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EFFECT OF ROTATION ON TRIPLE-DIFFUSIVE CONVECTION IN RIVLIN-ERICKSEN FLUID IN

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Abstract: This effect of rotation on triple- diffusive convection in Rivlin-Ericksen fluid in porous medium is considered in the presence of uniform vertical rotation. For the case of stationary convection, the stable solute gradients and rotation have stabilizing effect on the system, whereas the medium permeability has a destabilizing (or stabilizing) effect on the system under certain conditions. A linear stability analysis theory and normal mode analysis method have been carried out to study the onset convection. The kinematic viscoelasticity has no effect on the stationary convection. The solute gradients, rotation, porosity and kinematic viscoelasticity introduce oscillatory modes in the system, which were non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained.

Key Words: triple-diffusive convection, Rivlin-Ericksen, thermal convection, solute gradients, vertical magnetic field, rotation

1. Introduction

The theoretical and experimental results of the onset of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydrodynamics have been treated by Chandrasekhar [1] in his celebrated monograph. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [2]. The physics is quite similar to the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by solute gradients and free boundaries. The problem of the onset of ther-

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mal instability in the presence of solute gradients is of great importance because of its applications to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. The double-diffusive convection problems also arise in oceanography, limnology and engineering. With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The Rivlin-Ericksen is one such fluid. Johari [4] has discussed the viscoelastic Rivlin-Ericksen incompressible fluid under time dependent pressure gradient. Sisodia and Gupta [5] and Srivastava and Singh [6] have studied the unsteady flow of a dusty elastico- viscous Rivlin-Ericksen through channel of different cross-sections in the presence of the time dependent pressure gradient. In another study Garg et al; [7] have studied the rectilinear oscillations, of a sphere along its diameter in a conducting dusty Rivlin-Ericksen fluid in the presence of a uniform magnetic field. Sharma and Kumar [8] have studied the thermal instability of Rivlin-Ericksen elastico- viscous fluid acted on by a uniform rotation and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. In many astrophysical situations, the effect of rotation on thermosolutal convection in porous medium is also important. In recent years, the investigation of flow of fluids through the porous media has become an important topic. A great number of applications in Geophysics may be found in the book written by Philips [9].When the fluid permeates through a porous material, the gross effect is represented by the law. As a result of this macroscopic law,the usual viscous term in the equation of Rivlin-Ericksen fluid motion is replaced by the resistance term $[-\frac{1}{k_1}(\mu+\mu^{'}\frac{\partial}{\partial t})$q], where μ and $\mu^{'}$ are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid, K_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid. The problem of the thermosolutal convection in fluids in porous medium is of great importance in Geophysics, Soil Sciences, ground water Hydrology and Astrophysics. Generally, it is accepted that comets consists of a dust "snowball" made of mixture of frozen gases which, in the process of their journey, changes from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in astrophysical context McDonnel [10]. Out of large published work in pure fluid, the thermosolutal convection in porous medium has received only attention, because of its various engineering applications. A comprehensive review of the literature concerning thermosolutal convection in a fluid-saturated porous medium may be found in the book written by Nield and Bejan [11]. The thermal convection in Rivlin-Ericksen fluid has been studied by many authors [12-15]. A recent review of numerical techniques and their applications may be found in O'Sullivan et al; [16]. Oldenburg and Pruess [17] have developed a model for convection in a Darcy's porous medium, where the mechanism involves temperature, NaCl, $CaCl_2$ and KCl. Solar ponds are a particularly promising means of harnessing energy from the Sun by preventing convective overturning in a thermohaline system by salting from below. But we also appreciate the work of Bhattacharyya and Abbas

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[18] and Qin and Kaloni [19], they have considered the effect of rotation in angular momentum equation.

In the standard Bénard problem, the instability is driven by a density difference caused by a temperature difference between the upper and lower planes bounding the fluid. If the fluid, additionally has salt dissolved in it, then there are potentially two destabilizing sources for the density difference, the temperature field and salt field. The solution behavior in the double-diffusive convection problem is more interesting than that of the single component situation in so much as new instability phenomena may occur which is not present in the classical Bénard problem. When temperature and two or more component agencies, or three different salts, are present then the physical and mathematical situation becomes increasingly richer. Very interesting results in triply diffusive convection have been obtained by Pearlstein et al, [20]. The results of Pearlstein et al, are remarkable. They demonstrate that for triple diffusive convection linear instability can occur in discrete sections of the Rayleigh number domain with the fluid being linearly stable in a region in between the linear instability ones. This is because for certain parameters the neutral curve has a finite isolated oscillatory instability curve lying below the usual unbounded stationary convection one. Straughan and Walker [21] derive the equations for non-Boussinesq convection in a multi- component fluid and investigate the situation analogous to that of Pearlstein et al, but allowing for a density non linear in the temperature field. Lopez et al, [22] derive the equivalent problem with fixed boundary conditions and show that the effect of the boundary conditions breaks the perfect symmetry. In reality the density of a fluid is never a linear function of temperature, and so the work of Straughan and Walker applies to the general situation where the equation of state is one of the density quadratic in temperature. This is important, since they find that departure from the linear Boussinesq equation of state changes the perfect symmetry of the heart shaped neutral curve of Pearlstein et al.

In view of the recent increase in the number of non iso-thermal situations, we intend to extend our work to the problem of thermal convection in Rivlin-Ericksen fluid on triple-diffusive convection in the presence of rotation in porous medium.

2. Mathematical Formulation of the Problem

Here we consider an infinite, horizontal layer of thickness d of an electrically non-conducting incompressible Rivlin-Ericksen fluid heated and salted from below. The temperature T and solute concentrations C^1 and C^2 at the bottom and top surfaces $z = 0, d$ are T_0 and T_1; C_0^1 and C_1^1; and C_0^2 and C_1^2 respectively, and a uniform temperature gradient$\beta(= |\frac{dT}{dZ}|)$ and uniform solute gradients are $\beta'(= |\frac{dC^1}{dZ}|)$ and $\beta''(= |\frac{dC^2}{dZ}|)$ are maintained. Both the boundaries are taken to be free and perfect conductors of heat. The gravity field $\mathbf{g}(0,0,-g)$ and a uniform magnetic field $\mathbf{H}$ (0,0,H) pervade on the system. This fluid layer is assumed to be flowing through an

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isotropic and homogeneous porous medium of the porosity ε and the permeabilityk_1.

The equations expressing the conservation of momentum, mass, temperature, solute concentrations and equation of Rivlin-Ericksen fluid are

$$\frac{1}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial \mathrm{t}}+\frac{1}{\varepsilon}(\mathbf{q}.\nabla)\mathbf{q}\right] = -(\frac{1}{\rho_0})\nabla \mathrm{p}+g(1+\frac{\delta\rho}{\rho_0})-\frac{1}{k_1}(v+v^{'}\frac{\partial}{\partial t})\mathbf{q}+\frac{2}{\varepsilon}(\mathbf{q}\ \times\mathbf{\Omega}), \quad (1)$$

$$\nabla\mathbf{q}=0, \quad (2)$$

$$E\frac{\partial \mathbf{T}}{\partial t}+(\mathbf{q}.\nabla)T=\kappa\nabla^2 T, \quad (3)$$

$$E'\frac{\partial c^1}{\partial t}+(\mathbf{q}.\nabla)T=\kappa'\nabla^2 C^1, \quad (4)$$

$$E''\frac{\partial c^2}{\partial t}+(\mathbf{q}.\nabla)T=\kappa''\nabla^2 C^2. \quad (5)$$

In terms of temperature T and the concentrations $\mathrm{C}^1$and C^2, we suppose the density of the mixture is given by (known as density equation of state)

$$\rho=\rho_0[1-\alpha(T-T_{\mathrm{a}})+\alpha'(C^1-C^1_{\mathrm{a}})+\alpha''(C^2-C^2_{\mathrm{a}})], \quad (6)$$

where ρ,ρ_0, $\mathbf{q}$, t, g,v, $v^{'}\kappa$, κ', κ'', $\alpha\alpha'$, α'' are the fluid density, reference density, velocity, time, gravitational acceleration, the kinematic viscosity, the kinematic viscoelasticity, the thermal diffusivity, the solute diffusivity $\kappa'$and κ'', thermal coefficient of expansion, solvent coefficient of expansion α' and α''respectively. $\mathrm{T_a}$is the average temperature given by $\mathrm{T_a}$ = $(\mathrm{T_0}$+$\mathrm{T_1})/2$ where $\mathrm{T_0}$and $\mathrm{T_1}$ are the constant average temperatures of the lower and upper surfaces of the layer and $\mathrm{C^1_a}$ and $\mathrm{C^2_a}$are the average concentrations given by $\mathrm{C^1_a}$ = $(\mathrm{C^1_0}$+$\mathrm{C^1_1})/2$ and $\mathrm{C^2_a}$ = $(\mathrm{C^2_0}$+$\mathrm{C^2_1})/2$, where $\mathrm{C^1_0}$, $\mathrm{C^1_1}$ and $\mathrm{C^2_0}$, $\mathrm{C^2_1}$are the constant average concentrations of the lower and upper surfaces of the layer. In writing equation (1), we also use the Boussinesq approximation by allowing the density to change only in the gravitational body force term. When the permeability of the porous material is low, then the inertial force becomes relatively insignificant as compared with the viscous drag when flow is considered. And as we know$\frac{1}{\varepsilon}(\mathbf{q}.\nabla)\mathbf{q}$ term is generally small, so it seems best to drop it in numerical work.

A porous medium of very low permeability allows us to use the Darcy's model. For a medium of very large stable particle suspension, the permeability tends to be small justifying the use of Darcy's model. This is because the viscous drag force is negligibly small in comparison with Darcy's resistance due to the large particle suspension. Here E = ε+ (1-ε) $\frac{\rho_s C_s}{\rho_0 C_i}$ is a constant and E$'$, E $''$ are analogous to E but corresponding to solute rather than heat. ρ_s, c_s andρ_o, c_i stand for density and heat capacity of solid (porous) material and fluid respectively. The steady state solution is

$$\mathbf{q}\ =(0,0,0), \quad T=-\beta z+T_{\mathrm{a}}, \quad C^1=-\beta' z+C^1_{\mathrm{a}},$$

$$C^2 = -\beta'' z + C_{\mathrm{a}}^2, \quad \beta = (T_0 - T_1)/d,$$
$$\beta' = (C_1^1 - C_0^1)/d, \quad \beta'' = (C_1^2 - C_0^2)/d,$$
$$\rho_= - \rho_0(1 + \alpha\beta z - \alpha'\beta' z - \alpha''\beta'' z). \tag{7}$$

Here we use the linearized stability theory and the normal mode method. Consider a small perturbation on the steady state solution and let δp, $\delta\rho$, θ, γ, $\gamma^{'}$ $\mathbf{q}$(u, v, w) denote, respectively, the perturbation in pressure, densityρ, temperature T, solute concentrations C^1, C^2 and velocity $\mathbf{q}$ (0,0,0). The change in density$\delta\rho$, caused mainly by the perturbations $\theta, intemperatureand \gamma, \gamma^{'}$ in concentrations, is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma - \alpha''\gamma') \tag{8}$$

Then the linearized perturbation equations become

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial \mathrm{t}} = -(\frac{1}{\rho_0})\nabla\delta\mathrm{p} - g(\alpha\theta - \alpha'\gamma - \alpha''\gamma') - \frac{1}{k_1}(v + v^{'}\frac{\partial}{\partial t})\mathbf{q} + \frac{2}{\varepsilon}(\mathbf{q} \times \mathbf{\Omega}), \tag{9}$$

$$\nabla\mathbf{q} =, \tag{10}$$

$$E\frac{\partial\theta}{\partial t} = \beta\omega + \kappa\nabla^2\theta, \tag{11}$$

$$E'\frac{\partial\gamma}{\partial t} = \beta'\omega + \kappa'\nabla^2\gamma, \tag{12}$$

$$E''\frac{\partial\gamma'}{\partial t} = \beta''\omega + \kappa''\nabla^2\gamma', \tag{13}$$

3. Dispersion Relation

Analysing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[\omega, \theta\gamma\gamma', \zeta] = [W(z), \Theta(\mathrm{z}), Z(z), \Gamma(\mathrm{z}), \Psi\,(\mathrm{z})\,]\mathrm{expi}(\mathrm{i}k_x\mathrm{x} + \mathrm{i}k_y\mathrm{y} + \mathrm{nt}),$$

where k_x, k_y are the wave numbers along the x- and y-directions, respectively and $k = \sqrt{(}\ k_x^2 + k_y^2)$ is the resultant wave number and n is the growth rate which is, in general, a complex constant. $\zeta = \frac{\partial v}{\partial x}$ - $\frac{\partial u}{\partial y}$ stands for the z-component of vorticity.

Expressing the coordinates x, y, z in the new unit of length d and letting

$$a = kd, \quad \sigma = \frac{\mathrm{n}d^2}{v}, \quad p_1 = \frac{\mathrm{v}}{\kappa},$$

$$q_1 = \frac{\mathrm{v}}{\kappa'}, \quad q_2 = \frac{\mathrm{v}}{\kappa''}, \quad F = \frac{\mathrm{v}}{d^2},$$

$$P_l = \frac{k_1}{d^2}, \text{ and } D = \frac{d}{\mathrm{dz}}. \tag{14}$$

Equation (9)-(13), with the help of expression (14), in non dimensional form become

$$[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+\sigma F)](D^2-a^2)W + \frac{ga^2d^2}{v}(\alpha\Theta - \alpha'\Gamma - \alpha''\Psi) - \frac{2\Omega d^3}{\varepsilon \mathrm{v}}DZ = 0, \tag{15}$$

$$[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+\sigma F)]Z = (\frac{2\Omega d}{\varepsilon \mathrm{v}})DW, \tag{16}$$

$$(D^2-a^2-Ep_1\sigma)\Theta = -(\frac{\beta d^2}{\kappa})W, \tag{17}$$

$$(D^2-a^2-E'q_1\sigma)\Gamma = -(\frac{\beta d^2}{\kappa'})W, \tag{18}$$

$$(D^2-a^2-E''q_2\sigma)\Psi = -(\frac{\beta d^2}{\kappa''})W, \tag{19}$$

Consider the case where both the boundaries are free as well as perfect conductors of heat and concentrations. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which Equations (15)-(19) must be solved Chandrasekhar [1].

$$W = D^2W = 0, \quad \Theta = \Gamma = \Psi = 0, \quad DZ = 0, \tag{20}$$

at $z = 0$ and $z = 1$.

The case of two free boundaries, though a little artificial, is the most appropriate for stellar atmospheres Spiegel [23]. Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for z=0 and z=1 and hence, the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{21}$$

where W_0 is a constant.

Eliminating Θ,Γ,Ψ and Z between Equations (15)- (19) and substituting the proper solution $W = W_0 \sin \pi z$, in the resultant equation, we obtain the dispersion relation

$$R_1 = (\frac{1+x}{x})[\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}(1+i\sigma_1 F)](1+x+iEp_1\sigma_1) + T_{A_1}[\frac{1+x+iEp_1\sigma_1}{x\{\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}(1+i\sigma_1 F)\}}]$$
$$+ S_1[\frac{1+x+iEp_1\sigma_1}{(1+x+iEq_1\sigma_1}] + S_2[\frac{1+x+iEp_1\sigma_1}{(1+x+iEq_2\sigma_1)}], \tag{22}$$

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where $R_1 = \frac{g\alpha\beta d^4}{\upsilon\kappa\pi^4}$, $S_1 = \frac{g\alpha\beta d^4}{\upsilon\kappa'\pi^4}$, $S_1 = \frac{g\alpha\beta d^4}{\upsilon\kappa'\pi^4}$, $T_{A_1} = \frac{4\Omega^2 d^4}{\upsilon^2\pi^4} = (\frac{2\Omega d^2}{\upsilon\pi^2})^2$, $x = \frac{a^2}{\pi^2}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$ and $P = \pi^2 P_l$.

Equation (20) is the required dispersion relation including the effect of rotation, medium permeability, kinematic viscoelasticity and stable solute gradients on the triple- diffusive convection of Rivlin-Ericksen rotating fluid in the presence of porous medium.

4. Results and Discussion

4.1. The Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by σ =0. Putting σ =0, the dispersion relation (20) reduces to

$$R_1 = \frac{(1+x)^2}{\mathrm{xP}} + PT_{A_1}\frac{\mathbf{(1+x)}}{\mathbf{x}} + S_1 + S_2, \tag{23}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1} $s_{1,}s_2$ and P. The parameter F accounting for the kinematic viscoelasticity effect vanishes for the stationary convection.

To investigate the effects of stable solute gradients, rotation and medium permeability, we examine the behavior of$\frac{dR_1}{dS_1}$,$\frac{dR_1}{dS_2}$, $\frac{dR_1}{dT_{A_1}}$, $\frac{dR_1}{dP}$ analytically. Equation (22) yields

$$\frac{\mathrm{dR_1}}{\mathrm{dS_1}} = 1, \tag{24}$$

$$\frac{dR_1}{dS_2} = 1, \tag{25}$$

This implies that the stable solute gradients have a stabilizing effect on triple- diffusive convection in Rivlin-Ericksen fluid in porous medium. The reverse solute gradients have a destabilizing effect on the system and$\frac{dR_1}{dS_1}$,$\frac{dR_1}{dS_2}$ become negative. Equation (22) also yields

$$\frac{dR_1}{dT_{A_1}} = \frac{\mathbf{(1+x)}}{\mathbf{x}}P, \tag{26}$$

The rotation, therefore, has always a stabilizing effect on triple-diffusive convection in Rivlin-Ericksen fluid in porous medium. It is evident from (22) that

$$\frac{dR_1}{dP} = -\frac{\mathbf{(1+x)}}{\mathbf{x}}[\frac{\mathbf{(1+x)}}{\mathbf{P^2}} - T_{A_1}]. \tag{27}$$

In the absence of rotation ($T_{A_1}\rightarrow$), $\frac{\mathrm{dR_1}}{\mathrm{dP}}$ is given by

$$\frac{dR_1}{dP} = -\frac{\mathbf{(1+x)^2}}{\mathbf{xP^2}}, \tag{28}$$

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which is always negative. The medium permeability, therefore, has a destabilizing effect on triple-diffusive convection in Rivlin-Ericksen fluid in porous medium in the absence of rotation. In the presenceof rotation, the medium permeability has a destabilizing (or stabilizing) effect on the system if $T_{A_1}<(\text{or }>)\frac{\mathbf{1+x}}{\mathbf{P^2}}$. It has been observed that as the rotation parameter increases, the stabilizing effect of medium permeability also increases.

4.2. Stability of the System and Oscillatory Modes

Here we will examine the possibility of oscillatory modes, if any, in stability problem due to the presence of kinematic viscoelasticity, stable solute gradients, and rotation. Multiplying equation (15) by W*, the complex conjugate of W, and using (16) - (19) together with the boundary conditions (20), we obtain

$$[\frac{\sigma *}{\varepsilon}+\frac{1}{P_l}(1+\sigma^* F)]I_1-(\frac{\mathrm{g}\alpha\kappa\mathbf{a^2}}{v\beta})[I_2+Ep_1\sigma^* I_3]+(\frac{\mathrm{g}\alpha\kappa a^2}{v\beta'})[I_4+E'q_1\sigma^* I_5]$$
$$+\left(\frac{\mathrm{g}\alpha\kappa a^2}{v\beta''}\right)[I_6+E''q_2\sigma^* I_7]+d^2[\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}(1+\sigma^* F)]I_8=0. \quad (29)$$

Here

$$I_1=\int_0^1(|DW|^2+\mathrm{a}^2\,|W|^2)dz, \qquad I_2=\int_0^1(|D\Theta|^2+\mathrm{a}^2\,|\Theta|^2)dz,$$

$$I_3=\int_0^1(|\Theta|^2)dz, \qquad I_4=\int_0^1(|D\Gamma|^2+\mathrm{a}^2\,|\Gamma|^2)dz,$$

$$I_5=\int_0^1(|\Gamma|^2)dz, \qquad I_6=\int_0^1(|D\Psi|^2+\mathrm{a}^2\,|\Psi|^2)dz,$$

$$I_7=\int_0^1(|\Psi|^2)dz, \qquad I_8=\int_0^1(|Z|^2)dz, (30)$$

The integral $I_1, I_2, \ldots, I_8$ are all positive definite. Putting $\sigma=\sigma_\mathrm{r}+\mathrm{i}\ \sigma_\mathrm{i}$ and equating the real and imaginary parts of equation (29), we obtain

$$[(\frac{1}{\varepsilon}+\frac{F}{P_l})I_1-(\frac{\mathrm{g}\alpha\kappa\mathbf{a^2}}{v\beta})Ep_1I_3+(\frac{\mathrm{g}\alpha\kappa a^2}{v\beta'})E'q_1I_5+\left(\frac{\mathrm{g}\alpha\kappa a^2}{v\beta'}\right)E''q_2\sigma^* I_7+d^2(\frac{1}{\varepsilon}+\frac{F}{P_l})I_8]\sigma_\mathrm{r}$$
$$=-[\frac{I_1}{P_l}-\frac{\mathrm{g}\alpha\kappa\mathbf{a^2}}{v\beta}I_2+\frac{\mathrm{g}\alpha\kappa a^2}{v\beta'}I_4+\frac{\mathrm{g}\alpha\kappa a^2}{v\beta''}I_6+\frac{\mathbf{d^2}}{P_l}I_8], \quad (31)$$

$$[(\frac{1}{\varepsilon}+\frac{F}{P_l})I_1+(\frac{g\alpha\kappa\mathbf{a^2}}{v\beta})Ep_1I_3-(\frac{g\alpha\kappa a^2}{v\beta'})E'q_1I_5-\left(\frac{g\alpha\kappa a^2}{v\beta'}\right)E''q_2I_7 - d^2(\frac{1}{\varepsilon}+\frac{F}{P_l})I_8]\sigma_{\rm i}=0. \quad (32)$$

It is evident from (31) that $\sigma_{\rm r}$ is positive or negative. The system is, therefore, stable or unstable. It is clear from (32) that $\sigma_{\rm i}$ may be zero or non-zero, the modes may be non-oscillatory or oscillatory accordingly. The oscillatory modes are introduced due to the presence of kinematic viscoelasticity, stable solute gradients and rotation, which were non-existent in their absence.

4.3. The Case of Overstability

The present section is devoted to find the possibility that the observed instability may really be overstability. Since we wish to determine the Rayleigh number for the onset of instability through state of pure oscillations, it is suffices to find conditions for which (22) will admit of solutions with σ_1 real.

Equating real and imaginary parts of (22) and eliminating R_1 between them, we obtain

$$A_3c^3+A_2c_1^2+A_1c_1+A_0=0.(33)$$

Here:

$$c_1=\sigma_1^2 \text{ and } b=1+x,$$

and

$$A_3=b(E^{'}q_1E^{''}q_2)^2\left(\frac{1}{\varepsilon}+\frac{F}{P}\right)^2[\frac{Ep_1}{P}+b\left(\frac{1}{\varepsilon}+\frac{F}{P}\right) \quad (34)$$

$$A_0=b^5[\frac{\mathrm{b}}{P^2}\left(\frac{1}{\varepsilon}+\frac{F}{P}\right)+\frac{\mathrm{Ep_1}}{P^3}-T_{A_1}\left(\frac{1}{\varepsilon}+\frac{F}{P}\right)+T_{A_1}\frac{\mathrm{Ep_1}}{\mathrm{bP}}]+\frac{b^3}{P^2}(b-1)[S_1(Ep_1-E'q_1) + S_2(\mathrm{E}p_1-E^{''}q_2)]. \quad (35)$$

The coefficient A_2 and A_1 being quite lengthy and not needed in the discussion of overstability, so has not been written here.

Since σ_1 is real for overstability, the three values of $\mathrm{c}_1(=\sigma_1^2)$ are positive. The product of roots of equation (33) is $-\frac{A_0}{A_3}$, and if this is to negative, then A_3 and A_0 are of the same sign. Now the product of roots is negative if

$$Ep_1>E'q_1, Ep_1>E''q_2, Ep_1>P^3T_{A_1}\left(\frac{1}{\varepsilon}+\frac{F}{P}\right), Ep_1>E'q_1\left(1+\frac{\varepsilon F}{P}\right)$$

and

$$Ep_1>E''q_2\left(1+\frac{\varepsilon F}{P}\right), \quad (36)$$

which implies

$$E'\kappa < E\kappa', E''\kappa < E\kappa'', \kappa < \frac{\varepsilon v^3 d^3 \mathrm{E}}{4\Omega^2 k_1 (\pi^2 k_1^2 + \varepsilon v' d^2)}, E'\kappa \left(1 + \frac{\varepsilon v' d^2}{\pi^2 k_1^2}\right) < E\kappa'$$

and

$$E''\kappa \left(1 + \frac{\varepsilon v'' d^2}{\pi^2 k_2^2}\right) < E\kappa'.$$

Thus

$$\kappa < \frac{\varepsilon v^3 d^3 \mathrm{E}}{4\Omega^2 k_1 (\pi^2 k_1^2 + \varepsilon v' d^2)} and E'\kappa \left(1 + \frac{\varepsilon v' d^2}{\pi^2 k_1^2}\right) < E\kappa'$$

and

$$E''\kappa \left(1 + \frac{\varepsilon v'' d^2}{\pi^2 k_2^2}\right) < E\kappa'$$

are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

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