International Electronic Journal of Pure and Applied Mathematics

Volume 4 No. 2 2012, 79-82

ON PARTIAL BILATERAL AND IMPROPER PARTIAL BILATERAL GENARATING FUNCTIONS INVOLVING SOME BASIC CLASSICAL POPLYNOMIALS

Kishan Sharma

Department of Mathematics
NRI Institute of Technology and Management
Baraghata, Next to S.G. Motors
Jhansi Road, Gwalior-474001, INDIA

Abstract: In the present paper a group theoretic method is used to obtain more general class of generating functions from a given class of improper partial bilateral generating functions involving q-Hermite and q-Gegenbauer polynomials.

AMS Subject Classification: 44A60, 33A65
Key Words: bilateral generating function, q-Hermite polynomial, q-Gegenbauer polynomial

1. Introduction

The usual generating relation involving one basic classical function may be called linear or unilateral generating relation. By the term proper/bilateral generating function we mean a function which can be expanded in the powers of w in the following form

$$G(x, z, q, w) = \Sigma_{n=0}^{\infty} a_n f_n(q; x) g_n(q; z) w^n, \tag{1.1}$$

where is arbitrary that is independent of x and z and and are two different basic classical functions. In particular, when two basic classical functions are same that is, we call the generating relation as bilinear relation.

Definition 1.1 By the term proper partial bilateral generating relation for two basic classical polynomials, we mean the relation:

$$G(x, z, q, w) = \Sigma_{n=0}^{\infty} a_n w^n p_{m+n}^{(\alpha)}(q; x) q_{m+n}^{(\beta)}(q; z), \tag{1.2}$$

where the coefficients are quite arbitrary and are any two basic classical polynomials of order, and of parameters and respectively.

Received: June 4, 2011

2. Main Results

In this section, we will derive more general class of generating functions from a given class of improper partial bilateral generating functions involving q-Gegenbauer and q-Hermite polynomials.

Theorem 2.1. *If there exist the following class of improper partial bilateral generating functions for the q-Hermite and q-Gegenbauer polynomials by means of the relation*

$$G(x,z,q,w)=\Sigma_{n=0}^{\infty}a_n w^n H_{m+n}(q;x)C_{k+n}^{(\alpha)}(q;z) \tag{2.1}$$

where a_n is arbitrary, then the following general class of generating functions hold:

$$\exp_q(2wx-w^2)(1-2vz+v^2)^{-\alpha-k/2}G(x-w,\frac{z-v}{(1-2vz+v^2)^{1/2}},q,\frac{wv}{(1-2vz+v^2)^{1/2}})$$
$$=\Sigma_{n,r,s=0}^{\infty}a_n w^{n+s}v^{n+r}\frac{(k+n+1;q)_r}{s!r!}(H_{m+n+s}(q;x))(C_{k+n+r}^{(\alpha)}(q;z)), \tag{2.2}$$

where $\|2vz-v^2|<1, 0<|q|<1$.

Proof. Using the definitions of the q-Hermite and q-Gegenbauer polynomials (see [1]), multiplying both sides of (2.1) by $y^m t^k$ and replacing w by $wvyt$, we get

$$y^m t^k G(x,z,q,wvyt)=\Sigma_{n=0}^{\infty}a_n(wv)^n(H_{m+n}(q;x)y^{m+n})(C_{k+n}^{(\alpha)}(q;z)t^{k+n}). \tag{2.3}$$

Operators of one-parameter groups have been defined by Erdelyi et al [2] and McBride [3]. We extend the same technique to define the following operators

$$\begin{aligned}E_1 =&2xy-yB_{q,x},\\ E_2 =&(z^2-1)tB_{q,z}+zt^2B_{q,t}+(2\alpha+k)zt,\end{aligned} \tag{2.4}$$

where $B_{q,x}$ is the basic differential operator given by [1], with

$$\begin{aligned}E_1(H_{m+n}(q;x)y^{m+n}) =&H_{m+n+1}(q;x)y^{m+n+1},\\ E_2(C_{k+n}^{(\alpha)}(q;z)t^{k+n}) =&[k+n+1;q]C_{k+n+1}^{(\alpha)}(q;z)t^{k+n+1},\end{aligned} \tag{2.5}$$

The actions of one-parameter subgroups $exp_q(wE_1)$ and $exp_q(vE_2)$ on $f(x,y,q)$ and $f(z,t,q)$, respectively are given similarly to [4]:

$$\begin{aligned}exp_q(wE_1)(f(x,y,q)) =&exp_q(2wx-w^2y^2)(f(x-wy,y,q)),\\ exp_q(wE_2)(f(z,t,q)) =&exp_q(1-2vzt+v^2t^2)^{-\alpha}\\ &\times(f(\frac{z-vt}{(1-2vzt+v^2t^2)^{1/2}},\frac{t}{(1-2vzt+v^2t^2)^{1/2}},q)),\end{aligned} \tag{2.6}$$

where $\|2vzt-v^2t^2|<1$.

Now, on operating both sides of (2.3) by $exp_q(wE_1)exp_q(vE_2)$ and as a result of it, the relation(2.3) becomes

$$\exp_q(2wxy - w^2y^2)(1-2vzt+v^2t^2)^{-\alpha}y^m(\frac{t}{(1-2vzt+v^2t^2)^{1/2}})^k G(x-wy, \frac{z-vt}{(1-2vzt+v^2t^2)^{1/2}}, q, \frac{wvyt}{(1-2vzt+v^2t^2)^{1/2}}) = \Sigma_{n,r,s=0}^{\infty} a_n w^{n+s} v^{n+r} \frac{(k+n+1;q)_r}{s!r!} (H_{m+n+s}(q;x)y^{m+n+s})(C_{k+n+r}^{(\alpha)}(q;z)t^{k+n+r}). \tag{2.7}$$

Putting $y = t = 1$ in the above equation, we get

$$\exp_q(2wx-w^2)(1-2vz+v^2)^{-\alpha-k/2}G(x-w, \frac{z-v}{(1-2vz+v^2)^{1/2}}, q, \frac{wv}{(1-2vz+v^2)^{1/2}}) = \Sigma_{n,r,s=0}^{\infty} a_n w^{n+s} v^{n+r} \frac{(k+n+1;q)_r}{s!r!} (H_{m+n+s}(q;x))(C_{k+n+r}^{(\alpha)}(q;z)), \tag{2.8}$$

where $G(x,z,q,w)$ is given by (2.1).

It may be of interest to point out that for, the above theorem become nice general class of generating functions from the given class of proper partial bilateral generating functions, which need not be derived independently. We state that result in the form of the following theorem:

Theorem 2.2. *If there exist the following class of proper partial bilateral generating functions for the q-Hermite and q-Laguerre polynomials by means of the relation*

$$G(x,z,q,w) = \Sigma_{n=0}^{\infty} a_n w^n H_{m+n}(q;x) C_{m+n}^{(\alpha)}(q;z), \tag{2.9}$$

where a_n is arbitrary, then the following general class of generating functions hold:

$$\exp_q(2wx-w^2)(1-2vz+v^2)^{-\alpha-m/2}G(x-w, \frac{z-v}{(1-2vz+v^2)^{1/2}}, q, \frac{wv}{(1-2vz+v^2)^{1/2}}) = \Sigma_{n,r,s=0}^{\infty} a_n w^{n+s} v^{n+r} \frac{(m+n+1;q)_r}{s!r!} (H_{m+n+s}(q;x))(C_{m+n+r}^{(\alpha)}(q;z)), \tag{2.10}$$

where $\|2vz - v^2| < 1$, $0 < |q| < 1$.

If we take limit in Theorems 2.1 and 2.2, we get the generating relations, see [5].

Acknowledgments

The author is very thankful to the referees for giving several valuable suggestions in the presentation of the paper.

References

[1] H. Exton, *q-Hypergeometric Functions and Applications*, Ellis Horwood Limited, John Wiley and Sons, New York (1983).

[2] A. Erdely et al., *Higher Transcendental Functions*, Volume II, McGraw Hill Book Cor. Inc., New York (1953).

[3] E.B. McBride, *Obtaining Generating Functions*, SpringerVerlag, New York, Berlin (1971).

[4] W. Jr. Miller, *Lie Theory and Special Functions*, Academic Press, New York (1968).

[5] A.K. Sarkar, On partial bilateral and improper partial bilateral generating functions involving some special functions, *Filomat* (Nis), **18** (2004), 41-49.