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2-D NUMERICAL SOLUTION TO THE ADDED MASS AND DAMPING COEFFICIENTS FOR SYMMETRICAL SYSTEM OF HORIZONTAL CIRCLE CYLINDERS SUBMERGED BELOW THE FREE SURFACE OF INVISCID INFINITE DEPTH FLUID

Georgi Valkov[1] §, Radko Petkov[2]
[1]Dowell International Ltd.
10, Kosta Lulchev Str.
Sofia, 1113, BULGARIA
e-mail: ggv99mont@gmail.com
[2]National Institute of Meteorology and Hydrology
Bulgarian Academy of Sciences
66, Tsarigradsko Chaussee
Sofia, 1784, BULGARIA
e-mail: radolili@yahoo.com

Abstract: In the present paper, the hydrodynamic characteristics (the added mass and the damping coefficients) of systems of parallel horizontal cylinders are reviewed from the viewpoint of the small-amplitude waves theory. They are defined on the basis of the diffraction theory and the strip theory (2-D linear boundary-value problem), using the technique of distributing hydrodynamic source singularities over the submerged portion of the cylinders. The hydrodynamic characteristics are obtained through oscillatory velocity potential derived by the complex-valued Green's function in frequency domain approach.

A computer programme for calculating the hydrodynamic characteristics and forces upon multiple parallel horizontal cylinders with a vertical plane of symmetry has been devised in Fortran code. The hydrodynamic characteristics of three and four circular cylinders' system in infinite depth water have been computed and presented in graphical form.

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§Correspondence author

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1. Introduction

Determining the added mass and wave damping for bodies submerged into fluid is directly relevant to calculation of the hydrodynamically induced forces upon them due to wave agitation or due to constrained motion of the bodies in still fluid (from the mathematics point of view, both phenomena are identical). Studying of this problem has been conducted for a long time but results have been obtained mainly for single spatial bodies or plane figures. In need of hydrodynamic calculation for a system of bound bodies, the superposition method has been applied [1]. Nevertheless, this method reduces finding of the added mass and damping (respectively forces) of the system to simple superposition without accounting for hydrodynamic bodies' interaction. In the published subsequently works, determining the hydrodynamic characteristics of systems of two cylinders (*catamaran* type) crossing the free water surface [2], [3], [4], or submerged below it [5], [6] has shown that neglecting interaction between bodies could lead to significant quantitative and qualitative inaccuracies.

With progress of computers' development, solving the problem of hydrodynamic body interaction in systems of cylindrical members became possible numerically. However, solutions of hydrodynamic characteristics of multiple horizontal cylinders have been published mainly for catamaran type systems [7].

In the present paper, numerical solutions of 2-D added mass and damping coefficients for vertically axially symmetrical systems of three and four horizontal parallel circle cylinders are presented. The cylinders are considered of infinite length, oscillating harmonically below the free surface of inviscid fluid of infinite depth. Hypothesis of plane cross-sections as well as the method of source singularities of W. Frank in the linear free-surface theory [8] are applied, which reduces the problem to numerical solution of the Fredholm integral equations. The results are obtained in frequency domain codes. Analysis of factors influencing the hydrodynamic coefficients, such as level of submersion, numbers of cylinders, distance between axes, direction and frequency of *sway* and *heave* oscillation is performed.

2. Problem Formulation

Let examine a system of three horizontal circle cylinders of infinite length with equal radius R, submerged below the free surface of infinite depth fluid. The longitudinal cylinders' axes are parallel and in equal distance $2A$ among them, and lie in the same horizontal plane parallel to the free surface (see Figure 1).

Let choose right-handed Cartesian coordinate system $OXYZ$, oriented in a manner that plane XY coincides with the free surface plane, axis X is the intersection of the vertical plane of symmetry for the cylindrical system and the free surface plane, and axis Z is oriented vertically up. Let consider two types of oscillatory motions:

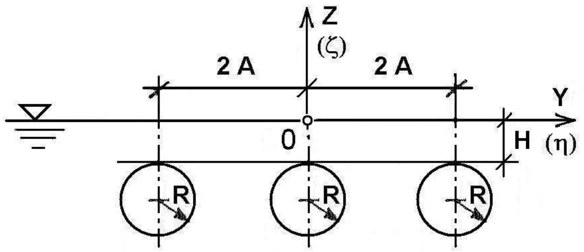

Figure 1: Cross-section of three circle cylinders' system submerged below the free surface of infinite depth fluid

vertical or *heave*, and horizontal or *sway*, supposing that the cylinders are forced to oscillate in still fluid (no swell presumed). Furthermore, the following assumptions are imposed:

- fluid is inviscid, gravitational, homogeneous and incompressible;
- fluid motion due to forced oscillation is vortex-free and intact;
- the motion amplitudes are supposed small enough comparing to the cylinder diameter $2R$;
- flow around the cylinders caused by oscillating is parallel-plane, i.e. occurring in the cross-section planes.

The first three assumptions allow using of the potential small-amplitude waves theory, and the fourth – the hypothesis of plane cross-sections. Hereby, the spatial wave-body interaction problem could be reduced to two-dimensional potential flow around circle cylinders.

3. Solution Method

Determining the added mass and damping resolves itself into finding the velocity potential of interaction of a forced in motion cylinders' system with the fluid. The velocity potential of the forced oscillation Φ_m will be searched among the complex functions:

$$\Phi_m(y,z;t) = \varphi_m(y,z)\,.e^{-i\,\sigma\,t}, \tag{1}$$

where y, z are complex coordinates of the field points in the fluid domain;

φ *(y,z)* – radiant function, the core of which is complex amplitude of the unsteady potential of forced oscillation;

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σ – angular frequency of oscillation;

t – time;

i – unit imaginary number;

m – subscript, accounting for oscillation direction (according to the conventional notation in hydrodynamics, $m= 2$ corresponds to *sway* oscillation, i.e. along axis Y ; and $m= 3$ – to *heave* motion, i.e. along axis Z).

Henceforth, let us agree when deal with complex functions to take into account only their real parts.

The radiant function must satisfy the following conditions:

- Laplace's equation in the field points of the fluid domain:

$$\Delta\varphi_m = 0 \tag{2}$$

- the linearized boundary condition of the small-amplitude waves theory on the free surface:

$$\frac{\partial\varphi_m}{\partial z} - K\varphi_m = 0 \quad \text{at} \quad z = 0, \tag{3}$$

  where $K = \sigma^2/g$ is the wave number for infinite depth fluid

- the boundary condition on the infinite depth:

$$\nabla\varphi_m = 0 \quad \text{at} \quad z \to -\infty \tag{4}$$

- the kinematic boundary condition on the cylinders' contours:

$$(\vec{n}\cdot\nabla)\varphi_m = -i\sigma n_m \quad \text{on} \quad C, \tag{5}$$

  where C is the wet perimeter of all cylinders in the system in rest position (in Figure 1 – the perimeter of the three circles); n_m – the m-th projection of the outward normal to contour C (at $m = 2$: $n_2 \equiv n_y$; at $m = 3$: $n_3 \equiv n_z$).

- the Sommerfeld radiation condition representing attenuation of the outgoing forced waves at infinity:

$$\lim_{y\to\pm\infty}\left(\frac{\partial}{\partial y}\varphi_m \mp iK\varphi_m\right) = 0 \tag{6}$$

In solution of the problem, it is presumed that an external harmonic exciting force is applied to the cylinders' system in order to set it in motion. Therewith, displacement of the cylinders s is going on harmonic law with unit amplitude:

$$s_m = e^{-i\,\sigma\,t} \tag{7}$$

The method of source singularities in modification of W. Frank is to be used in order to find the radiant function φ_m. According to this method, impact of the cylinders upon the ambient fluid is substituted for a system of hydrodynamic sources, distributed continuously over the submerged body contours in rest position.

The radiant function for m-th mode of oscillation φ_m is searched on the basis of Green's function for unit hydrodynamic sources, which strength of distribution q on the wet perimeter C is unknown function:

$$\varphi_m(y,z) = \int\limits_c q(\xi) G(x,\xi) dc \tag{8}$$

Above, $G(x,\xi)$ is complex potential in point $x = y + iz$, induced by unit hydrodynamic source, placed in point $\xi = \eta + i\zeta$; its core is Green's function (in Figure 1, axis η coincides with axis Y, and axis ζ – with axis Z); $q(\xi)$ – complex strength of source singularity distribution, which is function of its location on contour C.

Using symmetry conditions, integration over the three contours could be reduced to integrating over one-and-half contours (right or left to the axis Z in Figure 1). Symmetry conditions on the axis Z are stipulated as [5]:

$$\left| \begin{array}{l} \frac{\partial}{\partial y}\varphi_m(0,\mathrm{z}) = 0 \ \mathrm{at\,m} = 3, \\ \varphi_m(0,\mathrm{z}) = 0 \ \mathrm{at\,m} = 2. \end{array} \right. \tag{9}$$

Adjusted for symmetry conditions (9), equation (8) is transformed into:

$$\varphi_m(x) = \int\limits_{C^+} q^+(\xi^+) \cdot G^+(x^+,\xi^+) \cdot dc, \tag{10}$$

where C^+ is the wet perimeter of the right half (to the axis Z) of the symmetrical cylinders' contours (see Figure 1); superscript + shows positive abscissas (y-coordinates) of the Cartesian plane $Y0Z$.

Green's function for infinite depth fluid is obtained in work [9] ; using the symmetry conditions, it could be written as:

$$\begin{aligned} G^+(x,\xi) = \frac{1}{2\pi} Re\{\ln\frac{x-\xi}{x-\overline{\xi}} - (-1)^m \ln\frac{x+\overline{\xi}}{x+\xi} + 2P.V.\int\limits_0^\infty \frac{e^{-ik(x-\overline{\xi})}}{K-k}dk - \\ -(-1)^m 2P.V.\int\limits_0^\infty \frac{e^{-ik(x+\xi)}}{K-k}dk\} - iRe\left\{e^{-iK(x-\overline{\xi})} - (-1)^m e^{-iK(x+\xi)}\right\}, \end{aligned} \tag{11}$$

where $\overline{\xi} = \eta - i\zeta$ is complex conjugate source singularity in the upper semi-plane $\eta\zeta$; $P.V.$ – notation of the Cauchy principal value of integral.

Green's function G^+ in the form of (11) satisfies Laplace's equation (2) and the boundary conditions (3), (4), (6), except the kinematic boundary condition on the contour (5). The last is necessary for determining the source distribution strength $q^+(\xi)$. Considering the symmetry conditions, equations (5) must be satisfied on contour C^+ ; substituting (10) into (5) and separating the real and imaginary parts of the complex expressions, a system of the Fredholm integral equations of second kind is obtained:

$$\left|\begin{array}{l} (\vec{n}.\nabla)Re \int\limits_{C^+} q^+(\xi^+)\cdot G^+(x^+,\xi^+)\cdot dc = 0, \\ (\vec{n}.\nabla)Im \int\limits_{C^+} q^+(\xi^+)\cdot G^+(x^+,\xi^+)\cdot dc = -\sigma\, n_m^+. \end{array}\right. \tag{12}$$

Here, x^+, ξ^+ are the points of the fluid and the source, respectively, both adherent to contour C^+. The equations system (12) could be solved numerically by the Fredholm method, according to which contour C^+ is to be divided into N small segments, in the extent of which the source strength q^+, respectively the integrand, keep constant value. As a result, two-integral-equations system (12) could be reduced to a system of $2N$ algebraic linear equations with $2N$ variables q^+. Taking into account that $q^+(\xi)$ and $G^+(x,\, \xi)$ are complex functions, i.e.

$$q^+ = Re\left\{q^+\right\} + i\, Im\left\{q^+\right\},$$

and

$$G^+ = Re\left\{G^+\right\} - i\, Im\left\{G^+\right\} \qquad \text{(see (11))}$$

equations system (12) passes on to (13):

$$\left|\begin{array}{l} \sum\limits_{j=1}^{N} Re\left\{q_j^+\right\}\cdot I_{ij} + \sum\limits_{j=1}^{N} Im\left\{q_j^+\right\}\cdot J_{ij} = 0, \\ -\sum\limits_{j=1}^{N} Re\left\{q_j^+\right\}\cdot J_{ij} + \sum\limits_{j=1}^{N} Im\left\{q_j^+\right\}\cdot I_{ij} = -\sigma n_m^+\,|_{x=x_i}\,. \end{array}\right. \tag{13}$$

Here I_{ij} and J_{ij} are matrix influence coefficients on the i-th contour point due to the j-th contour segment in the m-th mode of oscillation, in phase with acceleration and velocity, respectively. They are expressed as:

$$\left|\begin{array}{l} I_{ij} = (\vec{n}\cdot\nabla)\int\limits_{C_j^+} Re\left\{G^+(x,\xi)\right\}\cdot dc\,|_{x=x_i}\,, \\ J_{ij} = (\vec{n}\cdot\nabla)\int\limits_{C_j^+} Im\left\{G^+(x,\xi)\right\}\cdot dc\,|_{x=x_i}\,. \end{array}\right. \tag{14}$$

Applying this approach, we can transform the radiant function φ_m from equation (10). For the purpose, the values of φ_m in the midpoint x_i of the ith segment are to be computed using the already found values of q^+:

$$\begin{aligned}\varphi_m(x_i) &= Re\left\{\varphi_m\right\}|_{x=x_i} + i\, Im\left\{\varphi_m\right\}|_{x=x_i}\\ &= \sum_{j=1}^{N}\Big[Re\left\{q_j^+\right\}\cdot A_{ij} + Im\left\{q_j^+\right\}\cdot B_{ij}\Big]\\ &\quad + i\sum_{j=1}^{N}\Big[Im\left\{q_j^+\right\}\cdot A_{ij} - Re\left\{q_j^+\right\}\cdot B_{ij}\Big]. \end{aligned} \tag{15}$$

Above,

$$\left|\begin{aligned} A_{ij} &= \int\limits_{C_j^+} Re\left\{G^+(x,\xi)\right\}\cdot dc\,|_{x=x_i}\,,\\ B_{ij} &= \int\limits_{C_j^+} Im\left\{G^+(x,\xi)\right\}\cdot dc\,|_{x=x_i}\,. \end{aligned}\right. \tag{16}$$

If the segments number N is sufficiently large, the curvilinear segments could be replaced with rectilinear ones. In this case, equations (14), (16) can be defined analytically [2], [8].

Hydrodynamic force, acting on the cylinders' system in the m-th mode of oscillation, is to be determined by integrating pressures on the contour:

$$F_m.e^{-i\sigma t} = -\int\limits_C P(y,z;t).n_m.dc. \tag{17}$$

In (17), the hydrodynamic pressures P along the cylinders' contours could be obtained from the velocity potential Φ_m by means of the linearized Bernoulli equation:

$$P(y,z;t) = -\rho\frac{\partial}{\partial t}\Phi_m(y,z;t) = i\rho\sigma\Phi_m(y,z;t). \tag{18}$$

Substituting (18) into (17) with consideration for (1), (19) is obtained:

$$F_m.e^{-i\sigma t} = -i\rho\sigma\, e^{-i\sigma t}\int\limits_C \varphi_m n_m dc. \tag{19}$$

The symmetry conditions for φ_m and n_m are given in the form of [10]:

$$\varphi_m(x^+) = -(-1)^m\varphi_m(x^-), \tag{20}$$

$$n_m\,|_{x^+} = -(-1)^m\; n_m|_{x^-} \tag{21}$$

(above, superscript – shows negative abscissas y of the Cartesian plane $Y0Z$).

Analysing the above expressions, it is evident that the product $(\varphi_m n_m)$ is always even function. Thus, we can pass to integrating of the right half of the contours:

$$F_m.e^{-i\sigma t} = -2i\rho\sigma\, e^{-i\sigma t}\int\limits_{C^+} \varphi_m\,(x^+).\,n_m\,|_{x=x^+}\cdot dc. \tag{22}$$

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On the other hand, hydrodynamic force is expressed through the added mass λ_{mm} and damping b_{mm} in the form of [2]:

$$F_m e^{-i\sigma t} = -\lambda_{mm} \frac{\partial^2}{\partial t^2} s_m - b_{mm} \frac{\partial}{\partial t} s_m. \tag{23}$$

Differentiating (7) and equalising the coefficients of *cos(σt)* and *sin(σt)* in the real parts of the complex expressions (22) and (23), and in respect of the contours symmetry, the following formulae for the added mass and damping are obtained:

$$\lambda_{mm} = \frac{2\rho}{\sigma} \int\limits_{C^+} Im\left\{\varphi_m\right\} \cdot n_m \cdot dc \,|_{x=x^+} \, , \tag{24}$$

$$b_{mm} = -2\rho \int\limits_{C^+} Re\left\{\varphi_m\right\} \cdot n_m \cdot dc \,|_{x=x^+} \, . \tag{25}$$

The same assumptions, allowing to replace integration over curvilinear segments with integration over rectilinear segments, are valid for formulae (24) and (25). Considering (15), expressions for λ_{mm} and b_{mm} are obtained in the form, convenient for numerical solution:

$$\lambda_{mm} = \frac{2\rho}{\sigma} \sum_{i=1}^{N} \left[\sum_{j=1}^{N} \left(Im\left\{q_j^+\right\} \cdot A_{ij} - Re\left\{q_j^+\right\} \cdot B_{ij} \right) \right] \cdot n_m \cdot \Delta l_i, \tag{26}$$

$$b_{mm} = -2\rho \sum_{i=1}^{N} \left[\sum_{j=1}^{N} \left(Re\left\{q_j^+\right\} \cdot A_{ij} + Im\left\{q_j^+\right\} \cdot B_{ij} \right) \right] \cdot n_m \cdot \Delta l_i \tag{27}$$

(here Δl_i is length of the i-th segment).

In conclusion, we can notice that the above given formulae are correct for any vertically axi-symmetrical plane system of n contours. Cross-section of a vertically symmetrical system of four circle cylinders with equal radius R is shown in Figure 2 as example.

4. Numerical Results and Analysis

The *boundary-element method* has been applied for numerical solution of 2-D potential problem of forced oscillation of multiple parallel horizontal cylinders in inviscid infinite depth fluid. A FORTRAN-based programme, named SHELF, for calculating the hydrodynamic characteristics of systems of three (*trimaran* type) and four (*tetramaran* type) circle cylinders with a vertical plane of symmetry has been devised (see Figures 1 and 2).

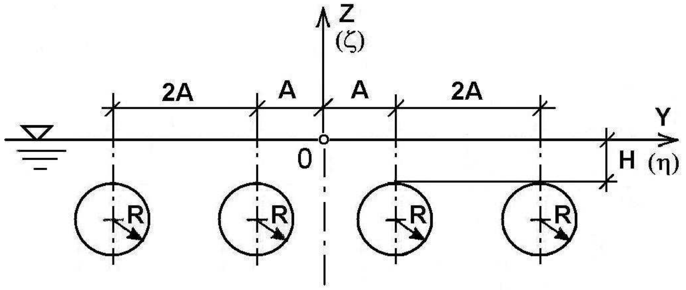

Figure 2: Cross-section of four circle cylinders' system submerged below the free surface of infinite depth fluid

For range of various non-dimensional wave numbers $K.R$, contours geometry and oscillating mode, dimensionless added mass and damping coefficients have been computed in the form:

$$\overline{\lambda}_{mm} = \frac{\lambda_{mm}}{\rho S}, \tag{28}$$

$$\overline{b}_{mm} = \frac{b_{mm}}{\rho \sigma S}, \tag{29}$$

where ρ is fluid density; S – total area of the cylinders cross-section.

Programme SHELF calculates hydrodynamic characteristics of axi-symmetric multiple cylinders in or below the free surface for all 2-D oscillation modes: *sway* ($\overline{\lambda}_{22}$ and $\overline{b}_{22}$), *heave* ($\overline{\lambda}_{33}$ and $\overline{b}_{33}$), and *roll* ($\overline{\lambda}_{44}$ and $\overline{b}_{44}$) motion, as well as mixed *sway-roll* added mass ($\overline{\lambda}_{24} = \overline{\lambda}_{42}$) and damping ($\overline{b}_{24} = \overline{b}_{42}$) coefficients. Moreover, SHELF computes the hydrodynamic forces acting upon the cylindrical system submerged in a heavy inviscid fluid, particularly into water.

From our numerical experiment and based on recommendations [11], [12], the optimum number of segments for each circle N_1 has been found as $N_1 = 24$ for computing (for the right half of the middle circle in Figure 1, $N_1/2 = 12$). Increase of this number has led to unimportant amelioration of solution precision at the expense of computing time augmentation

The *heave* and *sway* computed added mass and damping coefficients for the systems shown in Figures 1 and 2 are presented in graphical form as function of non-dimensional wave numbers $K.R$, level of submersion $H/2R$, and distance between axes A/R. The geometry of the cylindrical systems has been defined by following dimensionless parameters: level of submersion $H/2R$ (equal to 0.25, 0.5 and 0.75), and distance between axes A/R (equal to 1.5, 2, 3 and 4). All combinations of the above parameters have been inputted to compute the hydrodynamic characteristics

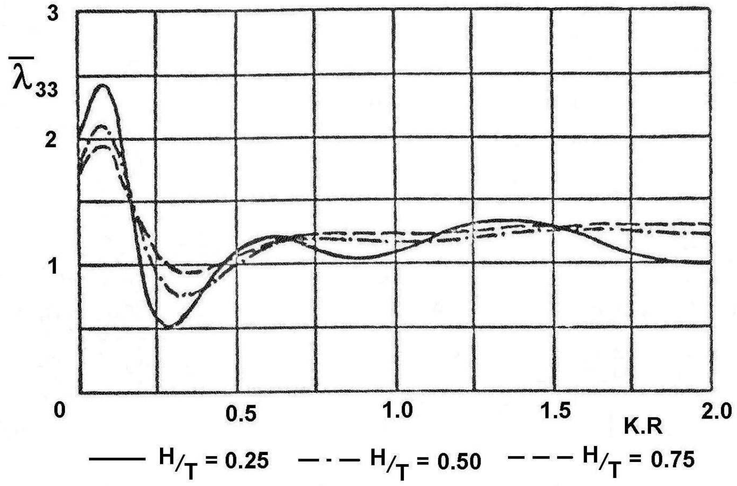

Figure 3: Heave water added mass coefficients $\overline{\lambda}_{33}$ of trimaran type circle cylinders at $A/R = 1.5$

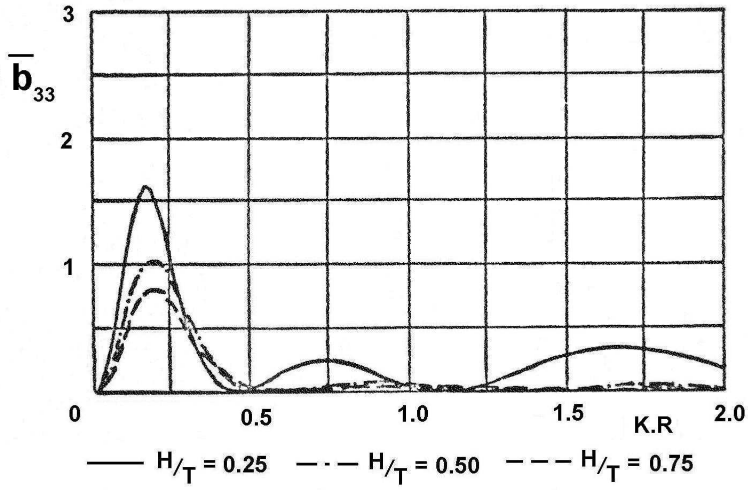

Figure 4: Heave water damping coefficients $\overline{b}_{33}$ of trimaran type circle cylinders at $A/R = 1.5$

of three and four cylinders' systems submerged into water (for brevity sake, only some of them are shown in graphical form in Figures 3, 4, 5, 6, 7, 8, 9 and 10).

Considering the computed variants, common analysis of the results has been performed and the following relations are observed.

- All graphs show oscillating character, in which the number of peaks corre-

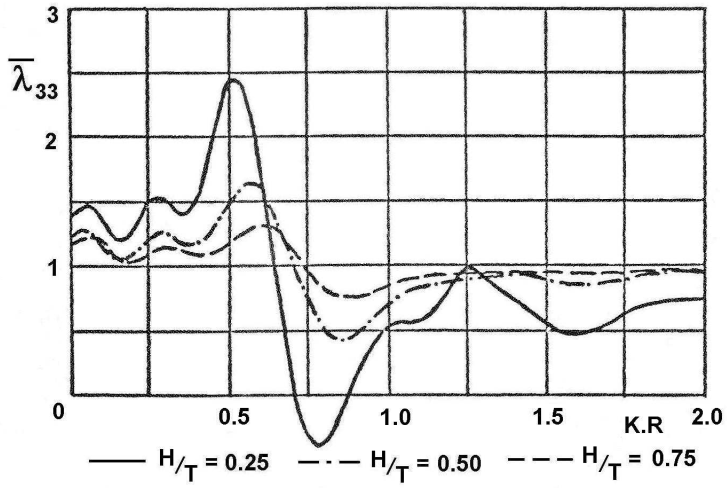

Figure 5: Heave water added mass coefficients $\overline{\lambda}_{33}$ of trimaran type circle cylinders at $A/R = 4$

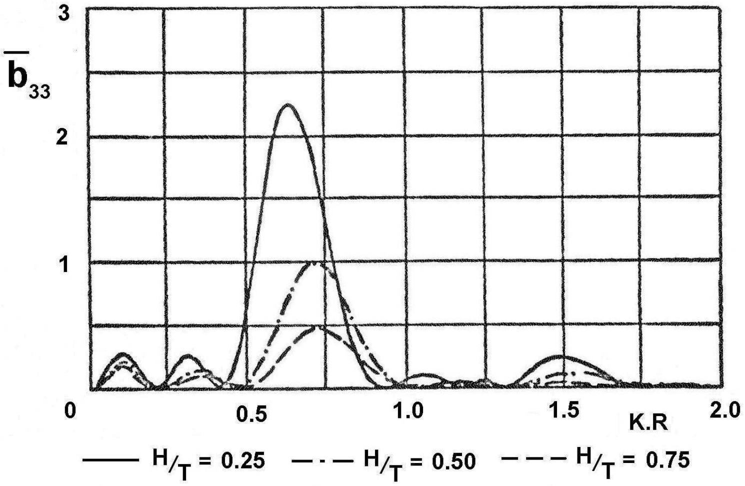

Figure 6: Heave water damping coefficients $\overline{b}_{33}$ of trimaran type circle cylinders at $A/R = 4$

sponds to the number of cylinders in the system (three or four). At small axes' distance, $A/R = 1.5$, they are slightly noticeable and become more delineated with enlarging of A/R (2 or 3); while at $A/R = 4$, a second series of peaks appears.

- Qualitatively, the graphs for *sway* oscillation $\overline{\lambda}_{22}$ and $\overline{b}_{22}$ repeat the depen-

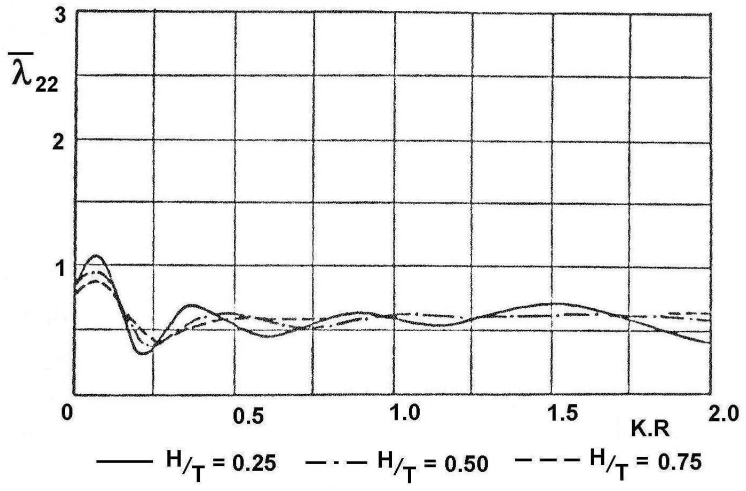

Figure 7: Sway water added mass coefficients $\overline{\lambda}_{22}$ of tetramaran type circle cylinders at $A/R = 1.5$

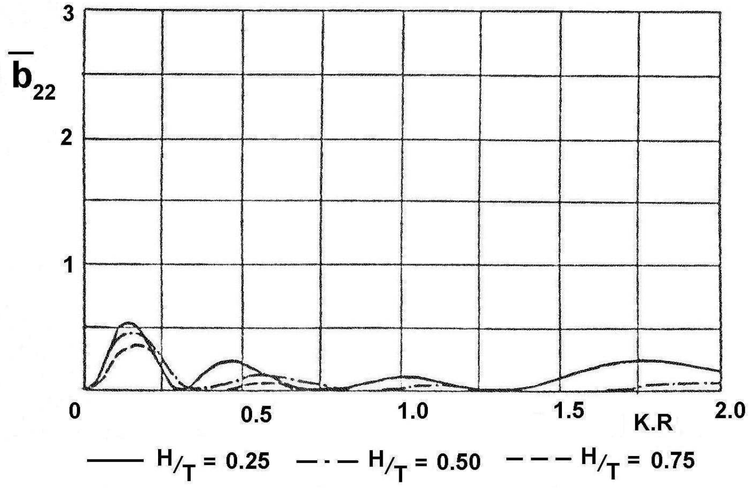

Figure 8: Sway water damping coefficients $\overline{b}_{22}$ of tetramaran type circle cylinders at $A/R = 1.5$

dence of $\overline{\lambda}_{33}$ and $\overline{b}_{33}$ for *heave* (their extrema are at the same frequencies) but quantitatively the *heave* extrema are larger (it is not evident from the graphs in Figures 3 to 10 because oscillations are presented for different system type – *trimaran* and *tetramaran*, for brevity sake). This tendency is well observed at small axes' distances ($A/R = 1.5$ or 2); diminishes at $A/R = 3$, and practically vanishes at $A/R = 4$.

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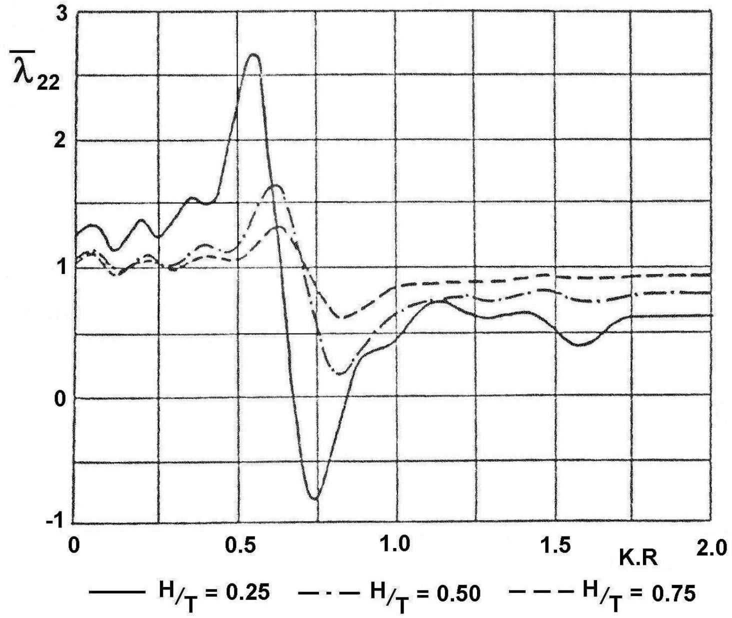

Figure 9: Sway water added mass coefficients $\overline{\lambda}_{22}$ of tetramaran type circle cylinders at $A/R = 4$

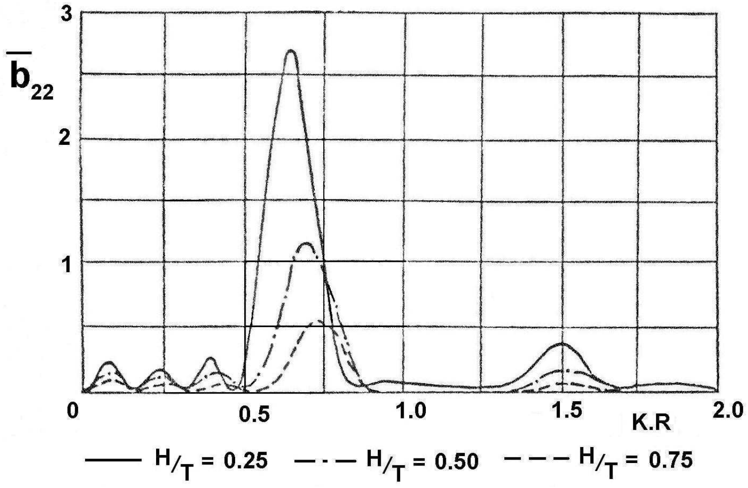

Figure 10: Sway water damping coefficients $\overline{b}_{22}$ of tetramaran type circle cylinders at $A/R = 4$

- At small distances between the cylinders axes, $A/R = 1.5$, the maximum value of the hydrodynamic characteristics is reached at the first peak. With enlarging the axes' distance, $A/R = 2$, the third peak starts to raise for three cylinders' system and the forth peak – for four cylinders' system; those peaks reach maximum at $A/R = 3$ or $A/R = 4$. At that, with growth of parameter A/R, a common shift of all the peaks to the range of low frequencies takes place. In result of such horizontal flattening of the peaks, the added mass coefficients for three and four contours can obtain negative values (see Figures 5 and 9). This phenomenon could be explained with the complex character of the hydrodynamic cylinders interaction as well as with the existence of the irregular frequencies effects [13], [14]. (It should be noted that negative values of the added mass have been computed also in the cases of two, three and four cylinders crossing the free water surface, [4], [11].)

- With increase of level of submersion $H/2R$, fading of the extrema and smoothing of the curves (without frequency shift) are observed. This tendency is well-marked at small axes' distances ($A/R = 1.5$ or 2) as well as at *sway* motion. In the limit of $H/2R \to \infty$, the added mass coefficients $\overline{\lambda}_{mm}$ tend to a constant value in all frequency range, equal to the value of the contours system in infinite fluid, while the damping coefficients $\overline{b}_{mm}$ tend to zero (there is no wave generation in infinite fluid).

- At equal other parameters ($H/2R$ and A/R), the common character of the curves modifies slightly with variation of the number of circle contours n (2, 3 or 4): qualitatively, insignificant lag occurs with increase of n due to appearance of small local extrema; quantitatively, the magnitudes of the maximal extrema amplify with increase of n that is well-marked at large axes' distances ($A/R = 3$ or 4).

Based on the abovementioned relations, we can conclude the following.

A) Main factor influencing quantitatively the added mass λ_{mm} and damping b_{mm} coefficients at predetermined frequency range is the level of submersion $H/2R$: with increase of $H/2R$ smoothing of the added mass λ_{mm} as well as approaching to zero of damping b_{mm} are observed. This tendency could be explained with diminishing of the free surface influence, i.e. in the limit of infinite fluid $\lambda_{mm} = \text{const}$ and $b_{mm} \equiv 0$. Other quantitatively influencing factors are the distance between axes A/R, the mode of oscillation m, and the number of contours in the cylindrical system n: at small axes' distances ($A/R = 1.5$ or 2), the oscillation mode is more influencing, while at large parameters A/R (3 or 4) – more significant factor is the number of contours in the cylindrical system.

B) In qualitative aspect, main factor of influence on the hydrodynamic characteristics is the axes' distance A/R. With enlargement of the latter, oscillation amplification and extrema magnification are evidenced. Similar trend, but to a lesser degree is found with increase of the contours number n (when $A/R =$ const) – this could be interpreted in way that with increasing of the contours, the distance between the outermost contours increases.

C) At $A/R = 4$, the added mass λ_{mm} and damping b_{mm} curves for *heave* and *sway* motion almost coincide. Such coincidence is typical for ordinary submerged circle. This observation implies the conclusion that at axes' distances $A/R > 4$, hydrodynamic interaction among the cylinders ceases. Hence, we recommend using computer programme SHELF in the range $A/R \leq 4$.

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