International Electronic Journal of Pure and Applied Mathematics

Volume 4 No. 4 2012, 297-320

MATHEMATICAL ANALYSIS OF A A HIV/AIDS MODEL WITH TREATMENT

Yves Emvudu[1 §], Bongor Danhree[2]

[1,2]Laboratory of Applied Mathematics
Department of Mathematics
Faculty of Science
University of Yaounde 1,
P.O. Box 812, Yaounde, CAMEROON

Abstract: In this paper, we present a deterministic model of the dynamics of HIV/AIDS in a population of size $N(t)$ at time t, consising of four classes of individuals or epidemiologic classes : class of healthy individuals, class of infected individuals with HIV, class of sick with AIDS and finally the class of HIV/AIDS restored after ARV-therapy. We firstly give the basic properties of the model and then we draw a mathematical analysis by starting with the sub-model: in the dynamic sub-model of infection of HIV/AIDS in the absence of treatment. The mathematical analysis performed on the dynamics of HIV/AIDS with ARV-therapy, allows us to highlight the phenomenon of bifurcation of the existing equilibrium points (disease-free equilibrium and endemic equilibrium) in the sub-model, therefore the model. This phenomenon, which destabilizes the system is explained by a small variation or disturbance of the data parameters passed by the system from one stable state to an unstable state. This state of stability or instability is governed by the basic reproduction number. Furthermore we show that in the absence of treatment of infected individuals with HIV/AIDS, the model admits a free disease equilibrium globally unstable. This shows that the HIV/AIDS is one of greater cause of death. In the presence of treatment, we seek and find an optimal therapy which consists of treating HIV/AIDS to infected individuals.

AMS Subject Classification: 34A34, 34D23, 92D30
Key Words: nonlinear dynamical systems, epidemiological models, HIV/AIDS model, stability

Received: March 1, 2012

§Correspondence author

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

1. Introduction

As the process by which a problem or phenomenon in the real world is described, interpreted and represented in terms of abstract symbols or models, modeling is applied for a long time in the fight against infectious diseases. The first modeling work in epidemiology was conducted in 1760 by Daniel Bernoulli [6] to evaluate the effectiveness of variolation: decrease mortality due to smallpox and gain in life expectancy. Since 1927 Kermak Mckendrik played a central role in the mathematical theory of epidemics [7]. Modeling consist of constructing numerically simulable models of real natural phenomena, a priori evaluation of strategies for intervention or control.

In order to properly model a phenomenon as the dynamics of HIV/AIDS, we have two major requirements: The need to integrate clinical medicine and biology in the modeling of epidemics on an other hand and the triple expertise: mathematical statistical and other information.

How mathematics can be used, for example, to get the reality of the dynamics of infection with a disease like AIDS to adopt a strategy of optimal control treatment or effective?

A model is only an approximation of reality and it is conceivable that several mathematical functions are likely to approach this reality. This is the question that we must provide solutions and show that taking ARV drugs by HIV infected/AIDS reduced the progression of this disease and we can achieve long-term zero new infections of AIDS. To do this we consider type of compartmental model, ie a model where the population is divided into four epidemiological classes may consist of healthy, HIV infected, AIDS patients and finally recovered after treatment.

We note that this model with four compartments was studied by Oluwaseun Sharom, N. Chandra Podder and Abba B. Gumel and B. Song (2007-2008) [1]. Our contribution will be to determine explicitly the dynamics of this infection and treatment strategy to achieve zero new infections.

In the model formulation, we will define the various flows of the transmission diffusion see of epidemiological behavior of individuals from one compartment to another compartment of our model. Our model will highlight the changes in numbers over time in each compartment. The time step is year.

Once the mathematical study of our model is made, we will be able with the help of numerical simulation to give the different states which follow and we easily s that the scale of a population's behavior or changes in healthy individuals, individuals undergoing primary infection or co-infection, in asymptomatic individuals and individuals suffering from this disease of the millennium: AIDS.

We then need to be able to design rules or a system of optimal control system which can lead to a desired state dynamics.

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

2. The Model

2.1. Description of the State Variables and Parameters of the Model

Variables	Descriptions
S (t)	Class of healthy individuals who may contract HIV
H_1 (t)	class of individuals with HIV
H_2 (t)	class of people sick with AIDS only
$W_H(t)$	Class of recovered after ARV-therapy of the HIV/AIDS H_1 and H_2
parameters	**Description**
Λ	recruitment rates of healthy
μ	Natural mortality rate
σ	The progression rate of HIV-infection to AIDS stage
C_2	multiplier of co-infection
θ	multiplier factor of up Of infection to the AIDS stage because of resistances
β_H	Forces of HIV infection,
δ_H	mortality due to HIV/AIDS
η_1 η_2 η_H η_D, η_r	numbers
τ_1, τ_2	The recovery rate of infectious individuals with HIV and AIDS Respectively

Table 1: Description of the variables and parameters of model

2.2. Diagram and Model

The diagram below allows to obtain model (1) of the dynamics of HIV/AIDS with treatment:

$$\begin{cases} \dot{S} = \Lambda - \lambda_H S - \mu S \\ \dot{H}_1 = \lambda_H S - (\mu + \sigma + \tau_1) H_1 \\ \dot{H}_2 = \sigma H_1 + \sigma\theta W_H - (\mu + \delta_H + \tau_2) H_2 \\ \dot{W}_H = \tau_1 H_1 + \tau_2 H_2 - (\mu + \sigma\theta) W_H \end{cases} \tag{1}$$

Where

$$\lambda_H = \beta_H \frac{H_1 + \eta_2 H_2 + \eta_H W_H}{N} \quad with \quad N = S + H_1 + H_2 + W_H$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

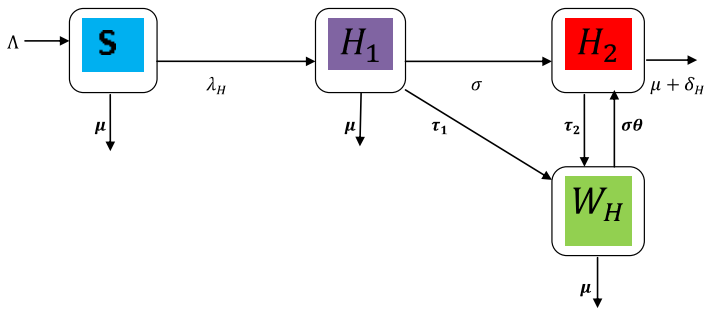


Figure 1: Compartmental diagram of HIV/AIDS infection with treatment

2.3. Basic Properties of the Model

2.3.1. Positivity of Solutions

Lemma 1. *The solutions of system (1) with initial conditions are non-negative for the duration of the epidemic.*

Proof. For the model to be epidemiologically plausible, it is necessary to prove that the solutions of system (1) with positive initial conditions, will remain positive at all times to come.

Let's us assume that $S(0) > 0$, $H_1(0) > 0$, $H_2(0) > 0$, $W_H(0) > 0$, and show that $\forall t > 0$, $S(t) > 0$, $H_1(t) > 0$, $H_2(t) > 0$, $W_H(t) > 0$. Whether $t_0 = sup\{t > 0 : S(t) > 0, H_1(t) > 0, H_2(t) > 0, W_H(t) > 0\}$. The first equation of system (1) provides:

$$\dot{S} = \Lambda - \lambda_H S - \mu S \tag{2}$$

$$\frac{d}{dt}[S(t)\exp\{\mu t + \int_0^t \lambda_H(\tau)d\tau\}] \geq \Lambda \exp \mu t + \int_0^t \lambda_H(\tau)d\tau$$

Passing to the integral of 0 from t_0, we have:

$$S(t_0)\exp\{\mu t_0 + \int_0^{t_0}[\lambda_H(\tau)d\tau\} - S(0) \geq \int_0^{t_0}(\Lambda \exp \mu x + \int_0^x \lambda_H(\tau)d\tau)dx$$

$$S(t_0) \geq S(0)\exp\{-(\mu t_0 + \int_0^{t_0}(\lambda_H(\tau))d\tau\} + \exp\{-(\mu t_0$$

$$+ \int_0^{t_0} \lambda_H \tau)d\tau\})(\int_0^{t_0} \Lambda \exp(\mu x + \int_0^x (\lambda_H(\tau)d\tau)dx).$$

Thus we get that $S(t_0) > 0$. That means that $S(t) > 0$. Similarly, one can show that $H_1(t) > 0$, $H_2(t) > 0$, $W_H(t) > 0$. It is concluded that the system admits positive solutions when all the constant parameters and variables are non-negative. □

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

2.3.2. Positive Invariance of the Bounded Domain of Solutions

Lemma 2. *Each of the solutions of system (1) is bounded, and for all $\varepsilon > 0$ the subset ω_ε of the positive region of $\mathbb{R}^4_+$ defined by:*

$$\Omega_\varepsilon = \{(S, H_1, H_2, W_H) \in \mathbb{R}^4_+ : N \leq \frac{\Lambda}{\mu} + \varepsilon\}$$

is a positively invariant region for system (1).

Proof. The total population of $N(t)$ at time t is given by:
$N(t) = S(t) + H_1(t) + H_2(t) + W_H(t)$ et $\dot{N}(t) = \Lambda - \mu N(t) - \delta_H H_2(t)$ then it follows that: $\dot{N}(t) < \Lambda - \mu N(t)$. Setting $M(t) = \Lambda - \mu N(t)$ We then obtain its derivative $\dot{M}(t) = -\mu \dot{N}(t) \geq -\mu M(t)$ whether

$$\dot{M}(t) \geq -\mu M(t) \Rightarrow M(t) \geq M(0)e^{-\mu t}$$

as $M(t) = \Lambda - \mu N(t)$ and $M(0) = \Lambda - \mu N(0)$
Then obtained for all $\varepsilon > 0$:

$$N(t) \leq \frac{\Lambda}{\mu} + N(0)e^{-\mu t} + \varepsilon$$

Letting $t \to +\infty$ we therefore obtain

$$N(t) \leq \frac{\Lambda}{\mu} + \varepsilon$$

Hence each solution of the system is bounded by: $\frac{\Lambda}{\mu} + N(0)e^{-\mu t} + \varepsilon$.

We now show that Ω_ε is a positively invariant region. Indeed, let h on a specific application $\mathbb{R}^4_+$ to $\mathbb{R}$ what more $x = (S, H_1, H_2, W_H)$ combines $h(x) = S + H_1 + H_2 + W_H$ h is a differentiable and its gradient is given by:

$$\nabla h = (1, 1, 1, 1) \qquad \forall x \ \in \mathbb{R}^4_+$$

more $\forall x \ \in h^{-1}(\frac{\Lambda}{\mu} + \varepsilon)$, the scalar product

$$< d(x), \nabla(h(x)) >= \dot{S} + \dot{H}_1 + \dot{H}_2 + \dot{W}_H \ \ with \ \ d(x) = (\dot{S}, \dot{H}_1, \dot{H}_2, \dot{W}_H)$$

whether

$$< d(x), \nabla(h(x)) >= \dot{N} = \Lambda - \mu N - \delta_H H_2$$

But $\forall x \in h^{-1}(\frac{\Lambda}{\mu} + \varepsilon)$, $N = \frac{\Lambda}{\mu} + \varepsilon$ so for $\varepsilon > 0$, we have

$$\forall \ \ x \in h^{-1}(\frac{\Lambda}{\mu} + \varepsilon), \qquad < d(x), \nabla(h(x)) >= \dot{N} = -\mu\varepsilon - \delta_H H_2 \leq 0$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

By the theorem of hence, it is concluded that:

$$\Omega_\varepsilon = \{(S, H_1, H_2, W_H) \in \mathbb{R}^4_+ : N \leq \frac{\Lambda}{\mu} + \varepsilon\}$$

is positively invariant for system (1) □

3. Mathematical Analysis of the Model

3.1. Local Stability of Equilibrium Without HIV and Basic Reproduction Rate

Proposition 3. *System (1) has a unique disease-free equilibrium (DFE):* $\varepsilon_0^{H\tau} = (\frac{\Lambda}{\mu}, 0, 0, 0)$ *and its basic reproduction rate* $R_0^{H\tau}$ *is*

$$R_0^{H\tau} = \frac{\beta_H\{(\mu+\delta_H)(\mu+\sigma\theta) + \sigma\eta_2[(\mu+\sigma\theta)+\theta\tau_1) + \eta_H[(\mu+\delta_H+\tau_2)\tau_1+\sigma\tau_2] + \mu\tau_2\}}{h_1(h_2h_3 - \sigma\theta\tau_2)}, \tag{3}$$

with

$$h_1(h_2h_3 - \sigma\theta\tau_2) = (\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta) + \mu\tau_2]$$

Proof. Applying the method of Van den Driessche and Watmough for calculating the basic reproduction rate $R_0^{H\tau}$ of model (1), we have:

$$\mathcal{F} = \begin{pmatrix} 0 \\ \lambda_H S \\ 0 \\ 0 \end{pmatrix} \quad \text{and,} \quad \mathcal{V} = \begin{pmatrix} \lambda_H S + \mu S - \Lambda \\ (\mu+\sigma+\tau_1)H_1 \\ (\mu+\delta_H+\tau_2)H_2 - \sigma H_1 - \sigma\theta W_H \\ (\mu+\sigma\theta)W_H - \tau_1 H_1 - \tau_2 H_2 \end{pmatrix}$$

$$F = \begin{pmatrix} \beta_H & \eta_2\beta_H & \eta_H\beta_H \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and,} \quad V = \begin{pmatrix} \mu+\sigma+\tau_1 & 0 & 0 \\ -\sigma & \mu+\delta_H+\tau_2 & -\sigma\theta \\ -\tau_1 & -\tau_2 & \mu+\sigma\theta \end{pmatrix}$$

Setting:

$$h_1 = \mu+\sigma+\tau_1, \quad h_2 = \mu+\delta_H+\tau_2, \quad \textit{and} \quad h_3 = \mu+\sigma\theta$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

$$\text{So, } V^{-1}=\begin{pmatrix} \frac{1}{h_1} & 0 & 0 \\ \frac{\sigma(h_3+\theta\tau_1)}{h_1(h_2h_3-\sigma\theta\tau_2)} & \frac{h_3}{h_2h_3-\sigma\theta\tau_2} & \frac{\sigma\theta}{h_2h_3-\sigma\theta\tau_2} \\ \frac{h_2\tau_1+\sigma\tau_2}{h_1(h_2h_3-\sigma\theta\tau_2)} & \frac{\tau_2}{h_2h_3-\sigma\theta\tau_2} & \frac{h_2}{h_2h_3-\sigma\theta\tau_2} \end{pmatrix} \text{ and}$$

$$FV^{-1}=\begin{pmatrix} \frac{\beta_H G}{h_1(h_2h_3-\sigma\theta\tau_2)} & \frac{\beta_H(h_3\eta_2+\tau_2\eta_H)}{h_2h_3-\sigma\theta\tau_2} & \frac{\beta_H(\sigma\theta\eta_2+h_2\eta_H)}{h_2h_3-\sigma\theta\tau_2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here

$$G=\sigma\eta_2(h_3+\theta\tau_1)+\eta_H(\sigma\tau_2+h_2\tau_1)+h_2h_3-\sigma\theta\tau_2$$

$$=(\mu+\delta_H)(\mu+\sigma\theta)+\sigma\eta_2[(\mu+\sigma\theta)+\theta\tau_1]+\eta_H[(\mu+\delta_H+\tau_2)\tau_1+\sigma\tau_2]+\mu\tau_2$$

The eigenvalues of FV^{-1} are: $R_1=R_2=0 \quad and \quad R_3=\dfrac{\beta_H G}{h_1(h_2h_3-\sigma\theta\tau_2)}$.

So the basic reproduction rate $R_0^{H\tau}$ of the sub-model (2.18) of the dynamics of HIV/AIDS-only in the presence of treatment is:

$$R_0^{H\tau}=\frac{\beta_H\{(\mu+\delta_H)(\mu+\sigma\theta)+\sigma\eta_2[(\mu+\sigma\theta)+\theta\tau_1)+\eta_H[(\mu+\delta_H+\tau_2)\tau_1+\sigma\tau_2]+\mu\tau_2\}}{h_1(h_2h_3-\sigma\theta\tau_2)},$$

with

$$h_1(h_2h_3-\sigma\theta\tau_2=(\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta)+\mu\tau_2]$$

□

Remark 4. We can see that:

When the rate τ_1 of HIV treatment in non-advanced (in other words, the treatment in the class (H_1) tends to infinity then the basic reproduction rate $R_0^{H\tau}$ of the model (1) of the dynamics of HIV/AIDS in the presence of treatment tends to $\dfrac{\beta_H[\eta_2\sigma\theta+\eta_H(\mu+\delta_H+\tau_2)]}{(\mu+\delta_H)(\mu+\sigma\theta)+\mu\tau_2}$ positive values.

$$\lim_{\tau_1\longrightarrow\infty} R_0^{H\tau}=\frac{\beta_H[\eta_2\sigma\theta+\eta_H(\mu+\delta_H+\tau_2)]}{(\mu+\delta_H)(\mu+\sigma\theta)+\mu\tau_2}>0$$

Similarly, when the rate τ_2 of AIDS treatment (that is to say the treatment in the class (H_2)) tends to infinity then the basic reproduction rate $R_0^{H\tau}$ of the model (1) of the dynamics of HIV/AIDS in the presence of treatment tends to $\dfrac{\beta_H[\mu+\eta_H(\sigma+\tau_1)]}{\mu(\mu+\sigma+\tau_1)}$ with positive values.

$$\lim_{\tau_2\longrightarrow\infty} R_0^{H\tau}=\frac{\beta_H[\mu+\eta_H(\sigma+\tau_1)]}{\mu(\mu+\sigma+\tau_1)}>0$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

Proposition 5. *The disease-free equilibrium (DFE) of model (1) of the dynamics of HIV/AIDS in the presence of treatment is locally asymptotically stable if* $R_0^{H\tau} < 1$ *and unstable if* $R_0^{H\tau} > 1$

Proof. At the disease-free equilibrium (DFE) $\varepsilon_0^{H\tau} = (\frac{\Lambda}{\mu}, 0, 0, 0)$, the Jacobian matrix of system (1) is given by:

$$J(\varepsilon_0^{H\tau}) = \begin{pmatrix} -\mu & -\beta_H & -\eta_2\beta_H & -\eta_H\beta_H \\ 0 & \beta_H - (\mu+\sigma+\tau_1) & \eta_2\beta_H & \eta_H\beta_H \\ 0 & \sigma & -(\mu+\delta_H+\tau_2) & \sigma\theta \\ 0 & \tau_1 & \tau_2 & -(\mu+\sigma\theta) \end{pmatrix} \tag{4}$$

The characteristic polynomial of $J(\varepsilon_0^{H\tau})$ is $P(\chi) = (\chi+\mu)(\chi^3 + a_2\chi^2 + a_1\chi + a_0)$ where the coefficients a_2, a_1 *and* a_0 are defined by:

$$\begin{cases} a_2 = 3\mu + \sigma + \delta_H + \tau_1 + \tau_2 + \sigma\theta - \beta_H \\ a_1 = 3\mu^2 + 2\mu\sigma + 2\mu\delta_H + 2\mu\tau_1 + 2\mu\tau_2 + 2\mu\sigma\theta + \sigma^2\theta + \sigma\theta\delta_H + \sigma\theta\tau_1 + \sigma\tau_2 \\ \qquad + \sigma\delta_H + \tau_1\delta_H + \tau_1\tau_2 - [2\mu + \delta_H + \tau_2 + \sigma\theta + \sigma\eta_2 + \tau_1\eta_H]\beta_H \\ a_0 = (\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta_t) + \mu\tau_2] - \beta_H\{(\mu+\delta_H)(\mu+\sigma\theta) \\ \qquad + \sigma\eta_2[(\mu+\sigma\theta) + \theta\tau_1) + \eta_H[(\mu+\delta_H+\tau_2)\tau_1 + \sigma\tau_2] + \mu\tau_2\} \end{cases} \tag{5}$$

If $R_0^{H\tau} > 1$ *then* $a_0 < 0$, because the expression of a_0 depending on that of $R_0^{H\tau}$ is:

$$a_0 = (1 - R_0^{H\tau})(\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta) + \mu\tau_2]$$
$$With \quad (\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta) + \mu\tau_2] > 0$$

According to Descartes's rule of signs, the polynomial $\chi^3 + a_2\chi^2 + a_1\chi + a_0$ admits at least one root with positive real part and therefore we conclude that $\varepsilon_0^{H\tau}$ is unstable if $R_0^{H\tau} > 1$.

If $R_0^{H\tau} < 1$ then $a_0 > 0$ and otherwise we have: $a_2 > 0$, in fact,

$$a_2 = 3\mu + \sigma + \delta_H + \tau_1 + \tau_2 + \sigma\theta - \beta_H$$

$$R_0^{H\tau} < 1 \Rightarrow a_0 > 0 \quad \Rightarrow \quad -\beta_H > -\frac{(\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta) + \mu\tau_2]}{G}$$

$$\Rightarrow a_2 > 3\mu + \sigma + \delta_H + \tau_1 + \tau_2 + \sigma\theta - \frac{(\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta) + \mu\tau_2]}{G}$$

$$\Rightarrow a_2 > \frac{(3\mu + \sigma + \delta_H + \tau_1 + \tau_2 + \sigma\theta)G - (\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta) + \mu\tau_2]}{G}.$$

By replacing G by its expression, we have:

$$\Rightarrow a_2 > \frac{(3\mu + \sigma + \delta_H + \tau_1 + \tau_2 + \sigma\theta)[\mu(\mu+\sigma\theta) + \delta_H(\mu+\sigma\theta) + \sigma\eta_2(\mu+\sigma\theta)}{(\mu+\delta_H)(\mu+\sigma\theta) + \sigma\eta_2[(\mu+\sigma\theta) + \theta\tau_1] + \eta_H[(\mu+\delta_H+\tau_2)\tau_1 + \sigma\tau_2] + \mu\tau_2}$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

$$+\frac{\eta_H\tau_1(\mu+\delta_H+\tau_2)+\sigma\tau_2\eta_H+\sigma\theta\eta_2\tau_1]-\mu[\mu(\mu+\sigma\theta)+\delta_H(\mu+\sigma\theta)+\mu\tau_2]}{(\mu+\delta_H)(\mu+\sigma\theta)+\sigma\eta_2[(\mu+\sigma\theta)+\theta_t\tau_1]+\eta_H[(\mu+\delta_H+\tau_2)\tau_1+\sigma\tau_2]+\mu\tau_2}$$

$$+\frac{-\sigma[\mu(\mu+\sigma\theta_t)+\delta_H(\mu+\sigma\theta)+\mu\tau_2]-\tau_1[\mu(\mu+\sigma\theta)+\delta_H(\mu+\sigma\theta)+\mu\tau_2]}{(\mu+\delta_H)(\mu+\sigma\theta)+\sigma\eta_2[(\mu+\sigma\theta)+\theta\tau_1]+\eta_H[(\mu+\delta_H+\tau_2)\tau_1+\sigma\tau_2]+\mu\tau_2}$$

Either we have after simplification:

$$a_2>\frac{(2\mu+\tau_2+\sigma\theta)[(\mu+\sigma\theta)(\mu+\delta_H+\sigma\eta_2)+\eta_H\tau_1(\mu+\delta_H+\tau_2)+\sigma\tau_1(\eta_H+\eta_2\theta)]}{(\mu+\delta_H)(\mu+\sigma\theta)+\sigma\eta_2[(\mu+\sigma\theta)+\theta\tau_1]+\eta_H[(\mu+\delta_H+\tau_2)\tau_1+\sigma\tau_2]+\mu\tau_2}$$

$$+\frac{(\sigma+\tau_1)[\mu\tau_2(\eta_H-1)+\sigma\eta_2(\mu+\sigma\theta)+\eta_H\tau_1(\mu+\delta_H+\tau_2)]}{(\mu+\delta_H)(\mu+\sigma\theta)+\sigma\eta_2[(\mu+\sigma\theta)+\theta\tau_1]+\eta_H[(\mu+\delta_H+\tau_2)\tau_1+\sigma\tau_2]+\mu\tau_2}$$

$$+\frac{\mu\sigma(\mu+\sigma\theta)(\eta_2-1)+\sigma\theta\eta_2\tau_1(\mu+\sigma+\tau_1)+\sigma\eta_H\tau_2(2\sigma+\tau_1)}{(\mu+\delta_H)(\mu+\sigma\theta)+\sigma\eta_2[(\mu+\sigma\theta)+\theta\tau_1]+\eta_H[(\mu+\delta_H+\tau_2)\tau_1+\sigma\tau_2]+\mu\tau_2}>0$$

$$\Rightarrow a_2>0$$

The conditions of stability: $a_0>0$ $a_2>0$ *et* $a_1a_2-a_0>0$ the Routh-Hurwitz criterion in the case of a polynomial of degree 3 are satisfied. then $\varepsilon_0^{H\tau}$ is locally asymptotically stable if $R_0^{H\tau}<1$ □

3.2. Global Stability of the Equilibrium without HIV/AIDS of Model (1)

Lemma 6. *The disease-free equilibrium (DFE) $\varepsilon_0^{H\tau}$ of model (1) is globally asymptotically stable when $R_0^{H\tau}<1$ and unstable if $R_0^{H\tau}>1$*

Proof. To apply the theorem of Castillo-Chavez (2002), we set:

$$\begin{cases}\dot{X}=F(X,Y)\\ \dot{Y}=G(X,Y), \qquad G(X,0)=0\end{cases} \tag{6}$$

Where

$$X=S\in\mathbb{R}_+ \ , \ Y=(H_1,H_2,W_H)\in\mathbb{R}_+^3$$

and

$$\varepsilon_0^{H\tau}=(X^*,0,0,0) \ \ with \ \ X^*=\frac{\Lambda}{\mu}$$

$$F(X,Y)=\big(\Lambda-\lambda_HS-\mu S\big) \quad \text{and} \quad G(X,Y)=\begin{pmatrix}\lambda_HS-(\mu+\sigma+\tau_1)H_1\\ \sigma H_1+\sigma\theta_tW_H-(\mu+\delta_H+\tau_2)H_2\\ \tau_1H_1+\tau_2H_2-(\mu+\sigma\theta)W_H\end{pmatrix}$$

As a result we have: $F(X,0)=\big(\Lambda-\mu S\big)$.

Check if the conditions C_1 and C_2 are satisfied:
•(C_1):$F(X^*,Y)=F(X^*,0)$,

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

•(C_2):$G(X,Y) = AY - \widehat{G}(X,Y)$, where

$$A = \begin{pmatrix} \beta_H - (\mu+\sigma+\tau_1) & \eta_2\beta_H & \eta_H\beta_H \\ \sigma & -(\mu+\delta_H+\tau_2) & \sigma\theta \\ \tau_1 & \tau_2 & -(\mu+\sigma\theta) \end{pmatrix},$$

$$\widehat{G}(X,Y) = \begin{pmatrix} \beta_H H_1 - \lambda_H S + \eta_2\beta_H H_2 + \eta_H\beta_H W_H \\ 0 \\ 0 \end{pmatrix}$$

By factoring the components of the column matrix $\widehat{G}(X,Y)$, we have

$$\widehat{G}(X,Y) = \begin{pmatrix} \beta_H(H_1 + \eta_2 H_2 + \eta_H W_H)(1-\frac{S}{N}) \\ 0 \\ 0 \end{pmatrix}$$

Setting

$$\widehat{G}_1(X,Y) = \beta_H(H_1 + \eta_2 H_2 + \eta_H W_H)(1-\frac{S}{N}), \quad \widehat{G}_2(X,Y) = 0 \ \ et \ \ \widehat{G}_3(X,Y) = 0$$

It follows that $\widehat{G}(X,Y) \geq 0$ for $(X,Y) \in \Omega_\varepsilon^{Hr}$.

Thus the conditions (C_1) *et* (C_2) are satisfied. Hence the disease-free equilibrium in the presence of treatment is globally asymptotically stable. □

3.3. Existence of Endemic Equilibrium

Proposition 7. *The endemic equilibrium of model (1) of the HIV/AIDS dynamic's in the presence of treatment is* $\varepsilon_{H\tau}^* = (S^*, H_1^*, H_2^*, W_H^*)$ *so that:*

$$\begin{cases} S^* = \dfrac{\Lambda}{\mu+\lambda_H^*} \\ H_1^* = \dfrac{\Lambda\lambda_H^*}{(\mu+\lambda_H^*)(\mu+\sigma+\tau_1)} \\ H_2^* = \dfrac{\sigma\Lambda\lambda_H^*(\mu+\sigma\theta+\theta\tau_1)}{(\mu+\lambda_H^*)(\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta)+\mu\tau_2])} \\ W_H^* = \dfrac{\Lambda\lambda_H^*[(\mu+\delta_H+\tau_2)\tau_1+\sigma\tau_2]}{(\mu+\lambda_H^*)(\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta)+\mu\tau_2]} \end{cases} \tag{7}$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

Proof. A point $\varepsilon^*_{H\tau} = (S^*, H^*_1, H^*_2, W^*_H)$ of $\Omega^{Hr}_{\varepsilon}$ is an endemic equilibrium solution if and only if the system:

$$\begin{cases} \Lambda - \lambda^*_H S^* - \mu S^* = 0 \\ \lambda^*_H S^* - (\mu + \sigma + \tau_1) H^*_1 = 0 \\ \sigma H^*_1 + \sigma\theta W^*_H - (\mu + \delta_H + \tau_2) H^*_2 = 0 \\ \tau_1 H^*_1 + \tau_2 H^*_2 - (\mu + \sigma\theta) W^*_H = 0 \end{cases} \tag{8}$$

Where:

$$\lambda^*_H = \beta_H \frac{H^*_1 + \eta_2 H^*_2 + \eta_H W^*_H}{N^*} \quad and \quad N^* = S^* + H^*_1 + H^*_2 + W^*_H \tag{9}$$

The first equation of system (8) gives:

$$S^* = \frac{\Lambda}{\mu + \lambda^*_H}$$

Consider the second equation of (8) and substituting S^* by its expression, we obtain H^*_1 given by:

$$H^*_1 = \frac{\Lambda \lambda^*_H}{(\mu + \lambda^*_H)(\mu + \sigma + \tau_1)}$$

Substituting H^*_1 by its expression above in the third and fourth equation of system (8), we find the following system of two equations:

$$\begin{cases} \dfrac{\sigma \Lambda \lambda^*_H}{(\mu + \lambda^*_H)(\mu + \sigma + \tau_1)} + \sigma\theta W^*_H - (\mu + \delta_H + \tau_2) H^*_2 = 0 \\ \dfrac{\tau_1 \Lambda \lambda^*_H}{(\mu + \lambda^*_H)(\mu + \sigma + \tau_1)} + \tau_2 H^*_2 - (\mu + \sigma\theta) W^*_H = 0 \end{cases} \tag{10}$$

Solving (10), allows us to obtain H^*_2 and W^*_H defined by:

$$H^*_2 = \frac{\sigma \Lambda \lambda^*_H (\mu + \sigma\theta + \theta\tau_1)}{(\mu + \lambda^*_H)(\mu + \sigma + \tau_1)[(\mu + \delta_H)(\mu + \sigma\theta) + \mu\tau_2])}$$

$$W^*_H = \frac{\Lambda \lambda^*_H [(\mu + \delta_H + \tau_2)\tau_1 + \sigma\tau_2]}{(\mu + \lambda^*_H)(\mu + \sigma + \tau_1)[(\mu + \delta_H)(\mu + \sigma\theta) + \mu\tau_2]}$$

Hence the results.

Whether $\varepsilon^*_{H\tau} = (S^*, H^*_1, H^*_2, W^*_H)$ considered as endemic equilibrium point of system (1). Injection of expressions $S^*, H^*_1, \quad H^*_2 \quad et \quad W^*_H$ in that of λ^*_H of the equation (9) allows us to obtain a quadratic equation in λ^*_H below:

$$\lambda^*_H (b_2 \lambda^*_H + b_1) = 0 \tag{11}$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

Where:

$$\begin{cases} b_2 = (\mu+\sigma\theta)(\mu+\sigma+\delta_H)+\tau_1(\mu+\delta_H+\sigma\theta)+\tau_2(\mu+\sigma+\tau_1), \\ b_1 = -\beta_H[(\mu+\delta_H)(\mu+\sigma\theta_t)+\sigma\eta_2[\mu+(\sigma+\tau_1)\theta] \\ +\eta_H\tau_1(\mu+\delta_H\tau_2)+\tau_2(\mu+\sigma\eta_H)]. \end{cases} \tag{12}$$

It is obvious that $b_1 < 0 \quad and\ b_2 > 0$ then according to the Descartes's rule of signs, equation (11) has a unique solution to this positive real part and parked the existence of this endemic equilibrium of the system.

Consider the second degree equation (11) verified by λ_H^* with coefficients as $b_2,\ b_1\ and\ b_0$ defined by the system (12) with $b_0 = 0$.

It was always $b_2 > 0$. and since $b_1 < 0$ and $b_0 = 0$, then equation (11) has a solution with zero real part and a solution to the positive real part . The solution to zero real part is associated with disease-free equilibrium $\varepsilon_0^{H\tau}$ of (1) and the other solution to the endemic equilibrium $\varepsilon_{H^*\tau}$. □

3.4. Analysis of the Bifurcation

For this analysis of bifurcation, we use the theorem of Castillo-Chavez and Song (2004):

Setting: $S = x_1,\ \ H_1 = x_2,\ \ H_2 = x_3\ \ et\ \ W_H^* = x_4$ and taking into account the expression of the parameter $\lambda_H = \beta_H \dfrac{H+\eta_2 H_2+\eta_H W_H}{N}$, we have

$N = x_1+x_2+x_3+x_4, \quad X = (x_1,x_2,x_3,x_4)^T, \quad and\ \ \frac{dX}{dt} = (g_1,g_2,g_3,g_4)$

Model (1) becomes as follows:

$$\begin{cases} \dot{x}_1 &= g_1 = \Lambda - \beta_H^* \dfrac{x_1x_2+\eta_2x_3x_1+\eta_Hx_4x_1}{x_1+x_2+x_3+x_4} - \mu.x_1 \\ \dot{x}_2 &= g_2 = \beta_H^* \dfrac{x_1x_2+\eta_2x_3x_1+\eta_Hx_4x_1}{x_1+x_2+x_3+x_4} - (\mu+\sigma+\tau_1)x_2 \\ \dot{x}_3 &= g_3 = \sigma x_2+\sigma\theta x_4-(\mu+\delta_H+\tau_2)x_3 \\ \dot{x}_4 &= g_4 = \tau_1x_2+\tau_2x_3-(\mu+\sigma\theta)x_4 \end{cases} \tag{13}$$

The Jacobian matrix in $\varepsilon_0^{H\tau}$ of system (13) is the same as (1) $J(\varepsilon_0^{H\tau})$ given by equation (4) . It also provides the same basic reproduction rate $R_0^{H\tau}$ as given by equation (3).

If we assume $\beta_H^* = \beta_H$ where β_H^* is a parameter of the bifurcation, its determination by resolution of $R_0^{H\tau} = 1$ gives:

$$\beta_H^* = \beta_H = \frac{(\mu+\sigma+\tau_1)[(\mu+\delta_H)(\mu+\sigma\theta)+\mu\tau_2]}{(\mu+\delta_H)(\mu+\sigma\theta+\eta_H\tau_1)+\sigma\eta_2(\mu+\sigma\theta+\theta\tau_1)+\eta_H(\mu+\tau_1)} \tag{14}$$

The coefficient of the bifurcation β_H^* can also be deduced from the Jacobian matrix of the linearized system of the model (1) around the equilibrium DFE $\varepsilon_0^{H\tau}$. The

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

Jacobian matrix $J(\ varepsilon_0^{H\ tau})$ see (4) of the system (1) linearized around disease free equilibrium
$x^* = x_{DFE} = \varepsilon_0^{H\tau} = (\frac{\Lambda}{\mu}, 0, 0, 0)$ when $\beta_H^* = \beta_H$ is given by:

$$J(x^*) = \begin{pmatrix} -\mu & -\beta_H^* & -\eta_2\beta_H^* & -\eta_H\beta_H^* \\ 0 & \beta_H^* - (\mu+\sigma+\tau_1) & \eta_2\beta_H^* & \eta_H\beta_H^* \\ 0 & \sigma & -(\mu+\delta_H+\tau_2) & \sigma\theta \\ 0 & \tau_1 & \tau_2 & -(\mu+\sigma\theta) \end{pmatrix} \tag{15}$$

It admits at least one simple eigenvalue with positive real part. This allows us then to use the technique of Castillo-Chavez and Song which the theorem is stated in the basic mathematical concepts. We have now to show that assertion A_1 of this theorem is true.

To verify the assertion A_2 Castillo-Chavez and Song theorem, we denote by $u = (u_1, u_2, u_3, u_4)^T$ the right eigenvector of the Jacobian matrix (15) associated with the eigenvalue zero. So we get:

$$\begin{pmatrix} -\mu & -\beta_H^* & -\eta_2\beta_H^* & -\eta_H\beta_H^* \\ 0 & \beta_H^* - (\mu+\sigma+\tau_1) & \eta_2\beta_H^* & \eta_H\beta_H^* \\ 0 & \sigma & -(\mu+\delta_H+\tau_2) & \sigma\theta \\ 0 & \tau_1 & \tau_2 & -(\mu+\sigma\theta) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{16}$$

Solving this matrix equation (16) gives:

$$u_1 = -\frac{\beta_H^*}{\mu}(u_2 + \eta_2 u_3 + \eta_H u_4),$$

$$u_2 = \frac{\beta_H^*}{(\mu+\sigma+\tau_1) - \beta_H^*}(\eta_2 u_3 + \eta_H u_4), \quad u_3 = u_3 > 0,$$

$$u_4 = \frac{\tau_1(\mu+\delta_H+\tau_2) + \sigma\tau_2}{\sigma(\mu+\sigma\theta+\theta\tau_1)} u_3.$$

The left eigenvector of the Jacobian matrix (15) associated with the zero eigenvalue when $\beta_H^* = \beta_H$ is $v = (v_1, v_2, v_3, v_4)^T$ so that:

$$\begin{pmatrix} v_1 & v_2 & v_3, v_4 \end{pmatrix} \begin{pmatrix} -\mu & -\beta_H^* & -\eta_2\beta_H^* & -\eta_H\beta_H^* \\ 0 & \beta_H^* - (\mu+\sigma+\tau_1) & \eta_2\beta_H^* & \eta_H\beta_H^* \\ 0 & \sigma & -(\mu+\delta_H+\tau_2) & \sigma\theta \\ 0 & \tau_1 & \tau_2 & -(\mu+\sigma\theta) \end{pmatrix}$$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}.$$

The matrix equation (17) gives the following resolution:

$$v_1 = 0, \quad v_2 = \frac{1}{(\mu+\sigma+\tau_1)-\beta_H^*}(\sigma v_3 + \tau_1 v_4), \quad v_3 = v_3,$$

$$v_4 = \frac{1}{\mu+\sigma\theta}(\beta_H^*\eta_2 v_2 + \sigma\theta v_3), \quad v_3 > 0$$

The calculations of the bifurcation coefficients a and b of the linearized system of (1) around equilibrium are based on the non-zero partial derivatives of g_i, $i = 1, 2, 3, 4.$ below the disease-free equilibrium x^*

$$\frac{\partial^2 g_2}{\partial x_2^2} = \frac{-2\beta_H^*\mu}{\Lambda}, \quad \frac{\partial^2 g_2}{\partial x_3^2} = \frac{-2\eta_2\beta_H^*\mu}{\Lambda}, \quad \frac{\partial^2 g_2}{\partial x_4^2} = \frac{-2\eta_H\beta_H^*\mu}{\Lambda},$$

$$\frac{\partial^2 g_2}{\partial x_2 \partial x_3} = \frac{-(1+\eta_2)\beta_H^*\mu}{\Lambda}, \quad \frac{\partial^2 g_2}{\partial x_2 \partial x_4} = \frac{-(1+\eta_H)\beta_H^*\mu}{\Lambda},$$

$$\frac{\partial^2 g_2}{\partial x_3 \partial x_4} = \frac{-(\eta_2+\eta_H\beta_H^*\mu}{\Lambda}$$

Using the above results in the expression of a in Castillo-Chavez and Song theorem, we obtain:

$$a = -\frac{2\mu\beta_H^*}{\Lambda}v_2[u_2^2 + \eta_2 u_3^2 + \eta_H u_4^2 + (1+\eta_2)u_2u_3 + (1+\eta_H)u_2u_4 + (\eta_2+\eta_H)u_3u_4]$$

$$= -\frac{2\mu\beta_H^*}{\Lambda}v_2[u_2(u_2+u_3+u_4) + (\eta_2 u_3 + \eta_H u_4)(u_2+u_3+u_4)]$$

After factorization, we finally get:

$$a = -\frac{2\mu\beta_H^*}{\Lambda}v_2(u_2+u_3+u_4)(u_2+\eta_2 u_3+\eta_H u_4) \tag{17}$$

It follows that: $a < 0$

To determine b, the expression of which is defined in the same theorem, we will use now the non-zero partial derivative below the $g_i \quad i = 1, 2, 3, 4.$ relatively at $\beta_H^* = \beta_H$:

$$\frac{\partial^2 g_2}{\partial x_2 \partial \beta_H^*} = 1, \qquad \frac{\partial^2 g_2}{\partial x_3 \partial \beta_H^*} = \eta_2, \qquad \frac{\partial^2 g_2}{\partial x_4 \partial \beta_H^*} = \eta_H$$

It follows that:

$$b = v_2(u_2 + \eta_2 u_3 + \eta_H u_4) > 0$$

The signs of the bifurcation coefficients a and b: $a < 0$ and $b > 0$ can be concluded in the light of Theorem Castillo-Chavez (2004) (iv) that the system (1) undergoes a bifurcation when $R_0^{H\tau} = 1$. So the study of bifurcation allows us to state the following result:

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

Proposition 8. *The model (2.18) of the transmission dynamics of HIV/AIDS-only in the presence of treatment, undergoes a bifurcation when the basic reproduction rate $R_0^{H\tau} = 1$. Which implies the existence of the unique endemic equilibrium locally asymptotically stable for $R_0^H = 1$.*

4. Numerical Simulations

For numerical simulations, we use the estimated parameters as Numerical simulation,

Parameters	values	references
Λ	50 000	[1]
μ	0.02	[1]
σ	1/33	[1]
c_2	1	[1]
θ	0.001	[1]
β_H	Variable	[1]
δ_H	0.01	[1]
η_2, η_H	1.2 0.001	[1]
τ_1, τ_2	Variables	[1]

Table 2: Numerical values of model parameters

according to the treatment, the basic reproduction rate $R_0^{H\tau}$ of the dynamics of HIV/AIDS is given below.

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

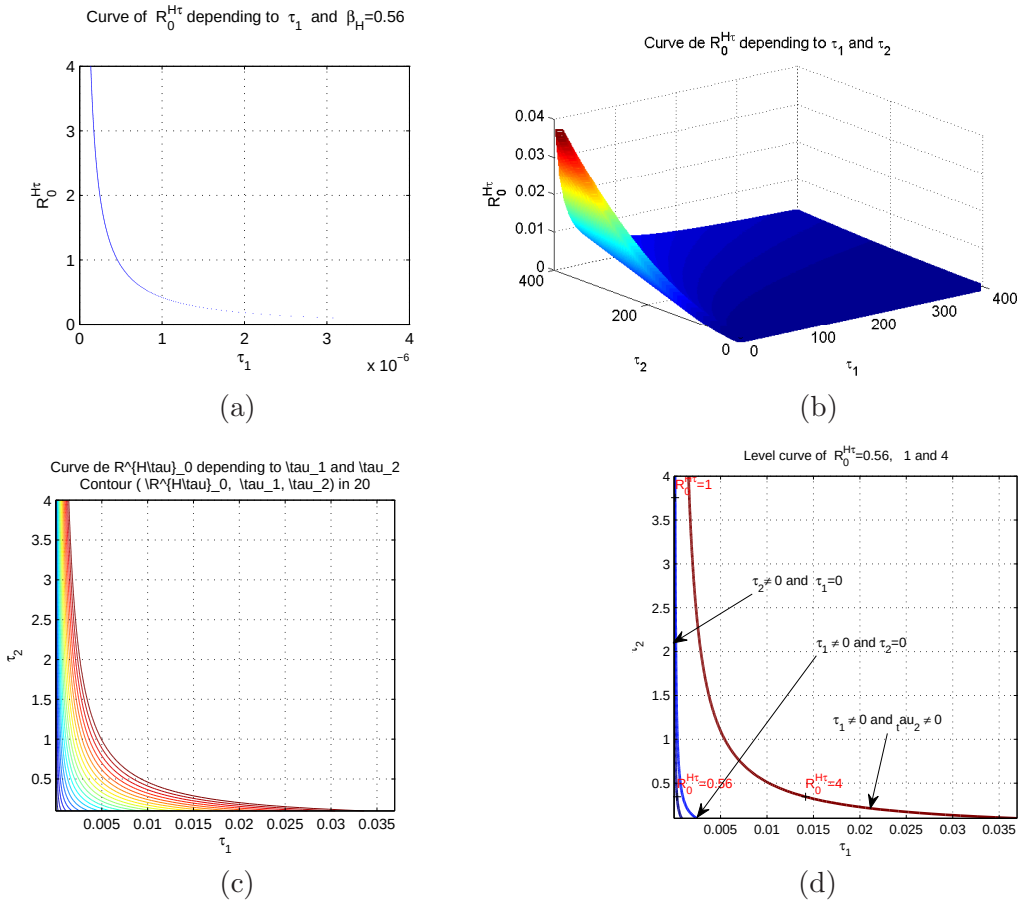


Figure 2: Curves of $R_0^{H\tau}$ depending to τ_1(a) and depending to τ_1 and τ_2 (b) ,(c): Contour$(R_0^{H\tau}, \tau_1, \tau_2)$ and (d): Level curve of $R_0^{T\tau} = (\tau_1, \tau_2) = C$ for $C = 0.56; 1$ *and* 4

We can observe on figures follow the trajectory and the successive trajectory of individuals of the dynamics HIV/AIDS-only model with treatment.

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

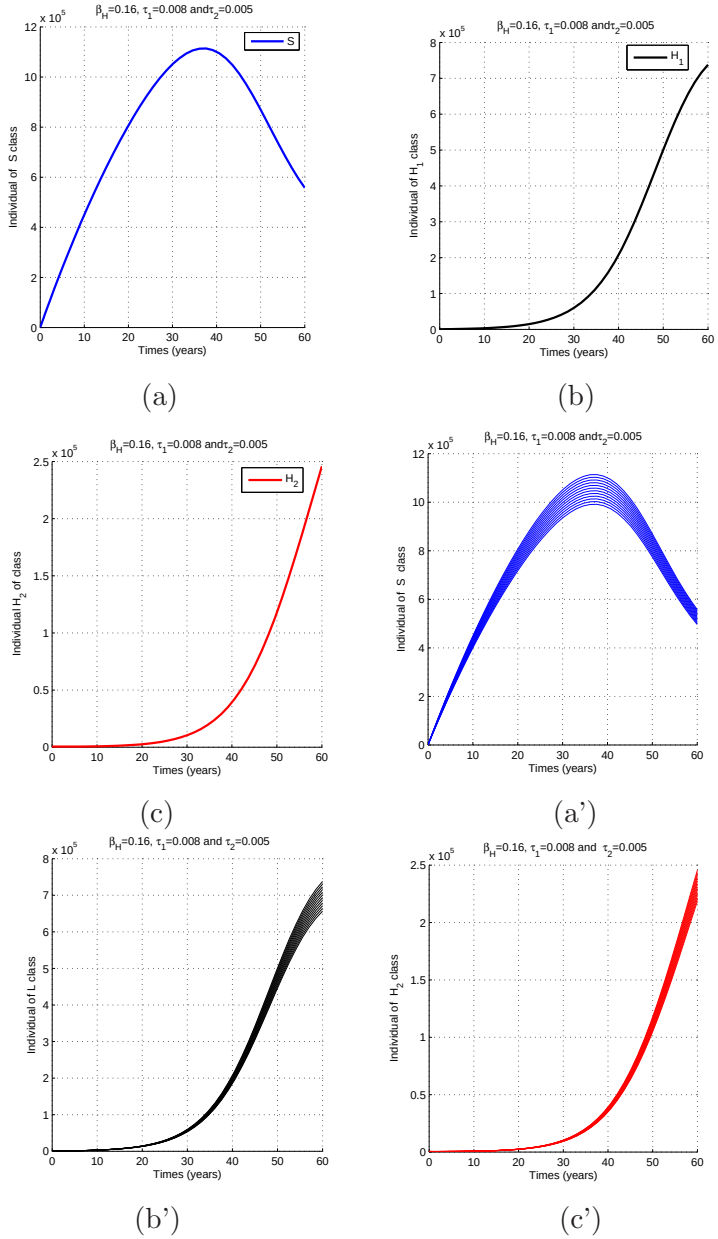
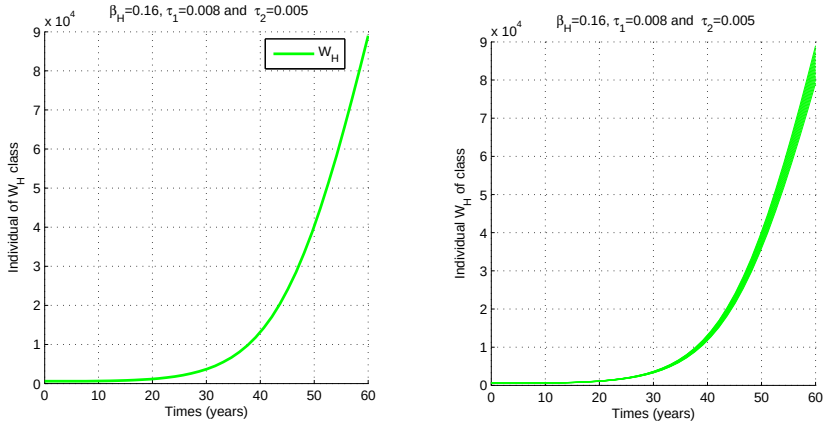


Figure 3: Simulation of the dynamics HIV/AIDS-only model with treatment. Trajectory of individuals for $\beta_H = 0.16$; $\tau_1 = 0.008$ and $\tau_2 = 0.005$

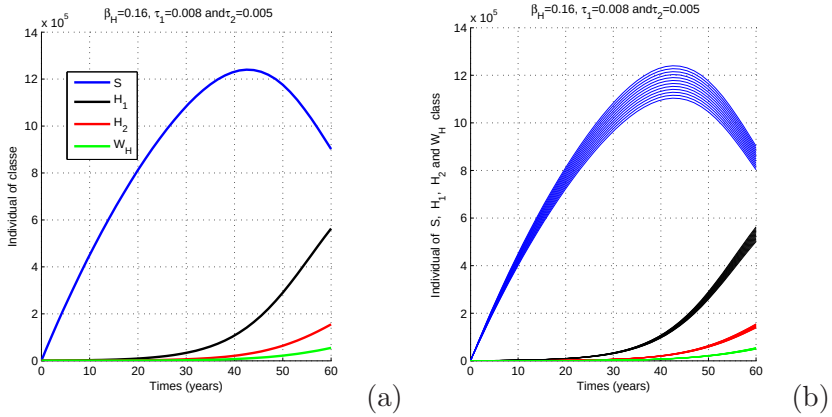
All these regrouped trajectories give two figures follow for the system of the

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)



(d) (d')

Figure 3: Continuation: Simulation of the dynamics HIV/AIDS-only model with treatment. Trajectory of individuals for $\beta_H = 0.16$; $\tau_1 = 0.008$ and $\tau_2 = 0.005$



(a) (b)

Figure 4: Simulation of the dynamics HIV/AIDS-only model with treatment. Trajectory (a) and successive trajectory (b) of individuals for β_H fixed one has the rate of treatment varied

HIV/AIDS-only model with treatment:

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

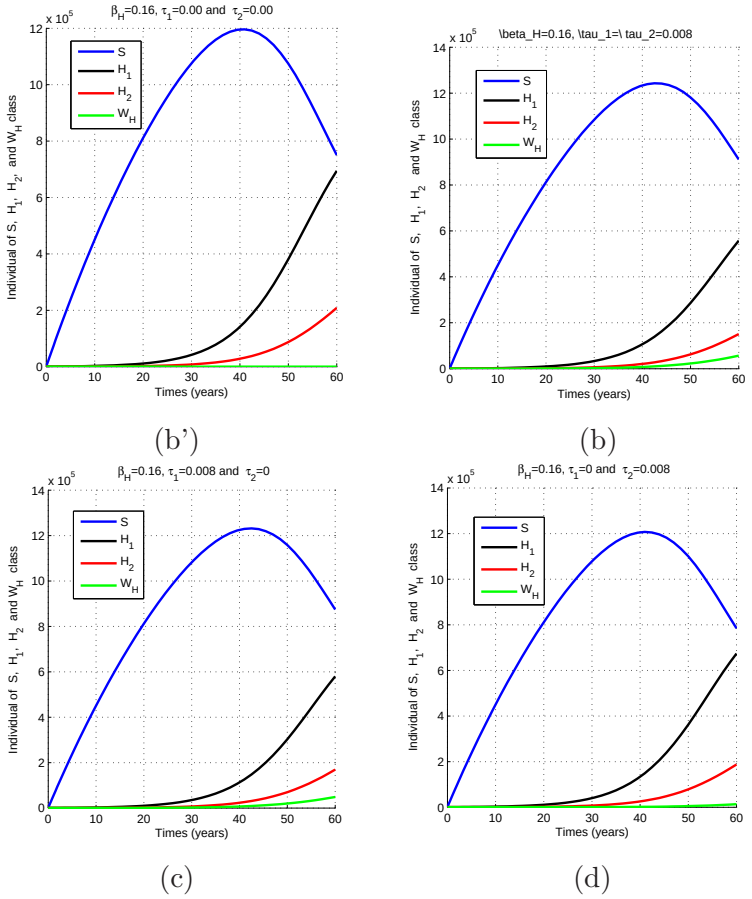


Figure 5: Simulation of the dynamics HIV/AIDS-only model with treatment.

(a): $\beta_H = 0.16,\ \tau_1 = \tau_2 = 0)$, (b): $\beta_H = 0.16,\ \tau_1 = \tau_2 = 0.008$
(c) : $\beta_H = 0.16,\ \tau_1 = 0/008\ and\ \tau_2 = 0$ (d): $\beta_H = 0.16,\ \tau_1 = 0\tau_2 = 0.008$

We observe changes of trajectories of the model of HIV/AIDS when one does the variation of values of treatment rates.

Simulations of the HIV/AIDS model with treatment of HIV-positive only (individuals (H_1)) without treating AIDS patients is to say, those of compartment (H_2)), give the curves (A), (B) and (C) below which show that for $(\tau_2 = 0)$ and $\beta_H = 0, 10$ estimated, the change in treatment rate (τ_1 from 0.01 to 0.08 is effectively reduce the number of infected with HIV/AIDS. By cons, if we take into account in the model (1), the treatment of AIDS only $(\tau_1 = 0)$, then for the same value of $\beta_H = 0.01$, the curves (D) (E) and (F) below show that an significant increase of (τ_2) from 0.01 more than 0.08 to reduce the number of infected.

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

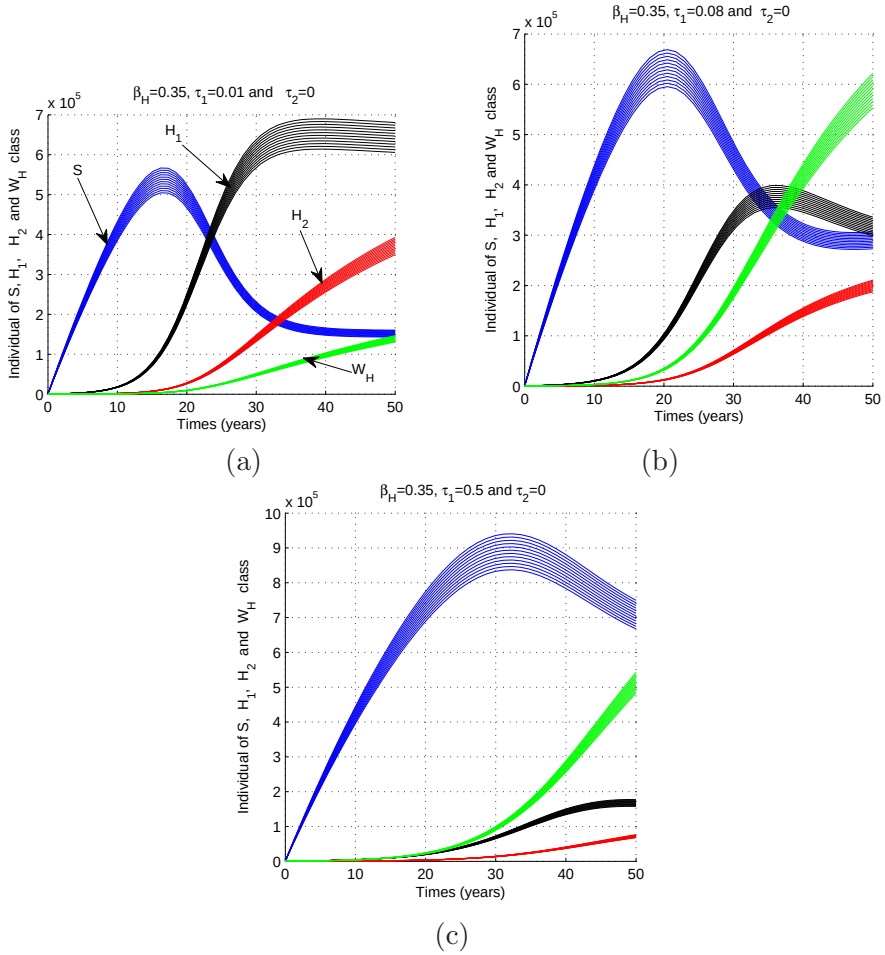


Figure 6: Simulation of the dynamics HIV/AIDS-only model with treatment . For β_H fixed ($\beta_H = 0.35$) and one has the rate of treatment varied (a): $\tau_1 = 0.01$ *and* $\tau_2 = 0$ (b): $\tau_1 = 0.0$ *and* $\tau_2 = 0$ (c): $\tau_1 = 0.5$ *and* $\tau_2 = 0$

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

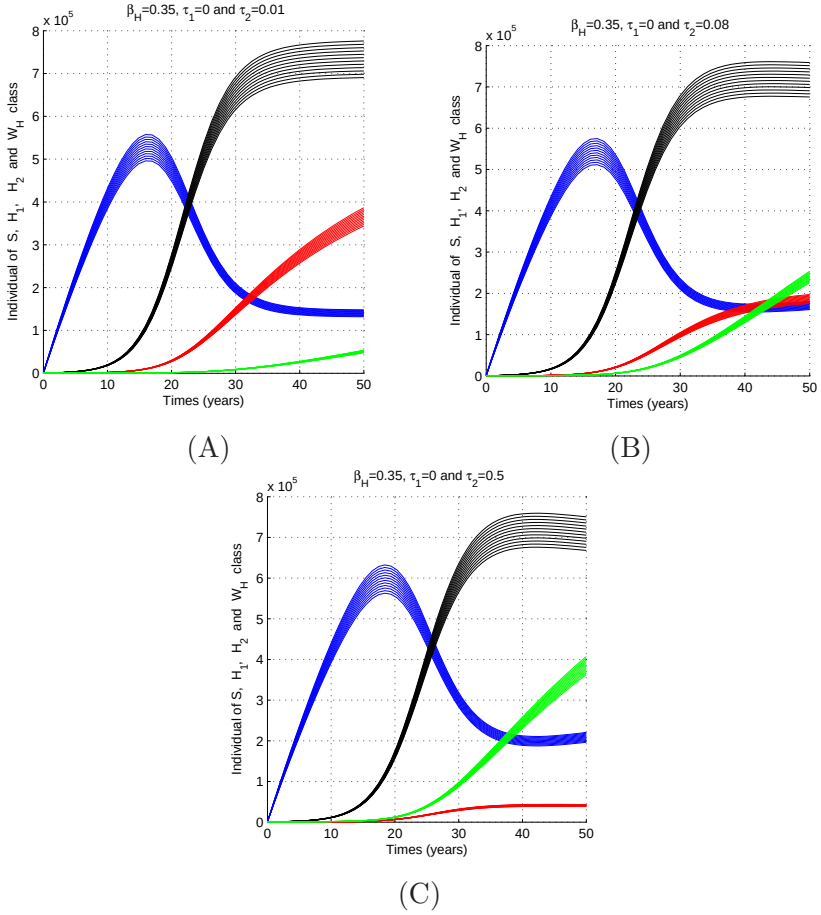


Figure 7: Simulation of the dynamics HIV/AIDS-only model with treatment.

For β_H fixed to 0.35 , $\tau_1 = 0$ and τ_2 variable.

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

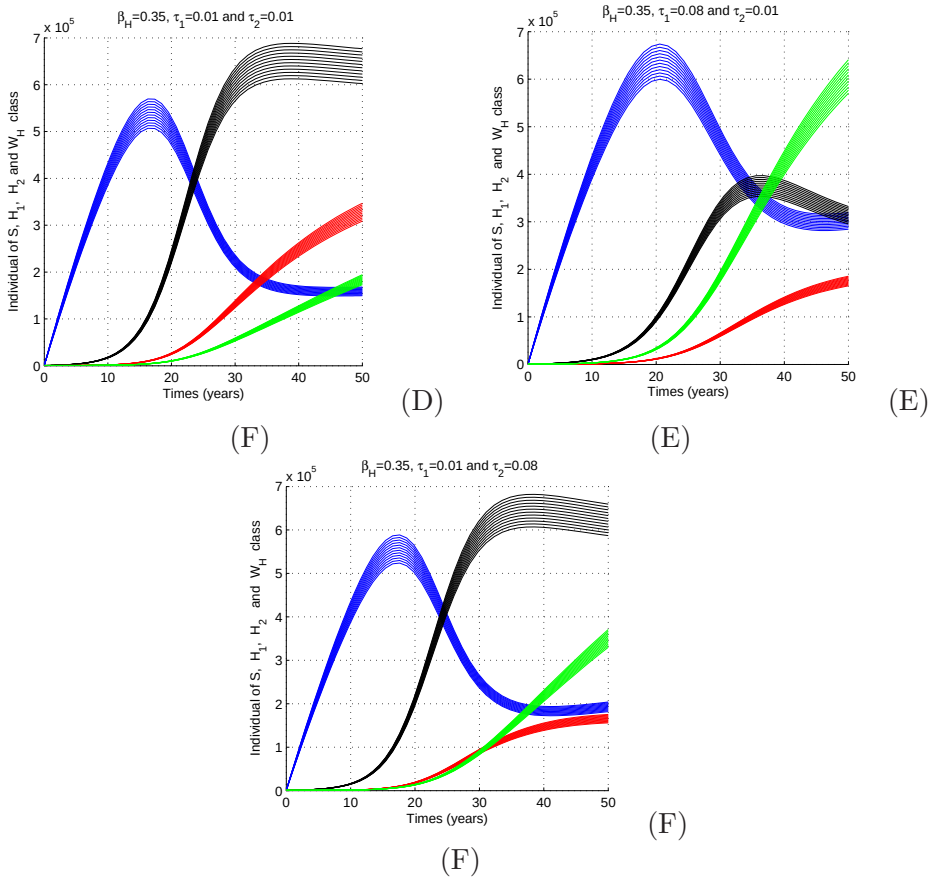


Figure 8: Simulation of the dynamics HIV/AIDS-only model with treatment.For β_H fixed to 0.35 , τ_1 and τ_2 variable.

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)

At the end of the numerical simulations of the dynamics of HIV/AIDS, we can say that a good strategy for treatment of infected HIV/AIDS requires support in advance of HIV: HIV-infected do not show and symptomatic signs of AIDS patients.

5. Conclusion

We have just made a detailed mathematical analysis of a model that describes the dynamics of infection HIV/AIDS. After having given basic properties of the model, the set limited of solutions and the initial conditions, we studied the dynamics of infection HIV/AIDS by means of some necessary mathematical tools and setting as Matlab, Maple. This model of the co- infection drifts a model very known model of co -infection tuberculosis - HIV/AIDS in presence of the treatment and already studied in article [1] of Gumel. We showed that it admits a equilibrium point, its basic reproduction rate that governs the stability of this equilibrium, is determined. While testing this model, with the realistic data, numerically one notes the impact of this infection on the propagation of HIV/AIDS creating a massive susceptible individual flux thus toward compartments of the tainted. The objective being to reduce mortality owed to bath illnesses in accordance with the national and international plan of struggles, we can make recourse to a means of control to know strategies of treatments of the illnesses to reduce the weight of this infection and to win in life expectancy of our populations. It is for that to make that we used in this paper, a model of infection in presence of the treatment that gives a good strategy of the treatment that consists in taking therapeutic in charge not only the tainted of HIV/AIDS, but also to found a control optima of treatment.

References

[1] O. Sharomi, C.N. Podder, A.B. Gumel et B. Song Mathematical Analysis of fhe Transmisssion Dynamics of HIV/TB Co-infection in the Presence of Treatment (20 september 2007 -1 january 2008)

[2] Birkhoff, G. and Rota, G. C. (1982). Ordinary differential equations. Needham Heights: Ginn.

[3] Castillo-Chavez, C., Feng, Z., and Huang, W(2002). On the computation of R0 and its role on global stability. math.la.asu.edu/chavez/2002/JB276.pdf

[4] P.Van Den Driesche et J. Watmough. 2002. Reproduction numbers and Subtreshold endemic equilibra for compartmental model of desease transmission. Math.Brosci

[5] C. Castillo-Chavez, Z. Feng, and W. Huang, " On the Compulation of R_0 and its role on global stability "

[6] D. Beroulli (1760)The mathematical modelling: tool of help to the decision in situation of uncertainty. Saint-Maurice(Fr) Sanitary eve institution, November 2008,

[7] Kermak et Mckendrick (1927). Surveillance et modelisation mathematique des maladies transmissibles. Antoine Flahault Iserm- Universite Pierre et Marie Curie UMR-S 707 16 juillet 2007

[8] Kirchner D. (1996) Using Mathematics to understand HIV immune Dynamics

[9] Samuel Bowong and Jean Jules Tewa. Parameter estimation and optimal control of the dynamics of transmission of tuberculosis with application to Cameroon 2010.

[10] C.P. Bhunu S. Mushayabasa J.M. Tchuenche: A Theoretical Assessment of the Effects of Smoking on the Transmission Dynamics of Tuberculosis Received: 2 February 2010 / Accepted: 22 June 2010 / Published: 20 August 2010 by Society for Mathematical Biology 2010

[11] Lavigne, M., Rocher, I., Steensma, C., Brassard, P. (2006). The impact of smoking on adherence to treatment for latent tuberculosis infection. BMC Public Health, 6(66).

[12] Berman. A, Plemmons, RJ:Nonegative matrices in the mathematical sciences. SIAM (Rev.35, 43-79(1993)

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 4, No. 4 (2012)