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AVERAGE EDGE-DOMINATION NUMBER IN GRAPHS

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Abstract: Domination number is one of the important graph vulnerability measures which have numerous applications in real life. Recently total domination and independent domination definitions are studied. In this work, the average edge-domination of a graph is given as a new vulnerability measure. Also, sub and upper bounds for average edge-domination are found and a polynomial time algorithm is given which calculates the average edge-domination of a graph.

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1. Introduction

Vulnerability is the most important concept in any communication network. Vulnerability measure is the resistance of a network after any disruption. In a network, this disruption not only takes place on its centers also sometimes on its links. In this case the vulnerability measures given on links (edges) take an important role while constructing communication networks. A communication network can be modeled by a graph whose vertices represent the stations and whose edges represent the lines of communication. A graph G is denoted by $G = (V(G), E(G))$, where $V(G)$ and $E(G)$ are vertices and edges sets of G, respectively. Also $V(G) = n$ and $E(G) = m$ are called order and size of G. All graphs in this paper are simple, finite, undirected, loopless and without multiple edges. In graph theory, many graph measures have been used widely in the past to describe the stability of a graph (see [4]). For exam-

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ple connectivity is one of the first and the most important measures of vulnerability. Latterly it is seen that these measures were not enough to compare two network models since if they have the same values of some of these parameters. For example the vertex connectivity of a path graph and a star which have the same order is equal. Also their edge connectivity values are equal. In this case we need some more measure to decide which model is more stable than the other. This situation oriented graph theory researchers to find new measures. Some of these measures are edge connectivity, edge integrity, domination and edge domination. Domination is previously studied in [3], [6], [10],[13]. Edge domination is given in [1], [8] and studied widely by [11], [9].

Recently many researchers in this field study not only the vertices and edges of graphs; they also study on vertices or edges which have a special property in graph. In this work, average edge-domination of a graph which is a new vulnerability measure given on edges of a graph is defined.

Let us imagine we want to construct a business center or a skyscraper. In every building, for emergency situation it is compulsory to put fire exits. In general, number of exits and their placements are only taken into consideration. Our aim is not either to determine the number of exits or adjust their places. We just need to increase the number of alternating ways to reach these exits. Why do we do this incase of increasing numbers of exits? In any business center or skyscraper, lots of people work or live in any floor of the building. For example, let us think a fire outbreak in a floor. In such cases people have a tendency to reach the nearest door usually. If there is only one way to the nearest door, there will be a chaos. To prevent this chaos, we can construct alternating ways to these doors or place doors at some point in which alternating ways intersect. While calculating the average edge-domination number, we are also looking for the alternating ways between edge pairs. Our algorithm is based on this.

Definition 1. Let the nodes of G be labeled as $v_1, v_2, v_3, ..., v_n$. The adjacency matrix $A = A(G) = [a_{ij}]$ of G is the binary matrix of order n (see [7]):

$$a_{ij} = \begin{cases} 1, \text{ if } v_i \text{ is adjacent with } v_j, \\ 0, \text{ otherwise.} \end{cases}$$

Definition 2. The length of a path is its number of edges (see [12]).

Definition 3. For a connected graph G, the distance $d(u, v)$ between two vertices u and v is defined as the minimum of the lengths of the $u - v$ paths of G (see [7]).

Definition 4. A clique of a graph is a maximal complete subgraph (see [12]).

Definition 5. Let $G = (V, E)$ be a finite and undirected graph without loops and multiple edges. An edge is said to dominate itself and any edge adjacent to it (see [1]).

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 1, No. 3 (2010)

Definition 6. Let e_1 and e_2 be two edges of the graph G. If they have one common vertex they are called neighbor edges of (see [1]).

If $e = (u, v) \in E$ then $N_G[e]$={$e' = (u', v') \in E|$ u=u', u=v' or v=u', v=v'} is called the closed neighborhood of e in G and $N_G(e) = N_G[e]\backslash\{e\}$ is the open neighborhood of e in G.

Definition 7. Let $e = (u, v)$ be an edge of connected graph G. Degree of e is defined as deg(e)=deg(u)+deg(v) $- 2$ (see [1]).

Definition 8. Let S be a set of edge-degrees of a connected graph G. Minimum edge-degree $\delta(G)$ is defined as $\min\{|S|\}$.

Definition 9. The degree of a vertex v is the number of edges adjacent to v and denoted by $d(v)$ (see [2]).

Definition 10. Let G be a connected graph, the number $d(G) = \frac{1}{|V|} \sum\limits_{v \in V} d(v)$ is called average degree of G (see [2]).

2. Average Edge-Domination Number

In this section the definition edge-distance, edge-distance matrix, pair edge-domination number, total pair edge-domination value, average edge-domination number of a graph and an example and some theorems are given.

Definition 11. Let G be a connected graph and $e_{1=}(u_1, v_1), e_2 = (u_2, v_2)$ be two edges of G, the distance between edges e_1 and e_2 is called *edge-distance* and defined as

$ed(e_1$,$e_2)$=min{ $d(u_1,\ u_2)$, $d(u_1,\ v_2)$, $d(v_1,\ u_2)$, $d(v_1,\ v_2)$} (see [5]).

If $ed(e_1, e_2) \geq 2$, then the pair neighborhood of these edges $N[e_1, e_2] = 0$.

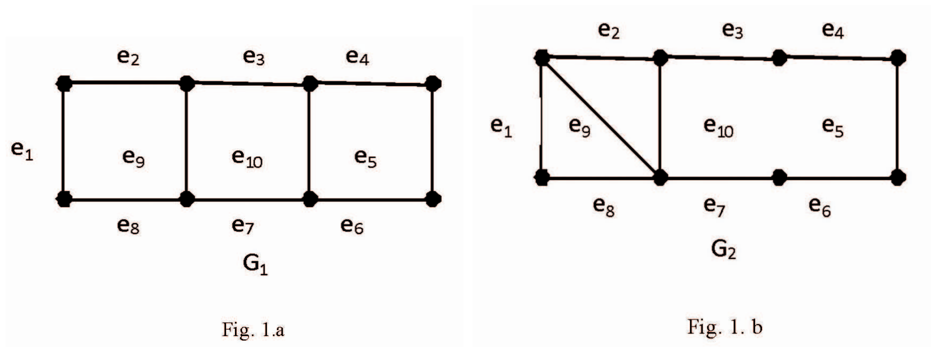
Definition 12. For a connected graph G, we define the $ED[i, j]$ *edge distance matrix* of the graph. Edge distance matrix consists real distance between edges. The value of the diagonal elements of the matrix is defined as infinity. This matrix is symmetric since distance matrix defined on vertices is symmetric (see [5]).

Definition 13. Let $e_{1=}(u_1, v_1)$ and$e_2 = (u_2, v_2)$ be two edges of graph then *pair edge-domination number* (PEDN) is defined as the number of the edges which are dominated by both e_1 and e_2 and having the following properties

e'=(u_1, v_2) or e'=(u_1, u_2) or e'=(v_1, u_2) or e'=(v_1, v_2) and it is denoted by *PEDN.*

Definition 14. The sum of *PEDN* of all pairs of edges of graph is defined

$$TPED = \sum_{e_i, e_j \in E(G)}^{PEDN(e_i, e_j)}$$



(a) (b)

Figure 1

where *total pair edge domination* value is denoted by *TPED.*

Definition 15. *Average edge-domination number* is defined as

$$AEDN = \frac{TPED}{\binom{r}{2}},$$

where $r = \binom{n}{2}$ is the number of maximum edge number of any n vertices graph.

For the graphs G_1 and G_2 given in Figure 1a and 1b whose orders, connectivity numbers $k(G_1) = k(G_2)$ and edge-connectivity numbers $k'(G_1) = k'(G_2)$ are the same, we consider the average edge-domination number of these two graphs.

By calculating *TPED* value for G_1 and G_2 graphs, the *AEDN* values are obtained as follows,

$$AEDN(G_1) = \frac{TPED}{\binom{r}{2}} = \frac{42}{\binom{28}{2}} = \frac{42}{378} \cong 0,111,$$

$$AEDN(G_2) = \frac{TPED}{\binom{r}{2}} = \frac{43}{\binom{28}{2}} = \frac{43}{378} \cong 0,113.$$

When we compare these two graph models we choose the one whose *AEDN* value is bigger. Since this means the communication in this graph is more stable than the other graph model.

Theorem 16. *For a connected graph G where $n \geq 3$, then*

$$0 < AEDN(G) < \frac{4\,(n-1)}{n+1}$$

Proof. In any graph G (where G is connected and not null) there should be at least one edge pair so *AEDN* value is greater than zero. Le tus take P_3. Since edge pair first occurs in P_3. There are 2 edges which make an edge pair. There is only one edge pair in P_3. *TPED*=1 and $AEDN = \frac{1}{\binom{3}{2}} = \frac{1}{3} > 0$.

In a complete graph K_n it is obvious that the number of the pair of edges will be maximum. And also the neighborhood between edge pairs will take its maximum value. When we examine the pair edges in K_n finally we have $\frac{4(n-1)}{n+1}$ (this value is calculated in detail in next section). Thus $0 < AEDN(G) < \frac{4(n-1)}{n+1}$. □

Theorem 17. *Let G be a connected graph. If an $e = (u, v)$, where u and v are nonadjacent elements of $V(G)$, is added to G then average edge-domination value of the new graph is bigger than graph G. In other words $AEDN(G) < AEDN(G+e)$.*

Proof. In a graph when a new edge is added, the number of the pairs of the edges increases. Due to this *TPED* value increases but the denominator remains the same. So this causes the inequality to be in the form

$$AEDN(G) < AEDN(G+e).$$ □

Theorem 18. *Let G_1 and G_2 be two graphs of having the same order p and size q. If G_1 includes a clique bigger than the clique which G_2 includes, then $AEDN(G_1) > AEDN(G_2)$.*

Proof. If any graph having a bigger clique, then any edge in this graph has more edge neighbors. So this means that the total pair edge domination number is bigger. From the definition of *AEDN* and since they have the same number of vertices *TPED* value is expected to be bigger in the graph whose clique is bigger so, $AEDN(G_1) > AEDN(G_2)$. □

Theorem 19. *Let G_1 and G_2 be two graphs of having the same order p. If $\delta'(G_1) < \delta'(G_2)$ then $AEDN(G_1) < AEDN(G_2)$.*

Proof. If $\delta'(G_1)$ is bigger than $\delta'(G_2)$ this means that the number of edges in G_1 is greater than the number of edges in G_2 since they have the same number of vertices. The edges examined in a graph whose number of edges is bigger, causes *PEDN* of each edge pair to be bigger. From the definition *AEDN* this will cause *TPED* to become bigger. Since their denominators are same $AEDN(G_1) > AEDN(G_2)$. □

Theorem 20. *Let H be a spanning sub graph of G, then*

$$AEDN(H) < AEDN(G).$$

Proof. Number of the vertices of H will remain the same as the graph itself. So the number of edges is decreased. In this case the number of edges to be examined and *TPED* will decrease. Finally as a result

$$AEDN(H) < AEDN(G). \qquad \square$$

Theorem 21. *Let G_1 and G_2 be two graphs of having the same order p (here d denotes the average degree of the graph). If $d(G_1) < d(G_2)$ then $AEDN(G_1) < AEDN(G_2)$.*

Proof. It is obvious that the number of the edges in a graph whose sum of its vertices degrees is big, will be more than the other graph having the same size. So the number of the examined edge pairs and *PEDN* in G_1 is less. From the definition of *AEDN* the denominator will remain same, since the number of vertices are same in G_1 and G_2. *TPED* in G_1 is less than in G_2, so

$$AEDN(G_1) < AEDN(G_2). \qquad \square$$

3. Basic Results

$P_n C_n, K_n, W_{1,n} K_{1,n} K_{m,n}$ are basic graph classes. In this section some results are given on.

Theorem 22. *For P_n, $AEDN(P_n) = \dfrac{2(n-3)+1}{\begin{pmatrix} \begin{pmatrix} n \\ 2 \end{pmatrix} \\ 2 \end{pmatrix}}$, $n \geq 3$.*

Proof. In path graph P_n there are n-1 edges. All the edges except the edge which is adjacent to one of the end vertices of the path and edges which are adjacent to this edge $(n-1)-2 = n-3$ have 2 value with the pairs of edges which are 0 and 1 distance far from these. The *PEDN* of n-3 edges is 2. There are no *PEDN* between edges whose distances between them are greater than 1. Then *TPED*=2.(n-3). But the edge adjacent to the edge which is adjacent to an end vertex has *PEDN*=1 with the pair it makes by the end edge.

Thus *TPED*=2(n-3)+1 and $AEDN(P_n) = \dfrac{2(n-3)+1}{\begin{pmatrix} \begin{pmatrix} n \\ 2 \end{pmatrix} \\ 2 \end{pmatrix}}$, $n \geq 3$. $\square$

Theorem 23. $AEDN\,(C_n) = \dfrac{2n}{\begin{pmatrix} \begin{pmatrix} n \\ 2 \end{pmatrix} \\ 2 \end{pmatrix}}$, $n \geq 3$.

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 1, No. 3 (2010)

Proof. In cycle graph C_n there are n edges. Every edge connects the two vertices of graph. Every vertex that constructs each edge has 2 vertices which have distance 0 and 1 from this vertex. Since each edge has two vertices it has two edge neighbors having distance 0 and two other edges having distance 1. In this case every edge has $PEDN$=4 for the pairs it makes with other edges. And also each neighbor is counted twice for all over the edge pairs, then $TPED$=$4n.\frac{1}{2} = 2n$ and $AEDN(C_n) = \dfrac{2n}{\binom{\binom{n}{2}}{2}}$, $n \geq 3$. □

Theorem 24. *For complete graph* K_n*,*

$$AEDN(K_n) = \frac{\binom{n}{2}(n-2).2 + \binom{n}{2}\binom{n-2}{2}.2}{\binom{\binom{n}{2}}{2}}.$$

Proof. (i) The edges which have common vertex. The number of edge pairs to be examined here is $\binom{n}{2}(n-2)$. Any edge taken from K_n has $(n-2).2$ (due to the end points of this edge) edge neighbors and since there are $\binom{n}{2}$ edges in K_n, then the number of edge pairs is $2.\binom{n}{2}(n-2)$. Since every edge pair is examined twice, we have $\binom{n}{2}(n-2)$. From the $AEDN$ definition every edge pair has $PEDN$=2. So for all these kind of edges $TPED$ $=2.\binom{n}{2}(n-2)$.

(ii) Edges which do not have any common vertex. Le tus take any edge from $\binom{n}{2}$. There are $\binom{n-2}{2}$ edges which do not have common vertex with this edge. Here we examine $\binom{n-2}{2}\binom{n}{2}\frac{1}{2}$ edge pairs. Each edge pair is included in C_4. So $PEDN$ is 4 for every edge pair. Then $TPED$=$4.\binom{n-2}{2}\binom{n}{2}\frac{1}{2}$. From all above

$$TPED = 2.(n-2)\binom{n}{2} + 2\binom{n-2}{2}\binom{n}{2}$$

.

Then $AEDN\ (K_n) = \dfrac{2.(n-2)\binom{n}{2} + 2\binom{n-2}{2}\binom{n}{2}}{\binom{\binom{n}{2}}{2}}$. □

Theorem 25. *Let $W_{1,n}$ be n+1 order wheel graph.*

$$AEDN(W_{1,n}) = \frac{\binom{n}{2} + 7n + n\left(6 + 2\left(n-4\right)\right)}{\binom{\binom{n+1}{2}}{2}}.$$

Proof. There exist 4 cases in a wheel graph to examine pair edge relations. These are the following:

(i) Edges in the outer cycle. In outer cycle there are n edges. The number of pairs of these edges is $\binom{n}{2}$. From the definition of $PEDN$ of C_n here $TPED$ of outer cycle is $2n$.

(ii) We examine C_3 s in $W_{1,n}$ since edge-domination occurs only in C_3 or C_4. In $W_{1,n}$ we must chose one edge in the outer cycle and two edges from inside to obtain a C_3. So there exist n different C_3 in $W_{1,n}$. Each edge in outer cycle makes to pair edges with 2 different edges from inside edges. Therefore the number of pairs of these edges is $2n$. In every C_3 $PEDN$ value is 2. So we have $TPED$ for these kind of edges, $2.2n$=$4n$.

(iii) The pair of inside edges is $\binom{n}{2}$. We examine here $\binom{n}{2}$edge pairs. Since all these edges dominate each other, each edge pair has $PEDN$=1 and from the edge pair they make with the edges of outer cycle $PEDN$= n, then $TPED$= $\binom{n}{2} + n$.

(iv) Edges which do not have any common vertex.

We examined the edges in outer cycle which do not have any common vertex. Here we examine the edge pairs between one edge from outer cycle and one edge from inside which do not have any common vertex. Let us take one edge from outer cycle in $W_{1,n}$. It is easy to see that this edge is adjacent to 4 edges (2 from outer cycle and 2 from inside). In $W_{1,n}$ there are $2n$ edges. So every edge in outer cycle has no any common vertex with ($2n$-5) edges. The n-3 edges of these $2n$-5 edges have been previously examined in (i). Then there are $2n$-5-(n-3)=n-2 edges have to be examined. Since there are n edges on outer cycle we have to examine n.(n-2) edge pairs.

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 1, No. 3 (2010)

Any edge in the outer cycle with two edges which are not adjacent to this edge is included in two different C_4s. In any C_4 in $W_{1,n}$ the pair edges which have no common vertex have $PEDN$=3.

We examined the pair edge-domination in two cycles so there left n-2-2=n-4 edges to be examined. Since n-4 edges are not included in any C_4 every edge pair inside has $PEDN$=2. So finally we have 2.3+2.(n-4) for each edge taken from outer cycle. There are n edges in outer cycle so $TPED = n.(2.3+2(n\text{-}4))$.

There are $2n$ edges in any $W_{1,n}$. So we have to show if there are $\binom{2n}{2}$ edges are examined or not.

$\binom{n}{2} + 2n + \binom{n}{2} + n(n-2) = \binom{2n}{2}$. It is obvious.

From all the 4 cases above the $TPED$=$\binom{n}{2}$+$7n$+$n\,(6 + 2\,(n-4))$ and finally average edge-domination number is $AEDN(W_{1,n}) = \dfrac{\binom{n}{2} + 7n + n\,(6 + 2\,(n-4))}{\binom{\binom{n+1}{2}}{2}}$.

□

Theorem 26. *For star graph $K_{1,n-1}$, $AEDN(K_{1,n-1}) = \dfrac{\binom{n-1}{2}}{\binom{\binom{n}{2}}{2}}$.*

Proof. In a star graph $K_{1,n-1}$, there are $(n-1)$ edges. Due to the structure of a star graph, the possible number of the pairs of edges is $\binom{n-1}{2}$. And from definition of $TPED$ is $\binom{n-1}{2}$and number of all the possible edges is $\binom{n}{2}$ then,

$$AEDN(K_{1,n-1}) = \frac{\binom{n-1}{2}}{\binom{\binom{n}{2}}{2}}.$$

□

Theorem 27. *For complete graph $K_{m,n}$,*

$$AEDN\left(K_{m,n}\right)=\frac{m.\binom{n}{2}+n\binom{m}{2}+4\binom{m}{2}\binom{n}{2}}{\binom{\binom{m+n}{2}}{2}}.$$

Proof. In a $K_{m,n}$ complete bipartite graph there are $m.n$ edges. We examine edge relations in 3 cases.

Let us take $V=V_1 \mathrm{U} V_2 \quad V_1=m$ and $V_2=n$

(i) The edge pairs that are made by the vertices of V_1

For the vertices of V_1 there are $\binom{n}{2}$. m edge pairs and each edge pair has $PEDN$=1 from the definition.

(ii) The edge pairs that are made by the vertices of V_2

For the vertices of V_2 there are $\binom{m}{2}$. n edge pairs and each edge pair has $PEDN$=1 from the definition. Each vertex of V_1 has n neighbor vertices so each vertex of V_1 makes $\binom{n}{2}$ edge. Since there are m vertices in V_1, then there exist $\binom{m}{2}.n$ edge pairs. It is the same for each vertex of V_2.

(iii) The number of C_4 is $\binom{m}{2}\binom{n}{2}$. Some edge pairs in C_4 have been previously examined in (i) and (ii), there left edges which do not have any common vertex. All the edges are included in 2 different C_4. So the number of the edges which will be examined is $\binom{m}{2}\binom{n}{2}.2$

In any C_4 the edge pairs which do not have common vertex always have $PEDN$=2.

So $TPED=2.\binom{m}{2}\binom{n}{2}.2+m.\binom{n}{2}+n.\binom{m}{2}$.

We obtain $AEDN=\dfrac{4.\binom{m}{2}\binom{n}{2}+m.\binom{n}{2}+n.\binom{m}{2}}{\binom{\binom{m+n}{2}}{2}}$

Now let us show that if we examined all the possible edge pairs or not.

$\binom{m}{2}\binom{n}{2}.2+m.\binom{n}{2}+n.\binom{m}{2}=\binom{m.n}{2}$ It can be easily seen. □

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 1, No. 3 (2010)

4. An Algorithm for Finding the Average Edge-Domination Number of a Graph

In final section, we give an algorithm whose complexity is $O(|V^3|)$. This algorithm works on the basis of adjacency matrix. By Floyd Marshall ($O(|V^3|)$) method distance matrix is calculated. Then the edge-distance matrix is calculated by the algorithm via distance matrix. It calculates the *TPED* of all edge pairs and the $AEDN$ of this graph.

$AEDN$ is calculated by the formula

$$TPED = \sum_{e_i,e_j\in E(G)}^{PEDN(e_i,e_j)}, \quad AEDN = \frac{TPED}{\binom{r}{2}},$$

where r is the maximum number of edges.

```
uses crt,graph;
type
koor=record
x,y :longint;
end;
const
n=4; {Number of vertices of the graph.}
m=3; {Number of edges }
var
    E : array[1..m] of koor;
d, A : array[1..n,1..n] of longint;
    ED : array[1..m,1..m] of longint ;{edge distance matrix}
    TPED : longint; {Total Edge Neighborhood}
k,i,j,q,u1,u2,v1,v2,enk,r,t : byte;
    AEDN, c,t1:real;
    BEGIN
write('Enter the adjacency matrix of the graph.');
q:=0;
for i:=1 to n do begin
for j:=1 to n do begin
write('A[ 'i,',',j,']=');
readln(A[i,j]);
if ( A[i,j]=1) and (i¡j) then begin
q:=q+1;
    E[q].x:=i;
    E[q].y:=j;
```

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 1, No. 3 (2010)

```
end;
end;
end;
    { Warshall - Floyd }
for i:=1 to n do
for j:=1 to n do
if A[i,j]> 0 then d[i,j]:=A[i,j]
else if i=j then d[i,j]:=0 else d[i,j]:=1000;
for k:=1 to n do
for i:=1 to n do
for j:=1 to n do
if d[i,j]>d[i,k]+d[k,j] then d[i,j]:=d[i,k]+d[k,j];
for r:=1 to q do begin
    ED[r,r]:=5000;
u1:=E[r].x;
v1:=E[r].y;
for t:=r+1 to q do begin
u2:=E[t].x;
v2:=E[t].y;
enk:=D[u1,u2];
if (u1=u2) or (u1=v2) or (v1=u2) or (v1=v2) then begin

    ED[r,t]:=0 ; ED[t,r]:=0;

end;
if (ED[r,t] <> 5000) or (Ed[r,t] <> 0) then begin
if enk >= D[u1,v2] then enk:=D[u1,v2];
if enk >= D[v1,u2] then enk:=D[v1,u2] ;
if enk >= D[v1,v2] then enk:=D[v1,v2];
    ED[r,t]:=enk; ED[t,r]:=enk;
end;
end; {t}
end; {r}
    TPED:=0;
for i:=1 to (q-1) do begin
u1:=E[i].x;v1:=E[i].y;
for j:=i+1 to q do begin
u2:=E[j].x; v2:=E[j].y;
if ED[i,j]=0 then TPED:=TPED+1;
if (ED[i,j]<> 0) and (d[u1,u2]=1) then TPED:=TPED+1;
if (ED[i,j]<> 0) and (d[u1,v2]=1) then TPED:=TPED+1;
if (ED[i,j]<> 0) and (d[v1,u2]=1) then TPED:=TPED+1;
```

International Electronic Journal of Pure and Applied Mathematics – IEJPAM, Volume 1, No. 3 (2010)

```
if (ED[i,j]<> 0) and (d[v1,v2]=1) then TPED:=TPED+1;
if (ED[i,j]=0) and (u1=u2) and (d[v1,v2]=1) then TPED:=TPED+1;
if (ED[i,j]=0) and (u1=v2) and (d[v1,u2]=1) then TPED:=TPED+1;
if (ED[i,j]=0) and (v1=u2) and (d[u1,v2]=1) then TPED:=TPED+1;
if (ED[i,j]=0) and (v1=v2) and (d[u1,u2]=1) then TPED:=TPED+1;
end; {j}
end; {i}
writeln('total pair edge dominating =',TPED );
t1:=(n*(n-1))/2; c:=(t1*(t1-1))/2;
    AEDN:=TPED/c;
writeln('average edge dominating number=',AEDN);
readln;
end.
```

5. Conclusion

In this work, a new vulnerability measure on graphs is given. This measure is different from other measures. When two graphs having the same number of vertices and edges are taken into consideration in any network design, the graph having bigger *AEDN*, is preferred to the other. Since the pair edge relations are stronger in the graph whose *AEDN* is bigger. This means if any failure happens in any link of the graph, there are left much more links still working between each other.

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