

**A REVISIT INTO THE INVERSE RELATION FOR
THE PARABOLA AND AN UNDERSTANDING BETWEEN
GIVEN AND INVERSE NOTATIONS – A FEW KEY POINTS**

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Within a classroom of learners, often, several methodologies and formulae are encountered and used to solve and plot a quadratic equation as a parabolic function. These methods are well understood and have been the core of most mathematics texts. However, the theoretical understanding of truly translating and evaluating a quadratic into the inverse often causes problems to most learners especially when not asked to interpret the given notation, say $f(x)$, $g(x)$, etc. This review reiterates some important key identifiers when solving parabolic functions.

Suppose we are given the relation $y = x^2$ and asked to define the domain and range of this relation and then to subsequently find the same for the inverse. The first step would not necessarily be to find its inverse because pertinent information about the inverse can be derived from the information obtained from the evaluation of the normal function. However, this is optional to the learner. Because the relation is a squared function, the domain would be real because all values of x would be squared and thus positive. Henceforth, the range would be 0 for the x value because the parabolic function would lie midpoint at point zero on the Cartesian plane, but would be positively infinite because the function is above the x axis, i.e. $[0; \infty)$.

In this case since the function is simple, the area of symmetry would be 0 for x and the turning point $(0; 0)$. From the information obtained about the normal function, the domain, range, axes of symmetry and turning points of the inverse function can be ascertained from it. This is because the domain of the normal relation becomes the range of the inverse function and hence, the range of the normal relation its inverse. An alternative well-known method of facilitating the understanding of the inverse relation, would be to obtain the inverse of the function, all x 's must be replaced by y 's and all y 's by x 's in the given notation. Therefore, the equation, domain, range, axis of symmetry and turning points of the inverse function, in that order, would be: $x = y^2$ (or $y = \pm\sqrt{x}$ or $\pm\sqrt{x} = y$), $[0; \infty)$, Real for all values of $x, y = 0, (0; 0)$.

The relation $y = x^2$ is also referred to as a function. Interestingly though, the reverse i.e. $x^2 = y$ is not noted as a function. An example that can be used to explain this discrepancy would be to find the inverse of $f : x \rightarrow (x - 5)^2 - 7$ in the form $f^{-1} = \{(x; y) : y = \dots\}$, where y is the subject of the formula (hence this is only an example that is in the form of $f : x \rightarrow (x - 1)^2$).

The inverse of $y = (x - 5)^2 - 7$, is

$$x = (y - 5)^2 - 7,$$

i.e.

$$x + 7 = (y - 5)^2,$$

i.e.

$$\pm\sqrt{x+7} = y - 5 \quad \text{or} \quad y = 5 \pm \sqrt{x+7}.$$

Therefore, $f^{-1} = \{(x; y) : y = 5 \pm \sqrt{x+7}\}$. At this point, learners should note that it is not correct to write $f : x \rightarrow 5 \pm \sqrt{x+7}$, because only the notation $f : x$ is used for functions and $y = 5 \pm \sqrt{x+7}$ is certainly not a function.

In conclusion, understanding the difference between normal and inverse notations is an important concept that needs to be mastered by learners. More importantly though, learners should be able to obtain information regarding inverse functions without wasting time translating whole given relations into their inverses. I consider this revisitation into the inverse of the parabola salient to all high school and university students because it forms the basis of understanding more multifaceted aspects of mathematics and mathematical literacy.