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ON EFFECTS OF A TRANSVERCE MAGNETIC FIELD ON AN UCM FLUID OVER A STRETCHING SHEET

Ch. Mamaloukas[1 §], M. Subhas Abel[2], J.V. Tawade[3], U.S. Mahabaleswar[4]

[1]Department of Informatics
Athens University of Economics and Business
76, Patision Str., Athens, 10434, GREECE
e-mail: mamkris@aueb.gr

[2,3]Department of Mathematics
Gulbarga University
Gulbarga, 585 106, Kamataka, INDIA
[2]e-mail: msabel2001@yahoo.co.uk
[3]e-mail: jagadish_maths@yahoo.co.in

[4]Department of Mathematics
Bangalore University
Mangalagangothri, Bangalore, 574 199, INDIA
e-mail: usmahabalesh@yahoo.co.in

Abstract: The effects of a transverse magnetic field within a boundary layer flow on an upper-convected Maxwell (UCM) fluid over a stretching sheet is examined in this paper. Similarity transformations are used to non-dimensionalize the governing boundary layer equations of motion. The resulting ordinary differential equations are then solved numerically by using efficient numerical shooting technique with fourth order Runge–Kutta algorithm [10]. The objective of the present work is to investigate the effect of elastic parameter β, and magnetic parameter Mn on the temperature field above the sheet. The results compared with the results of Hayat et al [7], presented through graphs and discussed.

AMS Subject Classification: 35Q30, 35A35

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[§]Correspondence author

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1. Introduction

The study of non-Newtonian fluid flows over a stretching sheet find applications in many manufacturing processes such as extrusion of molten polymers through a slit die for the production of plastic sheets, wire and fibre coating, foodstuff processing, etc. In a typical sheet production process, the extrudate starts to solidify as soon as it exits from the die. The quality of the final product depends on the rate of heat transfer and therefore cooling procedure has to be controlled effectively.

Sarpakaya [5] was the first researcher to study the MHD flow of a non-Newtonian fluid. Prandtl's boundary layer theory proved to be of great use in Newtonian fluids as Navier-Stokes equations can be converted into much simplified boundary layer equation which is easier to handle.

Crane [4] was the first among the others to consider the steady of two-dimensional flow of a Newtonian fluid driven by a stretching elastic flat sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point.

For the theoretical results to become of any industrial significance, realistic viscoelastic fluid models such as upper-convected Maxwell model or Oldroyd-B model should be invoked in the analysis. Because of the complexity of these fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids (see [6]-[13]).

The focal point in the present work is to investigate numerically the MHD flow and heat transfer of a UCM fluid above a semi-infinite stretching sheet. To achieve this goal, use will be made of a recent analysis carried out by Hayat et al. [7] in which the velocity field above the sheet was calculated for MHD flow of a Maxwell fluid with no heat transfer involved using homotopy analysis method (HAM).

2. Formulation of the Problem

For an MHD flow of an incompressible UCM fluid resting above a stretching sheet, which is being stretched linearly, the equations governing transport of heat and momentum can be written as (see [7], [15]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \lambda\left[u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right] - \frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where B_0, is the strength of the magnetic field, υ is the kinematic viscosity of the fluid and λ is the relaxation time parameter of the fluid.

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As boundary conditions, we have adopted the following two kinds of boundary heating:

$$\left.\begin{array}{llll}
\text{(i)} & \multicolumn{3}{c}{\text{Prescribed power-law surface temperature (PST)}} \\
u = Bx; \quad v = 0; & T = T_w\left(x\right) = T_0 - T_s\left(\frac{x}{L}\right)^2 & \text{at} & y = 0. \\
u \to 0; \quad T \to T_\infty & \text{as} & y \to \infty & \\
\text{(ii)} & \multicolumn{3}{c}{\text{Prescribed power-law heat flux (PHF)}} \\
u = Bx; & q_w = -k\left(\frac{\partial T}{\partial y}\right)_w = b\left(\frac{x}{L}\right)^2 & \text{at} & y = 0. \\
u \to 0; \quad T \to T_\infty & \text{as} & y \to \infty &
\end{array}\right\} \tag{4}$$

Here T_w is the wall temperature, T_∞ is the temperature far away from the sheet, T_0 is the melt temperature at the die exit and $T - T_s$ is the melt solidification temperature $(T < T_s < T_\infty)$. It has to be noted that in the above relationships, L is the distance between the die exit and the point at which the melt solidifies, k is the thermal conductivity of the fluid and b is a constant whose value also depends on the fluid.

3. Method of Solution

Introducing dimensionless variables and the similarity variable (see [9], [8]) as

$$\left.\begin{array}{c}
u = Bxf\,'\left(\eta\right), \quad v = \sqrt{vB}f\left(\eta\right), \quad \eta = \left(\frac{B}{v}\right)^{\frac{1}{2}} y, \\
\theta\left(\eta\right) = \frac{T-T_\infty}{T_w-T_\infty}, \qquad g\left(\eta\right) = \frac{T-T_\infty}{b\left(\frac{x}{l}\right)^2 \frac{1}{k}\sqrt{\frac{v}{b}}},
\end{array}\right\} \tag{5}$$

the governing equations (1)-(3) can be transformed exactly into a set of ordinary differential equations:

$$f\,''' - Mf\,' - \left(f\,'\right)^2 + f\,f\,'' + \beta\left(2f\,f\,'f\,'' - f\,^2 f\,'''\right) = 0, \tag{6}$$

$$\Pr\left[2f\,'\theta - \theta' f\right] = \theta'', \tag{7}$$

$$\Pr\left[2f\,'g - g' f\right] = g'', \tag{8}$$

and their associated boundary conditions are:

$$f = 0; \quad f\,' = 1; \quad \theta = 1; \quad g' = -1, \quad \text{at} \quad \eta = 0, \tag{9}$$

$$f\,' = 0; \quad \theta = 0; \quad g = 0, \quad \text{as} \quad \eta \to \infty, \tag{10}$$

where $M = \frac{\sigma B_0^2}{\rho B}$ is a magnetic parameter and $\beta = 2B$, is the elastic parameter.

The non-linear differential equations (6), (7) and (8) with appropriate boundary conditions given in (9) and (10) are first decomposed into a system of first order differential equations. The resulting initial value problem (IVP) then is solved

β	Hayat et. al (Porous sheet)		Present Results (Non-porous sheet)	
	M=0.0	M=0.2	M=0.0	M=0.2
0.0	-1.90250	-1.94211	-0.999962	-1.095445
0.4	-2.19206	-2.23023	-1.101850	-1.188270
0.8	-2.50598	-2.55180	-1.196692	-1.275878
1.2	-2.89841	-2.96086	-1.285257	-1.358733
1.6	-3.42262	-3.51050	-1.368641	-1.437369
2.0	-4.13099	-4.25324	-1.447617	-1.512280

Table 1: Comparison of values of skin friction coefficient $f''(0)$ for different elastic parameters with $M = 0.0$ and $M = 0.2$

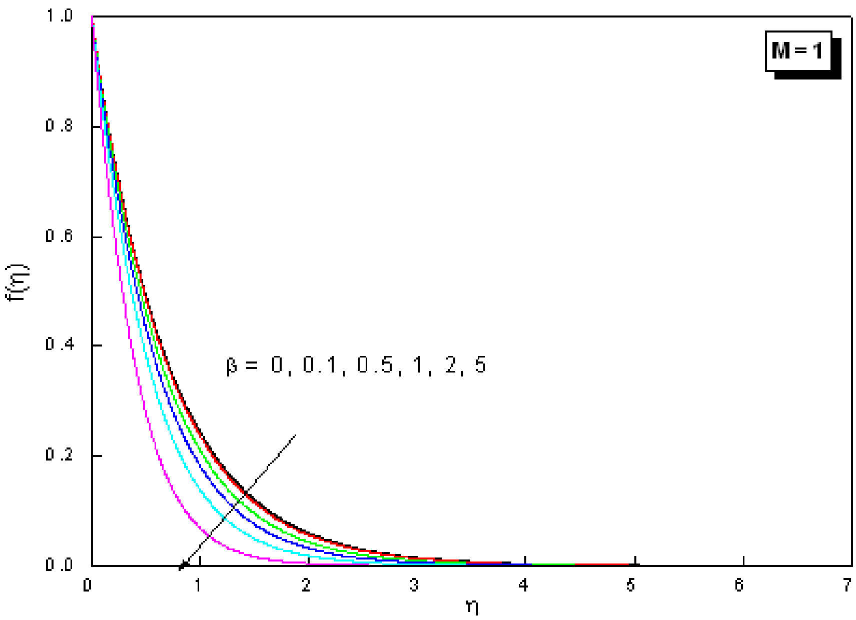

Figure 1: The effect of the elastic parameter on u-velocity component

numerically by the most efficient numerical shooting method technique. The convergence criterion largely depends on fairly good guesses of the initial conditions in the shooting technique. Once the convergence is achieved we integrate the resultant ordinary differential equations using standard fourth order Runge–Kutta method with the given set of parameters to obtain the required solution [10].

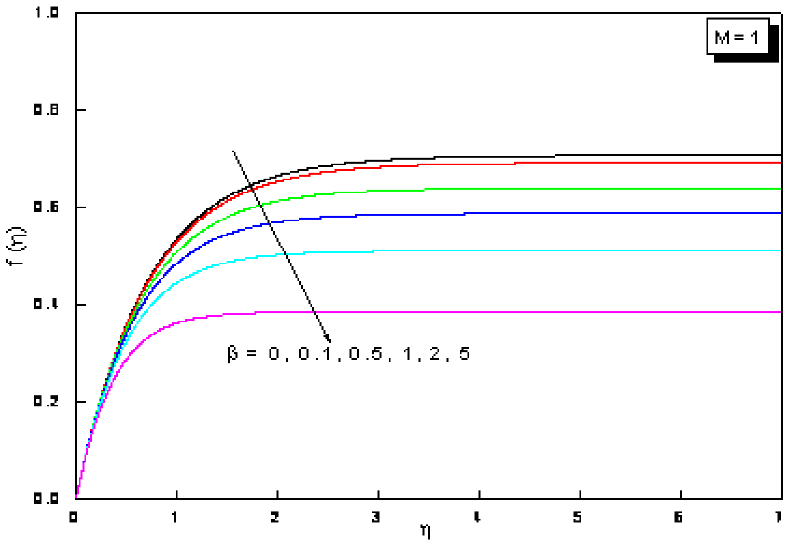

Figure 2: The effect of the elastic parameter on v-velocity component

4. Results and Discussion

The numerical results obtained in the present work are compared with some of the earlier published results in some limiting cases and are shown in Table 1.

Furthermore Figures 1 and 2 show the effect of magnetic parameter, M, in the absence of elastic parameter on the velocity profile above the sheet. An increase in the magnetic parameter leads in decrease of both u- and v- velocity components at any given point above the sheet. This is due to the fact that applied transverse magnetic field produces a drag in the form of Lorentz force thereby decreasing the magnitude of velocity. The drop in horizontal velocity as a consequence of the increase in the strength of magnetic field is observed.

Moreover, Figures 3 and 4 show the effect of magnetic parameter on the temperature profiles above the sheet for both PST and PHF cases. An increase in the magnetic parameter is seen to increase the fluid temperature above the sheet. That is, the thermal boundary layer becomes thicker for the larger magnetic parameter.

In conclusion, we observe that, when the magnetic parameter increases, the velocity decreases, also, for increase in elastic parameter, results in a decrease of velocity. The effect of the magnetic field and the elastic parameter on the UCM fluid above the stretching sheet is to suppress the velocity field, which in turn causes the enhancement of the temperature field.

A drop in skin friction as investigated in this paper has an important implication

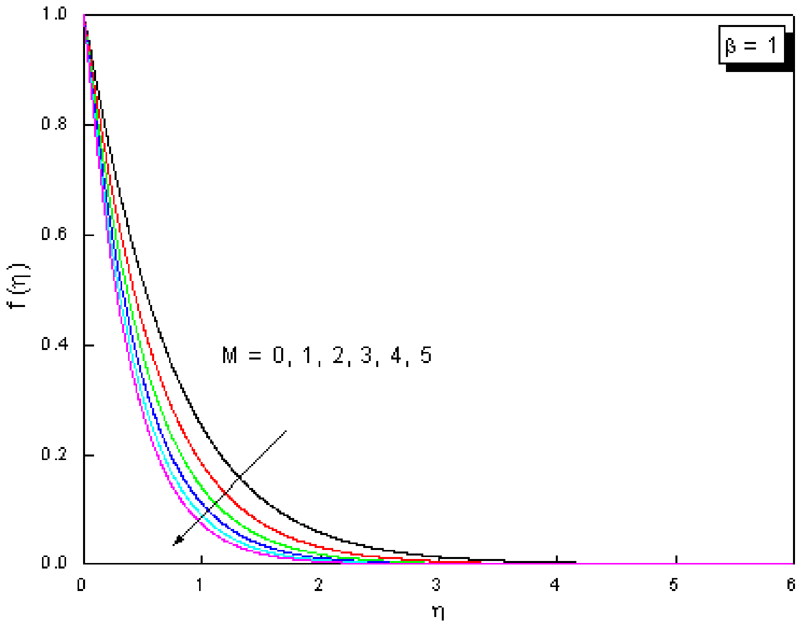

Figure 3: The effect of the magnetic parameter on u-velocity component

that in free coating operations, elastic properties of the coating formulations may be beneficial for the whole process. This means that less force may be needed to pull a moving sheet at a given withdrawal velocity or equivalently higher withdrawal speeds can be achieved for a given driving force resulting in, which increases in the rate of production [11]. A drop in the skin friction with an increase in the elastic parameter as observed in Table 1 gives the comparison of present results with that of Hayat et al [7]. Without any doubt, from this table, we can claim that our results are in excellent agreement with that of Hayat et al [7].

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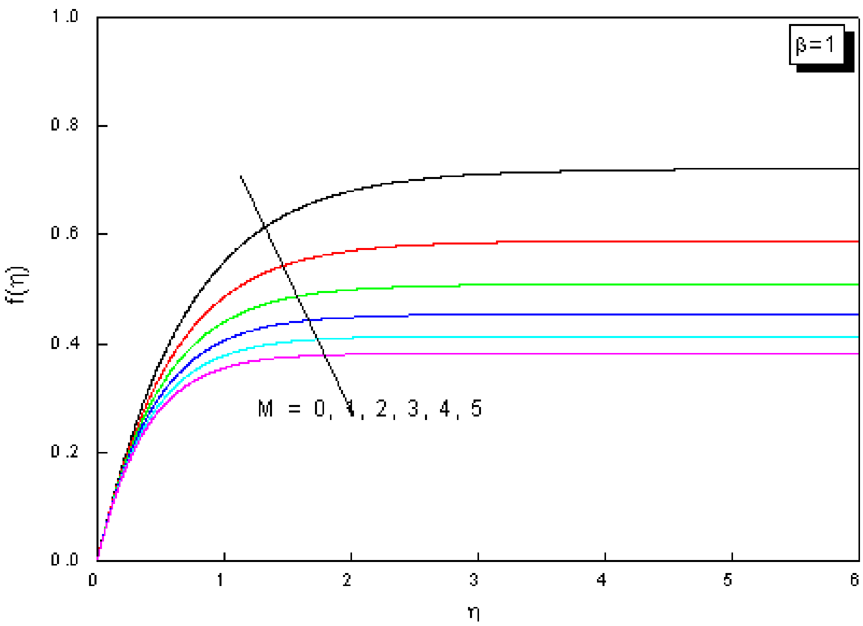

Figure 4: The effect of the magnetic parameter on v-velocity component

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