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USE OF MATHEMATICA PLATFORM FOR THE COMPUTATION $R^i_{j\ kl}$ OF CURVATURE TENSOR OF RANDERS METRIC ON SPHERE S^3

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Abstract: The article illustrates the computation of curvature tensor of randers metric on sphere S^3 using the facilities of *Mathematica* platform.

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1. Introduction

The purpose of this paper is to give components of Curvature Tensor of Randers metric in shortest time. We done this using directly the facilities *Mathematica*, see [2]. After a short introduction on coefficients of the Chern connection, Levi-Civita connection and coefficients of the Curvature Tensor in Section 2, see [1], We shall concentrate in Section 3 to the preparing the commands of *Mathematica* for computation components of Curvature Tensor of Randers metric on sphere S^3.

2. The Components of the Curvature Tensor of Randers Metric on S^3

We consider the sphere 3 of radius 1 in E^3:

$$S^3 = \{(x_1, x_2, x_3) | x_1^2 + x_2^2 + x_3^2 = 1\}.$$

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We parameterize S^3 as

$$(x_1, x_2, x_3) \longrightarrow (\varepsilon, x_1, x_2, x_3)/\sqrt{1 + x_1^2 + x_2^2 + x_3^2},$$

with $\varepsilon = \pm 1$.

A basis of $T_x S^3$ is as follows:

$$h_1 = \frac{(-\varepsilon x_1, 1 + x_2^2 + x_3^2, -x_1 x_2, -x_1 x_3)}{A^3},$$

$$h_2 = \frac{(-\varepsilon x_2, -x_1 x_2, 1 + x_1^2 + x_3^2, -x_2 x_3)}{A^3},$$

$$h_3 = \frac{(-\varepsilon x_3, -x_1 x_3, -x_2 x_3, 1 + x_1^2 + x_2^2)}{A^3},$$

where

$$A := \sqrt{1 + x_1^2 + x_2^2 + x_3^2}. \tag{1}$$

The induced metric g has the matrix:

$$\frac{1}{A^4} \begin{pmatrix} 1 + x_2^2 + x_3^2 & -x_1 x_2 & -x_1 x_3 \\ -x_2 x_1 & 1 + x_1^2 + x_3^2 & -x_2 x_3 \\ -x_1 x_3 & -x_2 x_3 & 1 + x_1^2 + x_2^2 \end{pmatrix} \tag{2}$$

The curvature tensor of randers metric has the following formula:

$$R^i_{j\ kl} = \frac{\delta \Gamma^i_{jl}}{\delta x^k} - \frac{\delta \Gamma^i_{jk}}{\delta x^l} + \Gamma^i_{hk} \Gamma^h_{jl} - \Gamma^i_{hl} \Gamma^h_{jk}, \tag{3}$$

where

$$\frac{\delta}{\delta x_j} = \frac{\partial}{\partial x_j} - N^i_j \frac{\partial}{\partial y_i}$$

$$N^i_j = \gamma^i_{jk} y_k - C^i_{jk} \gamma^k_{rs} y_r y_s \tag{4}$$

The coefficients of the Cartan connection as follows:

$$C^i_{jk} = \frac{A^i_{jk}}{F}$$

$$A^i_{jk} = \frac{F}{4} (F^2)_{y_i y_j y_k} \tag{5}$$

and the Levi-Civita coefficience is given by:

$$\gamma^i_{jk} = \frac{g^{is}}{2} \left(\frac{\partial g_{sj}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^s} + \frac{\partial g_{ks}}{\partial x^j} \right) \tag{6}$$

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We denote by (y_1, y_2, y_3) the components of an arbitrary tangent vector in the basis (h_1, h_2, h_3). The Randers function $F = \alpha + \beta$ for $\varepsilon = 1$ has the form:

$$F(x_1, x_2, x_3; y_1, y_2, y_3) =$$
$$\sqrt{(A^2 - x_1^2)y_1^2 + (A^2 - x_2^2)y_2^2 + (A^2 - x_3^2)y_3^2 - 2x_1x_2y_1y_2 - 2x_1x_3y_1y_3 - 2x_2x_3y_2y_3}$$
$$+\frac{(x_3A^2 - 2(x_1x_3 + x_2)x_1)y_1}{A} + \frac{(A^2 - 2(x_1x_3 + x_2)x_2)y_2}{A} + \frac{(x_1A^2 - 2(x_1x_3 + x_2)x_3)y_3}{A}.$$

Recall that $A = \sqrt{1 + x_1^2 + x_2^2 + x_3^2}$ (for details see [3]).

The coefficients of the Chern connection is defined as the following form:

$$\Gamma^i_{jk} = \frac{g^{is}}{2}(\frac{\delta g_{sj}}{\delta x^k} - \frac{\delta g_{jk}}{\delta x^s} + \frac{\delta g_{ks}}{\delta x^j}). \quad (7)$$

Here

$$g_{ij} = (\frac{F^2}{2})_{y_iy_j} = FF_{y_iy_j} + F_{y_i}F_{y_j}. \quad (8)$$

3. The Commands of Mathematica

We need to introduction index i, j, k, l for obtain components of Curvature Tensor of Randers metric. Then by *Mathematica* commands we have:

```
i = Input["i"]; j = Input["j"]; k = Input["k"]; l = Input["l"];
```

By the formula (1) we have:

```
a = \[Sqrt](1 + x[1]^2 + x[2]^2 + x[3]^2);
```

We input the Randers function by the *Mathematica* command as follows:

```
f =     (1/
         a^2)*(\[Sqrt]((a^2 - x[1]^2) y[1]^2 + (a^2 - x[2]^2)
         y[2]^2 + (a^2 - x[3]^2) y[3]^2 - 2 x[1]*x[2]*y[1]*y[2] -
         2 x[1]*x[3]*y[1]*y[3] - 2 x[2]*x[3]*y[2]*y[3])) +
         1/a ((x[3]*a^2 - 2 (x[1]*x[3] + x[2]) x[1])
         y[1] + (a^2 - 2 (x[1]*x[3] + x[2]) x[2])
         y[2] + (x[1]*a^2 - 2 (x[1]*x[3] + x[2]) x[3]) y[3]);
```

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By the formula (2), (8) and *Mathematica* commands we have:

```
g0[1, 1] = D[(f^2)/2, y[1], y[1]];
 g0[1, 2] = D[(f^2)/2, y[1],y[2]];
g0[1, 3] = D[(f^2)/2, y[1], y[3]];
 g0[2, 1] = D[(f^2)/2, y[2],y[1]];
g0[2, 2] = D[(f^2)/2, y[2], y[2]];
 g0[2, 3] = D[(f^2)/2,y[2],y[3]];
g0[3, 1] = D[(f^2)/2, y[3], y[1]];
 g0[3, 2] =D[(f^2)/2, y[3], y[2]];
g0[3, 3] = D[(f^2)/2, y[3], y[3]];
 Table[g0[i, j], {i, 3}, {j, 3}];
MatrixForm[%];
```

and

```
g1 = Inverse[Table[g0[i, j], {i, 3}, {j, 3}]];

g = (1/a^2)*{{1 + x[2]^2 + x[3]^2, -x[1]*x[2], -x[1]*x[3]}, {-x[1]*
      x[2], 1 + x[1]^2 + x[3]^2, -x[2]*x[3]}, {-x[3]*x[1], -x[3]*x[2],
      1 + x[1]^2 + x[2]^2}};
g2 = Inverse[g];
```

By the formula (5) we have:

```
b[i_, j_, k_] = (f/4)*D[f^2, y[i], y[j], y[k]]; c[i_, j_, k_] = b[i,
j, k]/f;
```

Moreover by the (6), (4), (7) and (3) respectively and *Mathematica* commands we have:

```
\[Gamma][i_, j_, k_] =
       \!\(\*UnderoverscriptBox[\(\[Sum]\), \(s =
       1\), \(3\)]\(\(\((g2[\([\)\(i, s\)\(]\)]/
       2)\)*\((D[g[\([\)\(s, j\)\(]\)], x[k]] -
       D[g[\([\)\(j, k\)\(]\)], x[s]] +
       D[g[\([\)\(k, s\)\(]\)], x[j]])\))\)\);


n[i_, j_] =
     \[Gamma][i, j, k]*y[k] - \!\(
     \*UnderoverscriptBox[\(\[Sum]\), \(r = 1\), \(3\)]\(
     \*UnderoverscriptBox[\(\[Sum]\), \(s =
```

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```
    1\), \(3\)]\((c[i, j, k]*\[Gamma][k, r, s]*y[r]*y[s])\)\)\);


\[CapitalGamma][i_, j_, k_] =
        \!\(\*UnderoverscriptBox[\(\[Sum]\), \(s =
        1\), \(3\)]\((\((g1[i, s]/
        2)\)*\((\((D[g0[s, j], x[k]] -
        n[i, k]*D[g0[s, j], y[i]])\) - \((D[g0[j, k], x[s]] -
        n[i, s]*D[g0[j, k], y[i]])\) + \((D[g0[k, s], x[j]] -
        n[i, j]*D[g0[k, s], y[i]])\))\))\)\);

R[i_, j_, k_, l_] =
      (D[\[CapitalGamma][i, j, l], x[k]] -
       n[i, k]*D[\[CapitalGamma][i, j, l], y[i]]) - (D[\[CapitalGamma][i,
       j, k], x[l]] -
       n[i, l]*D[\[CapitalGamma][i, j, k], y[i]]) + (\!\(
       \*UnderoverscriptBox[\(\[Sum]\), \(h = 1\), \(3\)]\[CapitalGamma][i,
       h, k]*\[CapitalGamma][h, j, l]\) - \!\(
       \*UnderoverscriptBox[\(\[Sum]\), \(h = 1\), \(3\)]\[CapitalGamma][i,
       h, l]*\[CapitalGamma][h, j, k]\))
```

4. Conclusion

In this paper after introduction the forms for obtain components of Randers Curvature, we use this forms for sphere S^3. we use the *Mathematica* platform to calculate the forms that obtain from the introductions.Because we discussion on sphere S^3, this platform with giving particular index and calculate needful formulas, finally obtains the components of Randers Curvature on sphere S^3.

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