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ON TWO PERSON ZERO-SUM FUZZY GAME WITH TOPSIS RANKING PROCEDURE

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Abstract: In this paper, TOPSIS for fuzzy data has been extended to determine the most preferable choice among all possible choices in the multi criteria decision making problem. Here, fuzzy numbers are involved in the TOPSIS procedure to rank the alternatives. The proposed method is applied to a two person zero sum game to analyze the existence of equilibrium which gives the player's choice of optimal strategies.

1. Introduction

A Game is a decision making situation with many players, having objectives that partly or completely conflict with each other. The attitude of the players such as moral or philosophical motives, which are important in taking the optimal decision, are better modeled using fuzzy set theory. The fuzzy approach introduced by L.A. Zadeh [10] is effective for modeling such multi criteria conflicting situations, since it works well with vague, ambiguous, imprecise, noisy and missing information. Using the notion of fuzzy sets, the payoff function in a game can also be fuzzified.

Fuzzy numbers are widely used in the research on fuzzy set theory [5], particularly in decision making problems. In order to identify the preference ranking of the fuzzy numbers, one fuzzy number needs to be evaluated and compared with the other numbers, which is not easy in practical situations. It is not easy to determine whether a fuzzy number is larger or smaller than another since they are represented by possibility distributions and they can overlap with each other.

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Technique for order performance by similarly to ideal solution(TOPSIS) [4], one of known classical MCDM methods, was first developed by Jwang and Yoon[3] for evolving a MCDM problem. In the process of TOPSIS, the performance ratings and the weights of the criteria are given as exact values. In this paper, TOPSIS is applied to rank the choice of alternatives of each player of a two person zero sum games. Using the multi criteria decision making [1,2], we analyse the existence of equilibrium which gives the player's choice of optimal strategies is found.

2. Preliminaries

2.1. Definition

A trapezoidal fuzzy number $\tilde{a} \approx (a_1, a_2, a_3, a_4)$is defined by the membership function,

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & if \quad a_1 \le x \le a_2; \\ 1, & if \quad a_2 \le x \le a_3; \\ \frac{x-a_4}{a_3-a_4}, & if \quad a_3 \le x \le a_4; \\ 0, & otherwise. \end{cases}$$

2.2. Operations on Fuzzy Numbers

For $\tilde{a} \approx (a_1, a_2, a_3, a_4)$ and $\tilde{b} \approx (b_1, b_2, b_3, b_4)$ the following operations are defined.

Addition: $\tilde{a} \oplus \tilde{b} \approx (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$.

Subtraction: $\tilde{a} \ominus \tilde{b} \approx (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$.

Multiplication:

$$\tilde{a} \odot \tilde{b} \approx \begin{cases} \left(\frac{a_1(b_1+b_2+b_3+b_4)}{4}, \frac{a_2(b_1+b_2+b_3+b_4)}{4}, \frac{a_3(b_1+b_2+b_3+b_4)}{4}, \frac{a_4(b_1+b_2+b_3+b_4)}{4}\right), & if \quad \tilde{b} \ge \tilde{0}; \\ \left(\frac{a_4(b_1+b_2+b_3+b_4)}{4}, \frac{a_3(b_1+b_2+b_3+b_4)}{4}, \frac{a_2(b_1+b_2+b_3+b_4)}{4}, \frac{a_1(b_1+b_2+b_3+b_4)}{4}\right), & if \quad \tilde{b} \le \tilde{0}; \end{cases}$$

Scalar Multiplication: If $k \neq 0$ is a scalar, $k\,\tilde{a}$ is defined as,

$$k\tilde{a} \approx \begin{cases} (ka_1, ka_2, ka_3, ka_4), & if \quad k > 0; \\ (ka_4, ka_3, ka_2, ka_1), & if \quad k < 0; \end{cases}$$

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3. TOPSIS Procedure

Step 1: For $\tilde{a} \approx (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number such that its ranking function $R(\tilde{a}) = \frac{a_1+a_2+a_3+a_4}{4}$.
Step 2: Normalize each fuzzy number$\tilde{A}_i$ into $\tilde{A}_i^N$ such that $\tilde{A}_i^N \approx (\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \frac{a_4}{k})$ where k denotes the maximum value of the universe of discourse. Let the normalized decision matrix be $\tilde{N}$.
Step 3: Find the weight of each fuzzy Number as follows: Consider each fuzzy number (a_1, a_2, a_3, a_4) as the vertices of a trapezium given by $[(a_1, 0), (a_2, 0), (a_3, 0), (a_4, 0)]$ and find the area of each trapezium. Divide the area of each trapezoidal fuzzy number by the maximum area among all the areas to fix the weight of each fuzzy number [7]. Associate a linguistic variable vector to each weight vector to form, the weighted matrix $\tilde{W} of \tilde{N}$.
Step 4: Calculate the weighted normalized decision matrix. The weighted normalized value

$$[\tilde{V}_{ij}] \approx \tilde{V} \approx \tilde{N} \odot \tilde{W}, i = 1, 2, \quad \cdot;\cdot \quad m \quad and \quad j = 1, 2, \quad \cdot;\cdot \quad n$$

.
Step 5: Let the positive ideal and negative ideal solution be $\tilde{I}^+ \approx (1, 1, 1, 1)$ and $\tilde{I}^- \approx (0, 0, 0, 0)$
Step 6: Calculate the separation measures $\tilde{d}_{ij}^+, \tilde{d}_{ij}^-$ using the distance function defined as follows: If $\tilde{a}$ *and* $\tilde{b}$are any two trapezoidal fuzzy numbers, the distance between them is given by

$$\tilde{d}(\tilde{a}, \tilde{b}) = \frac{1}{2} Max[| a_1 - a_2 | + | d_1 - d_2 |, | b_1 - b_2 | + | c_1 - c_2 |]$$

Step 7: Calculate the coefficient of closeness to the ideal solution. The closeness coefficient to the alternative$\tilde{A}_{ij}$is defined as

$$cc_{ij} = \frac{\tilde{d}_{ij}^-}{\tilde{d}_{ij}^+ + \tilde{d}_{ij}^-}$$

. Since $\tilde{d}_{ij} \geq 0\, clearly\, cc_{ij} \in [0, 1]$.
Step 8: According to closeness coefficient, the alternatives are ranked in the descending order and the best alternative is the one with the longest distance to the fuzzy positive ideal solution and with the shortest distance to the fuzzy negative ideal solution.

4. Numerical Example

Step 1: Consider the following fuzzy payoff matrix of a two person zero-sum game.

$$\tilde{A} \approx \begin{pmatrix} (2,4,6,8) & (0,2,4,6) & (1,5,9,13) \\ (1,5,9,13) & (3,7,11,15) & (0.25,0.75,1.25,1.75) \\ (7,9,11,13) & (3,5,7,9) & (0.5,1.5,2.5,3.5) \end{pmatrix}$$

Step 2: The corresponding normalized fuzzy decision matrix is given by,

$$\tilde{N} \approx \begin{pmatrix} (0.13,0.26,0.4,0.5) & (0,0.13,0.26,0.4) & (0.06,0.33,0.6,0.87) \\ (0.06,0.3,0.6,0.87) & (0.2,0.47,0.7,1) & (0.02,0.05,0.08,0.12) \\ (0.47,0.6,0.7,0.87) & (0.2,0.3,0.47,0.6) & (0.03,0.1,0.17,0.2) \end{pmatrix}$$

Step 3: The associated weighted fuzzy matrix using the linguistic variables is given by

$$\tilde{W} \approx \begin{pmatrix} (0.15,0.25,0.35,0.5) & (0.15,0.25,0.35,0.5) & (0.2,0.3,0.5,0.7) \\ (0.2,0.3,0.5,0.7) & (0.75,0.85,0.9,1) & (0,0,0,0.1) \\ (0.15,0.25,0.35,0.5) & (0.3,0.5,0.7,0.9) & (0,0,0,0.1) \end{pmatrix}$$

Step 4: The weighted normalized matrix $\tilde{V}$ is given by,

$$\tilde{V} \approx \tilde{N} \odot \tilde{W}$$
$$\approx \begin{pmatrix} (0.06,0.23,0.42,0.6) & (0.186,0.54,0.94,1.37) & (0.08,0.15,0.2,0.26) \\ (0.12,0.13,0.51,0.74) & (0.21,0.53,0.86,1.22) & (0.04,0.14,0.28,0.4) \\ (0.25,0.35,0.47,0.59) & (0.35,0.51,0.73,0.92) & (0.21,0.27,0.32,0.4) \end{pmatrix}$$

Step 5: Let the positive ideal and negative ideal solution be $\tilde{I}^+ \approx (1,1,1,1)$ and $\tilde{I}^- \approx (0,0,0,0)$

Step 6: Then the values of $\tilde{d}_{ij}^+$ and $\tilde{d}_{ij}^-$ can be found using $\tilde{d}_{ij}^+ = \tilde{d}(\tilde{a}_{ij}, \tilde{I}^+)$ and $\tilde{d}_{ij}^- = \tilde{d}(\tilde{a}_{ij}, \tilde{I}^-)$.

The values are as follows:

$\tilde{d}_{ij}$	$\tilde{a}_{11}$	$\tilde{a}_{12}$	$\tilde{a}_{13}$	$\tilde{a}_{21}$	$\tilde{a}_{22}$	$\tilde{a}_{23}$	$\tilde{a}_{31}$	$\tilde{a}_{32}$	$\tilde{a}_{33}$
$\tilde{d}_{ij}^+$	0.675	0.592	0.83	0.68	0.505	0.79	0.59	0.38	0.705
$\tilde{d}_{ij}^-$	0.33	0.778	0.175	0.43	0.715	0.22	0.42	0.635	0.305

Step 7: The corresponding closeness coefficients are given below:

$$\begin{array}{lll} cc_{11} = 0.328 & cc_{12} = 0.568 & cc_{13} = 0.174 \\ cc_{21} = 0.387 & cc_{22} = 0.586 & cc_{23} = 0.218 \\ cc_{31} = 0.416 & cc_{32} = 0.626 & cc_{33} = 0.302 \end{array}$$

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Step 8: Then the ranking order of the alternatives according to the closeness coefficient is given by,
$\tilde{A}_{32} \prec \tilde{A}_{22} \prec \tilde{A}_{12} \prec \tilde{A}_{31} \prec \tilde{A}_{21} \prec \tilde{A}_{11} \prec \tilde{A}_{33} \prec \tilde{A}_{23} \prec \tilde{A}_{13}$.
Hence the best alternative for player I is $\tilde{A}_{13}$ .Obviously, the best alternative for player II is $\tilde{A}_{32}$.

5. Conclision

In this paper, we have applied the TOPSIS method to rank the alternatives in a decision making problem, with fuzzy payoffs, aiming to identify the optimal strategies in the game. This method shows good results which are discriminative and close to the intuition. The same method can be applied to identify the optimal strategies in fuzzy Bi-matrix games also.

References

[1] R.E. Bellman, L.A. Zadeh, Decision making in a fuzzy environment, *Management Science*, **17**, No. 4 (1970), B-141-B-164.

[2] A. Billot, *Economic Theory of Fuzzy Equilibria: An Axiomatic Analysis*, New York, Springer-Verlag (1992).

[3] C.L. Hwang, K. Yoon, *Multiple Attribute Decision Making Methods and Applications*, Springer , Berlin Heidelberg (1981).

[4] Y.J. Lai, T.Y. Liu, C.L. Hwang, TOPSIS for MCDM, *European Journal of Operational Research*, **76**, No. 3 (1994), 486-500.

[5] H. Lee-Kwang, Lee Jee-Hyong, The method of ranking fuzzy numbers and its application to decision making, *IEEE Transactions on Fuzzy Systems*, **07**, No.6, 677-685.

[6] Martin Shubik, *The Uses and Methods of Game Theory*, New York, American Elsvier (1957).

[7] Paul R. Thie, *An Introduction to Linear Programming and Game Theory*, John Wiley and Sons, New York (1988).

[8] Prasun K. Nayak, Madhumangal Pal, Solutions of $m \times n$ rectangular fuzzy games by dominance.

[9] J. Van Neumann, O. Margenstern, *Theory of Games and Economic Behaviour*, Princeton, N.J. Princeton University Press (1944).

[10] L.A. Zadeh, Fuzzy sets, *Inform. and Control.*, **8** (1965), 338-352.

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