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A COMMON FIXED POINT THEOREM FOR A FAMILY OF HYBRID PAIRS OF MAPPINGS IN FUZZY METRIC SPACES

Amit Singh[1 §], Sandeep Bhatt[2]
[1,2]Department of Mathematics
H.N.B. Garhwal University
P.P. Box 100, Srinagar (Garhwal)
Uttarakhand, 246174, INDIA
[1]e-mail: singhamit841@gmail.com
[2]e-mail: bhattsandeep1982@gmail.com

Abstract: In the present paper we prove a unique common fixed point theorem for a family of hybrid pairs of mappings in fuzzy metric spaces by using the notion of weakly compatibility of mappings. Our result generalizes and extends some well known previous results.

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1. Introduction

In 1965, Zadeh [29] introduced the concept of fuzzy sets. Since then, by using this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. The concept of fuzzy metric space has been introduced and generalized in many ways ([5], [17]). George and Veeramani [10] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [18]. They also obtained a Hausdroff topology for this kind of fuzzy metric space which has very important applications in quantum particle physics particularly in connections with both string and E-infinity theory which were given and studied by El. Naschie [6, 7, 8, 9]. Many authors have proved fixed point theorems in fuzzy metric spaces (see, for instance, [11], [19], [22] and references thereof).

Hybrid fixed point theory for nonlinear single-valued and multivalued mappings is a new development in multivalued analysis (see, for instance, [1],[2], [3], [12], [16], [20], [21], [23], [24], [25], [26], [27], [28] and references thereof). Coincidence and fixed

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§Correspondence author

point theorems for hybrid mappings may be used to single functional inclusions and optimization problems (see, for instance, [4] and [17]).

In this paper, we establish a unique fixed point theorem for a family of hybrid pairs of mappings by using the concept of weakly compatible maps introduced by Jungck and Rhoades [14] which is more general than compatibility of maps. Our result extends and generalize the main result of Sedghi and Shobe [22] and others.

2. Preliminaries

Definition 1. A binary operation $* : [0,1]\times[0,1] \to [0,1]$ is continuous t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,

(ii) $*$ is continuous,

(iii) a $*$ 1 = a for all a $\in$ [0, 1],

(iv) $a * b \le c * d$ whenever a $\le$ c and b $\le$ d, for each a, b, c, d $\in$ [0, 1].

Two typical examples of continuous t-norm are a $*$ b = ab and a $*$ b = min (a, b).

Definition 2. (see [18]) A 3-tuple (X, M, $*$) is called a fuzzy metric space if X is an arbitrary nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions:

(i) M(x, y, t) $>$ 0,

(ii) M(x, y, t) = 1 if and only if x = y,

(iii) M(x, y, t) = M(y, x, t),

(iv) M(x, y, t) $*$ M(y, z, s) $\le$ M(x, z, t+s),

(v) M(x, y, $\cdot$) : [0, ∞) $\to$ [0, 1] is continuous, for each x, y, z $\in$ X and s, t $>$ 0.

Note that, M(x, y, t) can be thought as the definition of nearness between x and y with respect to t. It is known that M(x, y, $\cdot$) is nondecreasing for all x, y $\in$ X, see [10].

Let (X, M, $*$) be a fuzzy metric space. For t $>$ 0, the open ball B(x, r, t) with center x $\in$ X and radius $0 < r < 1$ is defined by

$$B(x,r,t) = \{y \in X : M(x,y,t) > 1-r\}.$$

The collection $\{B(x,r,t) : x \in X, 0 < r < 1, t > 0\}$ is a neighbourhood system for a topology τ on X induced by the fuzzy metric M. This topology is Hausdroff and first countable.

Definition 3. (see [10]) Let (X, M, $*$) be a fuzzy metric space. Then:

(i) a sequence $\{x_n\}$ in X is said to be convergent x in X if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x, t) \geq 1 - \epsilon$ for all $n \geq n_0$.

(ii) a sequence $\{x_n\}$ in X is said to be Cauchy if for each $\epsilon > 0$ and each t > 0, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all n, m $\geq n_0$.

(iii) A fuzzy metric space (X, M, $*$) is said to be complete if every Cauchy sequence is convergent. A subset A of X is said to be F-bounded if there exists t > 0 and $0 < r < 1$ such that M(x, y, t) > 1 - r for all x, y $\in$ A.

Lemma 4. (see [10]) *Let (X, M, $*$) be a fuzzy metric space. Then M(x, y, t) is non-decreasing with respect to t, for all x, y in X.*

Example 5. (Induced Fuzzy Metric, see [10]) Let (X, d) be a metric space. Denote $a * b = ab$ for all a, b $\in$ [0, 1] and let M_d be fuzzy set on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}.$$

Then (X, M_d, $*$) is a fuzzy metric space. We call this fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Example 6. (see [10]) Let X = N. Define a $*$ b = max $\{0, a + b - 1\}$ for all a, b $\in$ [0, 1] and let M be fuzzy set on $X^2 \times (0, \infty)$ as follows:

$$M(x, y, t) = \begin{cases} \frac{x}{y}, & \text{if } x \geq y, \\ \frac{y}{x}, & \text{if } y \geq x, \end{cases}$$

for all x, y $\in$ X and t > 0. Then (X, M, $*$) is a fuzzy metric space. Note that, in above example, there exists no metric d on X satisfying

$$M(x, y, t) = \frac{t}{t + d(x, y)}.$$

where M(x, y, t) is defined as in above example. Also note the above function M is not a fuzzy metric with the t-norm defined as $a * b = min\{a, b\}$.

Definition 7. Let (X, M, $*$) be a fuzzy metric space. M is said to be continuous on $X^2 \times (0, \infty)$ if

$$\lim_{n \to \infty} M(x_n, y_n, t_n) = M(x, y, t).$$

Whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0, \infty)$ converges to a point (x, y, t) $\in$ $X^2 \times (0, \infty)$ i.e.

$$\lim_{n \to \infty} M(x_n, x, t) = \lim_{n \to \infty} M(y_n, y, t) = 1 \text{ and } \lim_{n \to \infty} M(x, y, t_n) = M(x, y, t).$$

Lemma 8. (see [10]) *Let (X, M, $*$) be a fuzzy metric space. Then M is continuous function on $X^2 \times (0, \infty)$.*

Lemma 9. (see [22]) *Let (X, M, $*$) be a fuzzy metric space. If we define $E_{\lambda,M} : X^2 \to R^+ \cup \{0\}$ by*

$$E_{\lambda,M}(x, y) = \ inf\{t > 0 : M(x, y, t) > 1 - \lambda\}$$

(i) for each $\mu \in (0, 1)$ there exists $\lambda \in (0, 1)$ such that

$$E_{\mu,M}(x_1, x_n) \leq E_{\mu,M}(x_1, x_2) + E_{\mu,M}(x_2, x_3) + ... + E_{\mu,M}(x_{n-1}), x_n)$$

for any $x_1, x_2, ..., x_n \in X$.

(ii) The sequence $\{x_n\}_{n \in N}$ is convergent in fuzzy metric space (X, M, $*$) if and only if $E_{\lambda,M}(x_n, x) \to 0$. Also the sequence $\{x_n\}_{n \in N}$ is Cauchy sequence if and only if it is Cauchy with $E_{\lambda,M}$.

Lemma 10. (see [22]) *Let (X, M, $*$) be a fuzzy metic space. If*

$$M(x_n, x_{n+1}, t) \geq M(x_0, x_1, k^n t)$$

for some k > 1 and for every $n \in N$. Then the sequenc $\{x_n\}$ is a Cauchy sequence.

Definition 11. (see [14]) Let (X. M, $*$) be a fuzzy metric space. Then the mappings I: X $\to$ X and F : X $\to$ B(X) are said to be weakly compatible if they commute at their coincidence points, i.e., for each point u in X such that Fu = $\{Iu\}$, we have FIu = IFu. (note that the equation Fu = $\{Iu\}$ implies that Fu is a singleton set.

Throughout this paper, B(X) is the set of all empty bounded subsets of X. For every t > 0 let $\delta(A, B, t)$ be a function defined by

$$\delta(A, B, t) = \ inf\ \{M(a, b, t) : a \in A, b \in B\}.$$

If A consists of a single point a, we write $\delta(A, B, t) = \delta(a, B, t)$. If B is also consists of a single point b, we write $\delta(A, B, t) = M(a, b, t)$. It follows immediately from the definition that

$$\begin{aligned} \delta(A, B, t) &= \delta(B, A, t) \geq 0, \\ \delta(A, B, t + s) &\geq \delta(A, C, t) * \delta(C, B, t), \\ \delta(A, B, t) &= 1 \Longleftrightarrow A = B = \{a\}, \end{aligned}$$

for all A, B, C in B(X).

Lemma 12. *Let (X, M, $*$) be a fuzzy metric space. Then $\delta(A, b, t) \geq \delta(A, B, kt)$, for all A, B in B(X) with some k > 1.*

Lemma 13. *Let $\{A_n\}$ be a sequence in B(X) and y is a point in X such that $\delta(A_n, y, t) \to 1$. Then the sequence $\{A_n\}$ converges to the set $\{y\}$ in B(X).*

3. Main Result

Theorem 14. *Let (X, M, $*$) be a complete fuzzy metric space.Let $\{F_n\}_{n=1}^{\infty}$: X $\to$ B(X) and S, T : X $\to$ X satisfying:*

(i) $F_i(X) \subseteq T(X)$, $F_j(X) \subseteq S(X)$ for every x $\in$ X,

(ii) the pairs $\{F_i, S\}$ and $\{F_j, T\}$ are weakly compatible,

(iii) let ϕ : $[0,1]^3 \to [0,1]$ is a continuous function and increasing in any coordinate and ϕ(t, t, t) > t for every t $\in$ [0, 1),

(iv) $\delta(F_i(x), F_j(y), t) \geq \phi(M(Sx, Ty, kt), \delta(F_i(x), Sx, kt), \delta(F_j(y), Ty, kt))$, for every x, y in X, i = 2n - 1, j = 2n, (n $\in$ N), i $\neq$ j and some k > 1. Suppose that one of S(X) or T(X) is a closed subset of X, then there exists a unique point p $\in$ X such that $\{p\} = \{Sp\} = \{Tp\} = F_n(p)$.

Proof. Let x_0 be an arbitrary point in X. Since $F_i(X) \subseteq T(X)$, we choose a point x_1 in X such that $Tx_1 \in F_1x_0 = Y_0$. Also $F_j(X) \subseteq S(X)$, for this point x_1 there exists a point x_2 in X such that $Sx_2 \in F_2x_1 = Y_1$ and so on. Continuing in this manner, we can define a sequence $\{Y_n\}$ as follows

$$\begin{aligned} Y_{2n} &= Tx_{2n+1} \in F_{2n+1}(x_{2n}), \\ Y_{2n+1} &= Sx_{2n+2} \in F_{2n}(x_{2n+1}), \end{aligned}$$

for n = 0, 1, 2,... . For simplicity we put $V_n(t) = \delta(Y_n, Y_{n+1}, t)$, for n = 0, 1, 2,... . We prove that sequence $\{V_n(t)\}$ is an increasing and convergent to 1. Since

$$\begin{aligned} V_{2n}(t) &= \delta(Y_{2n}, Y_{2n+1}, t) \\ &= \delta(F_{2n+1}(x_{2n}), F_{2n}(x_{2n+1}), t) \\ &\geq \phi\{M(Sx_{2n}, Tx_{2n+1}, kt), \delta(F_{2n+1}(x_{2n}), Sx_{2n}, kt), \delta(F_{2n}(x_{2n+1}), \\ &\quad Tx_{2n+1}, kt)\} \\ &\geq \phi\{\delta(F_{2n}(x_{2n-1}), F_{2n+1}(x_{2n}), kt), \delta(F_{2n+1}(x_{2n}), F_{2n}(x_{2n-1}), kt), \\ &\quad \delta(F_{2n}(x_{2n+1}), F_{2n+1}(x_{2n}), kt)\} \\ &= \phi\{\delta(Y_{2n-1}, Y_{2n}, kt), \delta(Y_{2n}, Y_{2n-1}, kt), \delta(Y_{2n+1}, Y_{2n}, kt)\} \\ &= \phi\{V_{2n-1}(kt), V_{2n-1}(kt), V_{2n}(kt)\}. \end{aligned}$$

We prove that $V_{2n}(kt) \geq V_{2n-1}(kt)$. Now, if $V_{2n}(kt) < V_{2n-1}(kt)$ for some n $\in$ N, since ϕ is an increasing function, then by the above inequality we have

$$V_{2n}(t) \geq \phi(V_{2n}(kt), V_{2n}(kt), V_{2n}(kt) > V_{2n}(kt)$$

implying that $V_{2n}(t) > V_{2n}(kt)$, which is a contradiction by Lemma 12. Hence $V_{2n}(kt) \geq V_{2n-1}(kt)$. Similarly, we can show that

$$V_{2n+1}(kt) \geq V_{2n}(kt).$$

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Then we deduce that

$$V_0(t) \le V_1(t) \le V_2(t) \le \ldots .$$

Thus $\{V_n(t)\}$ is increasing sequence in [0, 1]. Therefore, it tends to a limit $a \le 1$. We claim that a = 1. If a < 1 and on making $n \to \infty$ in the following inequality,

$$\delta(Y_{2n}, Y_{2n+1}, t) = V_{2n}(t) \ge \phi(V_{2n-1}(kt), V_{2n-1}(kt), V_{2n}(kt)),$$

we get

$$a \ge \phi(a, a, a) > a, \ \textit{is a contradiction}.$$

Hence a = 1, i.e.,

$$V_n(t) = \delta(Y_n, Y_{n+1}) \to 1.$$

It is easily seen that

$$V_n(t) \ge V_{n-1}(kt) \ge \ldots V_0(k^n t) = \delta(Y_0, Y_1, k^n t).$$

If y_n is an arbitrary point in the set Y_n, then by the above inequality, it follows that

$$M(y_n, y_{n+1}, t) \ge M(y_0', y_1', k^n t)$$

for n = 0, 1, 2, . . ., $y_0' \in Y_0$ and $y_1' \in Y_1$. Thus for some $a \in A$ and $b \in B$ if we get

$$M(a, b, t) \ge \ inf \ \{M(x, y, t); x, y \in X\}.$$

Then $M(y_n, y_{n+1}, t) \ge M(a, b, k^n t)$. By Lemma 10, the sequence $\{y_n\}$ is a Cauchy sequence in X. Hence any sequence of $\{Y_n\}$ is a Cauchy sequence in X.

Suppose that T(X) is complete. Since $Tx_{2n+1} \in F_{2n+1}(x_{2n}) = Y_{2n}$ for n = 0, 1, 2, ... Therefore by the above inequality, the sequence $\{Tx_{2n+1}\}$ is Cauchy and hence $Tx_{2n+1} \to p = Tv \in T(X)$ for some v ∈ X. Also $Sx_{2n} \in F_{2n}(x_{2n-1}) = Y_{2n-1}$, then

$$M(Sx_{2n}, Tx_{2n+1}, t) \ge \delta(Y_{2n-1}, Y_{2n}, t) = V_{2n-1}(t) \to 1.$$

Consequently, $Sx_{2n} \to p$. Moreover, for n = 1, 2, 3, . . . we have

$$\begin{aligned} \delta(F_{2n+1}(x_{2n}), p, t) &\ge \delta(F_{2n+1}(x_{2n}), Sx_{2n}, \frac{t}{2}) * \delta(Sx_{2n}, p, \frac{t}{2}) \\ &\ge \delta(Y_{2n}, Y_{2n-1}, \frac{t}{2}) * \delta(Sx_{2n}, p, \frac{t}{2}). \end{aligned}$$

Therefore, $\lim_{n\to\infty} \delta(F_{2n+1}(x_{2n}), p, t) = 1 * 1 = 1$. Hence by Lemma 13, it follows that $\lim_{n\to\infty} F_{2n+1}(x_{2n}) = \{p\}$. In the same manner we can show that $\delta(F_{2n}(x_{2n-1}), p, t) \to 1$.

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Since for n = 1, 2, 3,..., we have

$$\delta(F_i(x_{2n}), F_j v, t) \geq \phi(M(Sx_{2n}, Tv, kt), \delta(Sx_{2n}, F_i(x_{2n}), kt), \delta(Tv, F_j v, kt)).$$

On letting $n \to \infty$ and if $F_j v \neq p$, then

$$\begin{aligned} \delta(p, F_j v, t) &\geq \phi(M(p, Tv, kt), \delta(p, p, kt), \delta(Tv, F_j, kt)) \\ &> \delta(p, F_j v, kt). \end{aligned}$$

Hence $\delta(p, F_j v, t) > \delta(p, F_j v, kt)$, which is a contradiction by Lemma 12. Thus $F_j v = \{p\} = \{Tv\}$. Since $F_j(X) \subseteq S(X)$, there exists a point $u \in X$ such that $\{Su\} = F_j v = \{Tv\}$. Now if $F_i u \neq F_j v$, then

$$\begin{aligned} \delta(F_i u, F_j v, t) &\geq \phi(M(Su, Tv, kt), \delta(Su, F_i u, kt), \delta(Tv, F_j v, kt)) \\ &> \delta(F_i u, F_j v, kt), \text{ is a contradiction.} \end{aligned}$$

So we have $F_i u = F_j v$. Hence $F_i u = F_j v = \{Su\} = \{Tv\}$. Since $F_i u = \{Su\}$ and the pair $\{F_i, S\}$ is weakly compatible, then we obtain $F_i p = F_i Su = SF_i u = \{Sp\}$. If $F_i p \neq \{p\}$, then we have

$$\begin{aligned} \delta(F_i p, p, t) &= \delta(F_i p, F_j v, t) \\ &\geq \phi(M(Sp, Tv, kt), \delta(Sp, F_i p, kt), \delta(Tv, F_j v, ky)) \\ &> \delta(F_i p, p, kt), \text{ is a contradiction.} \end{aligned}$$

It follows that $F_i p = \{Sp\} = \{p\}$. Similarly, $F_j p = \{Tp\} = \{p\}$. Therefore, $F_i p = F_j p = \{Sp\} = \{Tp\} = \{p\}$. Hence p is a common fixed point of F_n, S and T. Finally, in order to prove the uniqueness of p, suppose that q is another common fixed point of F_n, S and T. Then

$$\begin{aligned} M(p, q, t) &\geq \delta(F_i p, F_j q, t) \\ &\geq \phi(M(Sp, Tq, kt), \delta(Sp, F_i p, kt), \delta(Tq, F_j q, kt)) \\ &> M(p, q, kt), \text{ is a contradiction.} \end{aligned}$$

Hence p = q. Therefore p is a unique common fixed point of F_n, S and T respectively. □

Corollary 15. *Let (X, M, ∗) be a complete fuzzy metric space.Let F, G : X → B(X) and S, T : X → X satisfying:*

(i) $F(X) \subseteq T(X)$, $G(X) \subseteq S(X)$ *for every* $x \in X$,

(ii) the pairs $\{F, S\}$ *and* $\{G, T\}$ *are weakly compatible,*

(iii) let $\phi : [0, 1]^3 \to [0, 1]$ *is a continuous function and increasing in any co-ordinate and* $\phi(t, t, t) > t$ *for every* $t \in [0, 1)$,

(iv) $\delta(Fx, Gy, t) \geq \phi(M(Sx, Ty, kt), \delta(Fx, Sx, kt), \delta(Gy, Ty, kt))$, *for every x, y in X and some k > 1. Suppose that one of S(X) or T(X) is a closed subset of X, then there exists a unique point* $p \in X$ *such that* $\{p\} = \{Sp\} = \{Tp\} = F(p) = G(p)$.

Corollary 16. *Let (X, M, ∗) be a complete fuzzy metric space.Let F: X → B(X) and S: X → X satisfying:*

(i) $F(X) \subseteq S(X)$ *for every x ∈ X,*

(ii) the pair $\{F, S\}$ *is weakly compatible,*

(iii) let $\phi : [0,1]^3 \to [0,1]$ *is a continuous function and increasing in any coordinate and* $\phi(t, t, t) > t$ *for every t ∈ [0, 1),*

(iv) $\delta(Fx, Fy, t) \geq \phi(M(Sx, Sy, kt), \delta(Fx, Sx, kt), \delta(Fy, Sy, kt))$, *for every x, y in X and some k > 1. Suppose that S(X) is a closed subset of X, then there exists a unique point p ∈ X such that* $\{p\} = \{Sp\} = F(p)$.

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