1, 2 \ldots, m, \quad b_{\mathrm{i}} \geq 0,$$

$$x_{\mathrm{j}} \geq 0 = j = 1, 2, \ldots, n. \tag{3}$$

Definition 2.2. (Feasible Solution) In a L.P.P. any solution which satisfies the non negative condition i.e., $x_{\mathrm{j}} \geq 0$, $j = 1, 2, \ldots, n$ is called feasible solution and is usually observed as Feasible Solution.

(Optimal Feasible Solution) Any Feasible Solution which optimizes the objective function is called an Optimal Feasible Solution.

Definition 2.3. (Spanning Set) A set of vectors a_{11},a_2 .... a_n from t^n is said to span or generate E^n if every vectgor in E^n can be written as a Linear Combination of a_1, a_2, ...a_n

(OR)

A Set of vectors α_1, α_2,........ α_n from V(F) is said to be span or generated by VCF. If every vector in V can be expressed a Linear combination of the vectors(α_1, α_2 ...... α_n).

Definition 2.4. (Basis) A set of vectors (α_1, α_2 ...... α_n) from the vector space VCF is said to be basis for VCF if:

(i) (α_1, α_2 ...... α_n) are Linear Independent

(ii) (α_1, α_2 ...... α_n) Spans VCF

(OR)

A basis for E^n which spans the entire space.

Definition 2.5. (Basic Solution) Give a set of m simultaneous Linear equations in n unknowns, $A_x = b$, m <n and m is rank of Matrix A. If all the 'n – m' variables that are not associated with the columns of the matrix B are set equals to zero then the solution to the resultant system of equations is called a Basis Solution

Definition 2.6. (Basic Feasible Solution (Bfs)) A basic solution is a BFS if none of the basic variables is negative.

Definition 2.7. Optimal Basic Feasible Solution:

A BFS to an L.P.P. is said to be an optimal or optimum if it optimizes (Maximize or Minimize) the objective function.

Definition 2.8. (Slack Variables) Suppose an inequality constraint contains the sign $\leq$ (with bi ≥ 0) for example the inequality is

$$a_{i1}x_1 + a_{i2}x_2 + \ldots\ldots + a_{in}x_n \leq b_i$$

then we introduce a new variable x* ≥ 0 and we convert the inequality into equality.

$$a_{i1}x_1 + a_{i2}x_2 + \ldots\ldots + a_{in}x_n + x* = b_i,$$

where x* ≥ 0 is called as slack variable.

Definition 2.9. (Surplus Variables) Suppose an inequality constraint contains the sign $\geq$ (with bi ≥ 0) for example the inequality is

$$a_{i1}x_1 + a_{i2}x_2 + \ldots\ldots + a_{in}x_n \geq b_i,$$

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then we introduce a new variable $x^1 \geq 0$ and we convert the inequality into equality.

$$a_{i1}x_1 + a_{i2}x_2 + \ldots\ldots + a_{in}x_n - x^1 = b_i,$$

here $x^1 \geq 0$ is called as Surplus variable.

Definition 2.10. (Fundamental Theorem of Linear Programming) Given a set of m simultaneous linear equation in n unknowns, $m \leq n$, $Ax = b$, with rank of $A = m$. If there is a Feasible solution $X \geq 0$ then there exist a Basic Feasible Solution.

Definition 2.11. (Unbounded Solution) Given any basic feasible solution to LPP I_f for this solution there is some column a_j not in the basis for which $Z_i - C_j < 0$ and $y_{ij} \leq$ (i = 1, 2, ...m) then there exist feasible solutions in which m + 1 variables can be different from zero, with the value of the objective function being arbitrary large. In such a case, the problem has an unbounded solution if the objective function is to be maximize.

Similarly if for some a_j, $Z_i - C_j > 0$ & $y_{ij} \leq 0$ (I = 1,2,......m) then there exist feasible solutions, in which m + 1 variables can be different from zero, with the value of the objective function being arbitrarily small. IN such a case, the problem has an unbounded solution if z is to be maximized.

Definition 2.12. (Optimality Conditions) Given a basic feasible solution XB $= B^{-1}b$ with $Z_0 = C_B X_B$ to Linear Programming problem
$Ax = b$, $x \geq 0$ Max $z = Cx$ $\exists$ $Z_j - C_j \geq 0$ for every column aj in A. Then Z_0 is the maximum value of Z subject to the constraints and the BFS is an optimal BFS.

Given a BFS $X_B = B^{-1}b$ with $Z_0 = C_B X_B$ to the LPP $A^x = b$, $x \geq 0$, Min Z $= Cx \rightarrow Z_j - C_j \leq 0$ for every column a_j in A Then Z_0 is the minimum value of Z subject to the constraints and the BFS is an optimal BFS.

Definition 2.13. (Artificial Variable) One type of variable introduced in a linear program model in order to find an initial basic feasible solution; an artificial variable is used for equality constraints and for greater-than or equal inequality constraints.

Definition 2.14. (Simplex Method) For simplicity, in this course we solve \by hand" only the case where the constraints are of the form _and the right-hand-sides are nonnegative. We will explain the steps of the simplex method while we progress through an example.

Example 2.1. Solve the linear program

$$\max x_1 + x_2,$$

$$2x_1 + x_2\_4,$$

$$x_1 + 2x_2\_3,$$

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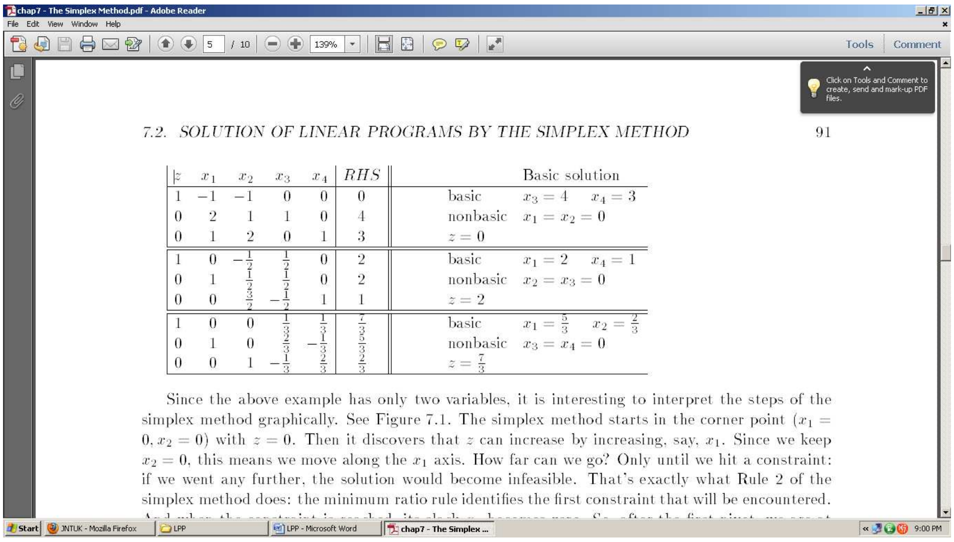

7.2. SOLUTION OF LINEAR PROGRAMS BY THE SIMPLEX METHOD 91

z	x_1	x_2	x_3	x_4	RHS	Basic solution		
1	−1	−1	0	0	0	basic	$x_3 = 4$	$x_4 = 3$
0	2	1	1	0	4	nonbasic	$x_1 = x_2 = 0$	
0	1	2	0	1	3	$z = 0$		
1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	basic	$x_1 = 2$	$x_4 = 1$
0	1	$\frac{1}{2}$	$\frac{1}{2}$	0	2	nonbasic	$x_2 = x_3 = 0$	
0	0	$\frac{3}{2}$	$-\frac{1}{2}$	1	1	$z = 2$		
1	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{7}{3}$	basic	$x_1 = \frac{5}{3}$	$x_2 = \frac{2}{3}$
0	1	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{5}{3}$	nonbasic	$x_3 = x_4 = 0$	
0	0	1	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$z = \frac{7}{3}$		

Since the above example has only two variables, it is interesting to interpret the steps of the simplex method graphically. See Figure 7.1. The simplex method starts in the corner point ($x_1 = 0, x_2 = 0$) with $z = 0$. Then it discovers that z can increase by increasing, say, x_1. Since we keep $x_2 = 0$, this means we move along the x_1 axis. How far can we go? Only until we hit a constraint: if we went any further, the solution would become infeasible. That's exactly what Rule 2 of the simplex method does: the minimum ratio rule identifies the first constraint that will be encountered.

$$x_1 \geq 0, \quad x_2 \geq 0.$$

First, we convert the problem into standard form by adding slack variables x3 ≥ 0 and x4 ≥ 0.

$$\max x_1 + x_2,$$

$$2x_1 + x_2 + x_3 = 4,$$

$$x_1 + 2x_2 + x_4 = 3,$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0.$$

3. Solution of Linear Programs by the Simplex Method

Since the above example has only two variables, it is interesting to interpret the steps of the simplex method graphically. if we went any further, the solution would become infeasible. That's exactly what Rule 2 of the simplex method does: the minimum ratio rule identifies the first constraint that will be encountered. And when the constraint is reached, its slack x3 becomes zero. So, after the first pivot, we are at the point ($x_1 = 2$, $x_2 = 0$). Rule 1 discovers that z can be increased by increasing x2 while keeping $x_3 = 0$. This means that we move along the boundary of the feasible region $2x_1 + x_2 = 4$ until we reach another constraint! After pivoting, we reach the optimal point $x_1 = \frac{5}{3}, x_2 = \frac{2}{3}$.

Definition 2.15. (Revised Simplex Method) Original simplex method calculates and stores *all* numbers in the tableau – many are not needed.

Revised Simplex Method (more efficient for computing)

Used in all commercially available packages. (e.g. IBM MPSX, CDC APEX III)
Max Z = $\underline{c\ x}$
subject to A x $\leq$ b
$\underline{x} \geq \underline{0}$
Initially constraints become (standard form):
$[\underline{A}\ \underline{I}] \left(\frac{x}{x_s}\right) = \binom{b}{}$
$\underline{x}_s$ = slack variables (Initially $\underline{B} = \underline{I}$)
Where B is Basis matrix: columns relating to basic variables.

3.1. At any Iteration Non-Basic Variables =0

$\underline{B\ x_B} = \underline{b}$.
Therefore $\underline{x}_B = \underline{B}^{-1}\underline{b}$ $B^{-1} \rightarrow$ inverse matrix.
$Z = \underline{c}_B \underline{x}_B$ where $\underline{c}_B$ = objective coefficients of basic variables.

Example 2.16.
Max $Z = 3X_1 + 5X_2$
s.t. $X_1 \leq 4$
$2X_2 \leq 12$
$3X_1 + 2X_2 \leq 18$
$X_1, X_2 \geq 0$
Standard form of constraints:
$X_1 + S_1 = 4$
$2X_2 + S_2 = 12$
$3X_1 + 2X_2 + S_3 = 18$
$X_1, X_2, S_1, S_2, S_3 \geq 0$

$$\underline{x}_B = \underline{B}^{-1}\underline{b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}$$

$\underline{c}_B = [0]$

$$Z = 0\ 0\ 0 \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix} = 0$$

Determine entering variable, X_j, with associated vector $\underline{P}_j$.

— compute $\underline{Y} = \underline{c}_B \underline{B}^{-1}$

— compute $z_j - c_j = \underline{Y\ P}_j - c_j$ for all non-basic variables.

Determine leaving variable, X_r, with associated vector $\underline{P}_r$.

— compute $\underline{x}_B = \underline{B}^{-1}\underline{b}$ (current R.H.S.)

— compute current constraint coefficients of entering variable:
$\alpha^j = \underline{B}^{-1}\underline{P}_j$

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X_r is associated with
$\theta = \text{Min}\ \{(\underline{x}_B)_k / \alpha_k^j, \alpha_k^j > 0\}$
Determine new $\underline{B}^{-1}$
Solution after one iteration:
$\underline{x}_B = \underline{B}^{-1}\underline{b}$
Compute $\underline{Y} = \underline{c}_B\underline{B}^{-1}$
- compute $\underline{Y} = \underline{c}_B\underline{B}^{-1}$
$\underline{Y} = 0\ 5\ 3\ \underline{B}^{-1} = 0\ 3/2\ 1$
- compute $z_j - c_j = \underline{Y}\ \underline{P}_j - c_j$ for all non-basic variables (S_2 and S_3):-

$$S_2:\ z_4 - c_4 = [0\ 3/2\ 1] \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - = 3/2$$

$$S_3:\ z_5 - c_5 = [0\ 3/2\ 1] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - 0 = 1$$

No negatives. Therefore stop.
Optimal solution:
$S_1{}^* = 2$
$X_2{}^* = 6$
$X_1{}^* = 2$

$$Z^* = \underline{c}_B\underline{x}_B = [0\ 5\ 3] \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix} = 36$$

Definition 2.17. (Multiple Optimal Solutions) The linear programming problems discussed in the previous section possessed unique solutions. This was because the optimal value occurred at one of the extreme points (corner points). But situations may arise, when the optimal solution obtained is not unique. This case may arise when the line representing the objective function is parallel to one of the lines bounding the feasible region. The presence of multiple solutions is illustrated through the following example.

$$\text{Maximize } z = \mathbf{x_1} + \mathbf{2x_2},$$

subject to

$$x_1 \leq 80,$$

$$x_2 \leq 60,$$

$$5x_1 + 6x_2 \leq 600,$$

$$x_1 + 2x_2 \leq 160,$$

$$x_1, x_2 \geq 0.$$

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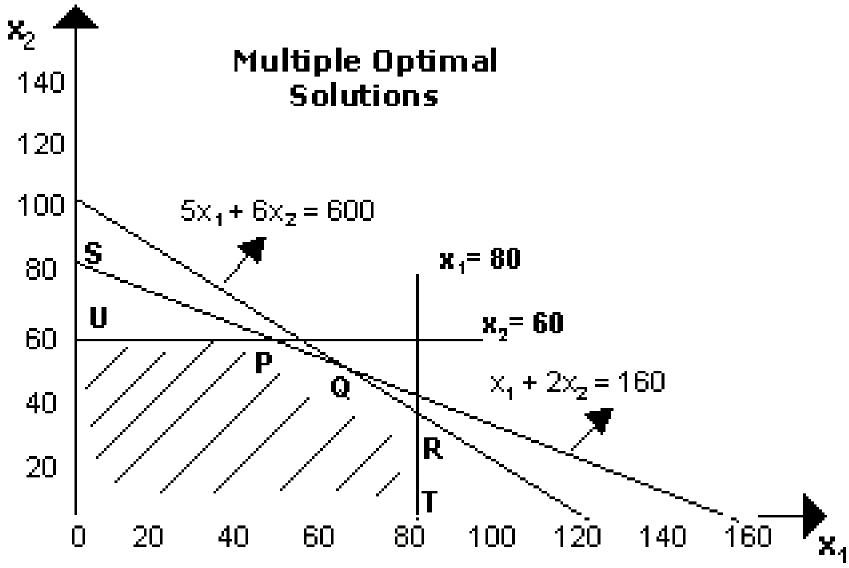

In the above figure, there is no unique outer most corner cut by the objective function line. All points from P to Q lying on line PQ represent optimal solutions and all these will give the same optimal value (maximum profit) of Rs. 160. This is indicated by the fact that both the points P with co-ordinates (40, 60) and Q with co-ordinates (60, 50) are on the line $x_1 + 2x_2 = 160$. Thus, every point on the line PQ maximizes the value of the objective function and the problem has multiple solutions.

Definition 2.18. (Infeasible Problem) In some cases, there is no feasible solution area, i.e., there are no points that satisfy all constraints of the problem. An infeasible LP problem with two decision variables can be identified through its graph. For example, let us consider the following linear programming problem.

$$\text{Minimize}\, z = \mathbf{200x_1} + \mathbf{300x_2}$$

subject to

$$2x_1 + 3x_2 \geq 1200,$$

$$x_1 + x_2 \leq 400,$$

$$2x_1 + 1.5x_2 \geq 900,$$

$$x_1, x_2 \geq 0.$$

The region located on the right of PQR includes all solutions, which satisfy the first and the third constraints. The region located on the left of ST includes all solutions, which satisfy the second constraint. Thus, the problem is infeasible because there is no set of points that satisfy all the three constraints.

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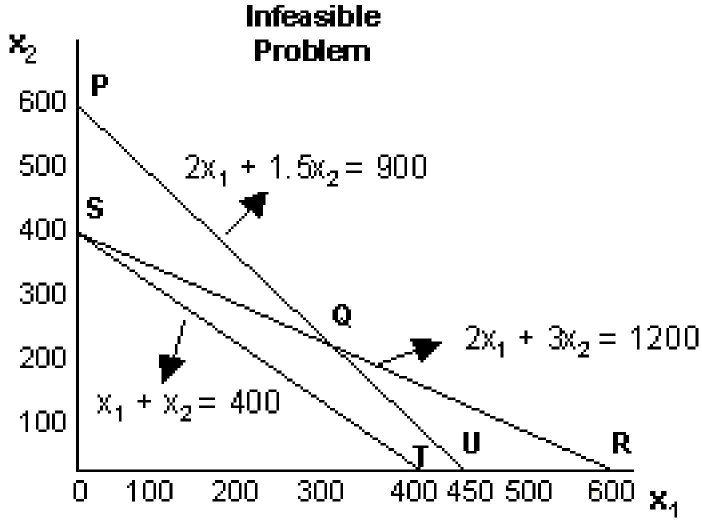

Definition 2.19. (Unbounded Solutions) It is a solution whose objective function is infinite. If the feasible region is unbounded then one or more decision variables will increase indefinitely without violating feasibility, and the value of the objective function can be made arbitrarily large. Consider the following model:

$$\text{Minimize}, z = \mathbf{40x_1} + \mathbf{60x_2},$$

subject to

$$2x_1 + x_2 \geq 70,$$

$$x_1 + x_2 \geq 40,$$

$$x_1 + 3x_2 \geq 90,$$

$$x_1, x_2 \geq 0.$$

The point (x_1, x_2) must be somewhere in the solution space as shown in the figure by shaded portion.

The three extreme points (corner points) in the finite plane are:
P = (90, 0); Q = (24, 22) and R = (0, 70). The values of the objective function at these extreme points are:
Z(P) = 3600, Z(Q) = 2280 and Z(R) = 4200.

In this case, no maximum of the objective function exists because the region has no boundary for increasing values of x_1 and x_2. Thus, it is not possible to maximize the objective function in this case and the solution is unbounded.

Optimization Methods: Linear Programming- Graphical Method 1

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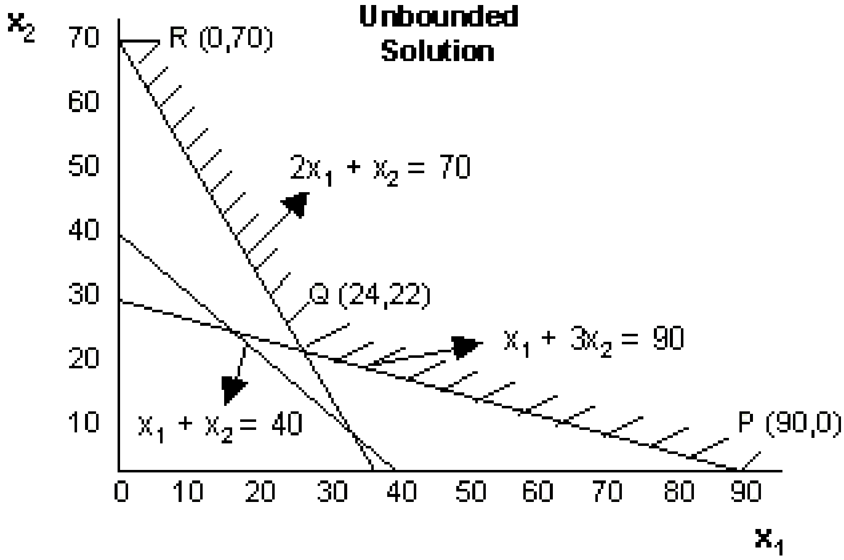

4. General Concepts on Cost-Benefit Analysis

Cost–benefit analysis (CBA), sometimes called **benefit–cost analysis** (BCA), is a systematic process for calculating and comparing benefits and costs of a project, decision or government policy (hereafter, "project"). CBA has two purposes:

1. To determine if it is a sound investment/decision (justification/feasibility),
2. To provide a basis for comparing projects. It involves comparing the total expected cost of each option against the total expected benefits, to see whether the benefits outweigh the costs, and by how much.

BA is related to, but distinct from cost-effectiveness analysis. In CBA, benefits and costs are expressed in money terms, and are adjusted for the time value of money, so that all flows of benefits and flows of project costs over time (which tend to occur at different points in time) are expressed on a common basis in terms of their "net present value."

The value of a cost–benefit analysis depends on the accuracy of the individual cost and benefit estimates. Comparative studies indicate that such estimates are often flawed, preventing improvements in Pareto and Kaldor-Hicks efficiency. Causes of these inaccuracies include:

1. Over reliance on data from past projects (often differing markedly in function or size and the skill levels of the team members)
2. Use of subjective impressions by assessment team members

3. Inappropriate use of heuristics to derive money cost of the intangible elements
4. Confirmation bias among project supporters (looking for reasons to proceed)

The reduction of total cooling size of cool freezer than the reduction of cooling size of condenser, convective surface and the first and second super cooler for the change of cost sets 1 to 3. These cause the growth of their relative portion at total cooling size of the cool freezer. However, the reduction of total cooling size of cool freezer is lower than the reduction of cooling size of economizer and air cooler because their relative portion decreases. The changes of cost set from 1 to 3 cause the decrease of the total cost of the cool freezer. In relation to the first set the total cost at the second set is lower for 13% and at the third set for 22%.

5. Main Results

The linear programming model ran for two scenarios: A and B.

The first scenario A was carried out for three types of coal lignite dark coal and stone coal coal cost Cj = 0.06, investment costs in CL = 27500, C_{SPP} = 22500, Cppp = 20000, Ckp = 21000, Ceko = 15000, C_{ZW} = 12500.

The optimal total cost characteristic value of the cool freezers for different mass flow rate at the cool freezer exit and different type of coal. A greater mass of flow at the cool freezer exit causes a higher total characteristic value of the cool freezer $Z = \Sigma$ Zk which means a greater cooling size of the steam freezer. The lowest value exist when the cool freezer‘s fuel is lignite, and the highest value exists when the fuel is stone coal. Reductions of total cooling size of the cool freezer, in relation to the case when coal is lignite, are up to 18% for dark coal and 23% for the stone coal. Total costs are reduced up to 25% for dark coal and 30% for stone coal.

The second scenario B was carried out when $M_{out} = 8$ and fuel is dark coal. The first set is the same as in the previous scenario A. In the second and third set investment cost are higher per 50% while the cost of coal is reduced per 20% in relation to the previous scenario.

The point (x_1, x_2) must be somewhere in the solution space as shown in the figure by shaded portion.

The three extreme points (corner points) in the finite plane are:
P = (90, 0); Q = (24, 22) and R = (0, 70)
The values of the objective function at these extreme points are:
Z(P) = 3600, Z(Q) = 2280 and Z(R) = 4200

In this case, no maximum of the objective function exists because the region has no boundary for increasing values of x_1 and x_2. Thus, it is not possible to maximize the objective function in this case and the solution is unbounded.

Optimization Methods: Linear Programming- Graphical Method 1

There are two standard methods for handling artificial variables within the simplex method:

- The Big M method
- The 2-Phase Method

Although they seem to be different, they are essentially identical. However, methodologically the 2-Phase method is much superior. We shall therefore focus on it.

The 2-Phase method is based on the following simple observation: Suppose that you have a linear programming problem in canonical form and you wish to generate a feasible solution (not necessarily optimal) such that a given variable, say x_3, is equal to zero. Then, all you have to do is solve the linear programming problem obtained from the original problem by replacing the original objective function by x_3 and setting opt=min.

If more than one variable is required to be equal to zero, then replace the original objective function by the sum of all the variables you want to set to zero.

Observe that because of the non-negativity constraint, the sum of any collection of variables cannot be negative. Hence the smallest possible feasible value of such a sum is zero. If the smallest feasible sum is strictly positive, then the implication is that it is impossible to set all the designated variables to zero.

Applying this simple idea to artificial variables we obtain the following recipe:

To set all the artificial variables to zero, solve a linear programming problem derived from the canonical form of the original problem by replacing the original objective function by the sum of all the artificial valriables and setting opt=min. If the optimal value of the modified objective function is not equal to zero, then the problem (system of constraints) is not feasible.

Example. The modeling aspects of this approach let us re-examine the following little example:

$$x_1 + 3x_2 + 7x_3 >= 25,$$

$$-3x_1 - 2x_2 + 7x_3 = -5,$$

$$2x_1 + x_2 + 4x_3 <= 10,$$

$$x_2, x_3 >= 0.$$

Taking case of its violations of the standard form we obtain the following canonical form:

There are two artificial variables, namely $\underline{x}_6$ and $\underline{x}_7$. Thus, Phase 1 of the 2-Phase method involves the following linear programming problem:

x_1	-	x_2	+	$3x_3$	+	$7x_4$	-	x_5	+	$\underline{x}_6$					=	25
$3x_1$	-	$3x_2$	+	$2x_3$	-	$7x_4$	+					$\underline{x}_7$			=	5
$2x_1$	-	$2x_2$	+	x_3	+	$4x_4$	+							x_8	=	10
$x_j >= 0$, j=1,...,8																

$w^* := \min w := \underline{x}_6 + \underline{x}_7$																
x_1	-	x_2	+	$3x_3$	+	$7x_4$	-	x_5	+	$\underline{x}_6$					=	25
$3x_1$	-	$3x_2$	+	$2x_3$	-	$7x_4$	+					$\underline{x}_7$			=	5
$2x_1$	-	$2x_2$	+	x_3	+	$4x_4$	+							x_8	=	10
$x_j >= 0$, j=1,...,8																

If we now incorporate the objective function in the constraint in the usual manner and place the new variable, namely w last, we obtain the following system:

Note that this system is not in a canonical form because the columns of the artificial variables are not elementary columns. To obtain a canonical form we have to add the rows of the artificial variables to the last row. This yields:

This is then the system that will be used to initialize the simplex algorithm for Phase 1 of the 2-Phase method. Of course, the column of w will not appear in the tableau.

We can distinguish between two cases as far as the end of Phase 1 is concerned, namely:

- Case 1: $w^* > 0$: The optimal value of w is greater than zero. And Case 2: $w^* = 0$: The optimal value of w is equal to zero.

In Case 1 we conclude that the LP problem under consideration does not have a feasible solution whereas Case 2 implies that the constraints are feasible, hence the problem under consideration possesses a feasible solution.

The following final simplex tableau is an example of Case 1:

One artificial variable (x_6) is in the basis and is not equal to zero. The problem does not have a feasible solution. It should be noted that although Case 2 implies

x_1	-	x_2	+	$3x_3$	+	$7x_4$	-	x_5	+	$\underline{x}_6$						=	25
$3x_1$	-	$3x_2$	+	$2x_3$	-	$7x_4$	+					$\underline{x}_7$				=	5
$2x_1$	-	$2x_2$	+	x_3	+	$4x_4$	+							x_8		=	10
									-	$\underline{x}_6$	-	$\underline{x}_7$			+ w	=	0
$x_j >= 0$, j=1,...,8, w>= 0																	

x_1	-	x_2	+	$3x_3$	+	$7x_4$	-	x_5	+	$\underline{x}_6$						=	25
$3x_1$	-	$3x_2$	+	$2x_3$	-	$7x_4$	+					$\underline{x}_7$				=	5
$2x_1$	-	$2x_2$	+	x_3	+	$4x_4$	+							x_8		=	10
$4x_1$	-	$4x_2$	+	$5x_3$	+		-	x_5	+						w	=	30
$x_j>= 0$, j=1,...,8, w>= 0																	

x_1	x_2	x_3	x_4	x_5	$\underline{x}_6$	$\underline{x}_7$	w		RHS
1	-1	2	7	-1		2			25
	-3	1	-7		1	1			5
	-2	-1	-4			-1	1		5

that all the artificial variables are equal to zero, this does not mean that they are all out of the basis.

So it is necessary to consider Case 1 in more detail, namely:

- Case 2.1: All the artificial variables are non-basic.
- Case 2.2: some of the artificial variables are in the basis

In Case 2.1 we proceed to Phase 2 of the 2-Phase method replacing the objecitve function of Phase 1 with the original objective function. This will typically violate the canonical form of the problem and thus pivot operations may have to be used to restore the canonical form.

The following is a typical example of Case 2.1

x_1	x_2	x_3	x_4	x_5	$\underline{x}_6$	$\underline{x}_7$	w		RHS
1	-1	2	7			2			25
	-3	1	-7	1	1	1			5
	-2	-1			-3	-1	1		0

The two artificial variables are not in the basis; hence w is equal to zero.

Case 2.2 represents a degenerate basis, namely a situation where one or more of the basic variables are equal to zero. Here is an example:

To take a degenerate artificial variable out of the basis we pivot on any non-artificial variable whose coefficient in the row of the artificial is not equal to zero and enter it into the basis.

Fore example, in the table above, we can replace x_6 by either x_2 or x_3 or x_4.

If the coefficients of all the non-artificial variables in that row are zeros, then the conclusion is that the constraint is redundant and thus can be ignored.

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x_1	x_2	x_3	x_4	x_5	$\underline{x}_6$	$\underline{x}_7$	w		RHS
1	-1	2	7	-1		2			25
	-3	1	-7		1	1			0
	-2	-1	-4			-1	1		0

6. Conclusion

The paper shows that it is possible to use linear programming model to find optimal cooling size and consumption of coal for the minimal total cost of cool freezer. Depending on the mass flow rate of cool at the cool freezer exit and we get optimal results for different types of coal. Also, the paper presents the change of relative portion of minor cooling sizes at total cooling size of the cool freezer for three cost sets. These results can be used for optimization of the cool freezer in order to avoid losses of energy and money.

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